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Students' Elementary Algebra

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FOR

HIGH SCHOOL CLASSES

BY

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PREFACE

A change in the public opinion as to the relative merit of different subjects to meet the fast changing demands of society has been responsible for the reorientation of the courses of study in the most important subjects of the High School and other examinations in Rajputana, the Uttar Pradesh and elsewhere. Some subjects, compulsory till recently, appear in the list of the optionals and vice versa.

The present work is designed to serve the two-fold purpose, Firstly, sufficient subject matter and examples are embodied in the text to guide and give practice to the scholars for whom Algebra is yet a compulsory subject. The entire course of Algebra according to the syllabus of the Rajputana Board is thoroughly covered. Secondly, in anticipation of an extension of the course in very near future, as has been already done, in the United Provinces, fresh chapters dealing with such topics as 'The Law of Index', 'The surds' etc. have been added. This practice, usually prevalent in advanced text books has the great advantage of whetting the bright students' appetite for the subject.

The entire course, old and new, has been treated in a realistic and lucid way. The author has brought to bear his 28 years long experience on dealing with individual difficulties framing graded examples and setting up a standard in the subject. The common 'pit falls' facing the pupils in understanding the principles and working out examples have been successfully tackled and explained.

Many solved examples of varied nature furnish a strong back-ground for new exercises where necessary solution of examples are given along with unsolved exercises. This way is considered 'Modern' and is, in fact more useful in as much as a pupil is able to recapitulate at first hand what he has to do with regard to example just following the solutions. The average pupil has therefore not to grope in the dark as to the line of attack.

The exercises are numerous providing sufficient material for all the three types of pupils—the average, below the average and above the average. Some exercises are followed by Miscellaneous Exercises containing harder sums, very useful for recapitulation and tests.

It is, therefore, expected that an intelligent grasp of and interest in the subject will follow a careful study of the principles and working out of examples as given in the following pages.

Papers of the Rajputana University and other examining bodies have been incorporated in the end to add to the utility of the book.

—The Author

PREFACE TO THE SECOND EDITION

This is the second edition of Students' Elementary Algebra, a book that has been accorded very popular reception both by students and teachers alike of this subject.

The Answers have carefully been scrutinised and slight changes have also been made where found necessary.

The book in its present form is hoped to prove very useful to the teachers and the students alike.

July 1, 1953. —Publishers.

CONTENTS

CHAPTER I

FACTORS

1. *Product* :—When a is multiplied by b, we get ab. ab is the *product* of a and b.

Factors :—Each of the quantities a and b is a factor of ab. Similarly $a(b+c)$ is the *product* of a and $(b+c)$ and each of the quantities a and $(b+c)$ is a factor of the quantity $a(b+c)$.

Therefore to get the factors of a quantity we should obtain such other quantities as when multiplied together, give the given quantity as the result. $(a+b)$ and $(a-b)$ are the factors of a^2-b^2 since $(a+b)$ and $(a-b)$ multiplied together result in (a^2-b^2).

Common Factors :—

2. The factors of ab and ac are respectively a, b and a, c. In ab and ac there is a quantity a common to both, a is called the common factor of ab and ac. Similarly, a is the common factor of $a(b+c)$, and $a(c+d)$, while other factors are $(b+c)$ and $(c+d)$ respectively.

Common Factor is therefore a quantity which is a factor of each of two or more quantities.

3. a is a common factor of ab and ac. In order to get other factors of ab and ac we have to divide each ab and ac by a. The respective results, b and c are the other factors of ab and ac. Similarly, a is the common factor of $a(b+c)$ and $a(c+d)$. The other factors if $(b+c)$ and $(c+d)$ are the respective quotients when $a(b+c)$ and $a(c+d)$ are respectively divided by a.

If there is a common factor of two or more quantities other factors of those quantities are obtained by dividing those quantities by the common factor.

4. *Solved Examples.*

Resolve into simple factors the following :—

(i) xy, xyz, $x(a+b)$.

Solution : Of all these quantities x is a common factor.

$\therefore$ $xy \div x = y$,

$xyz \div x = yz$,

and $x(a+b) \div x = a+b$.

$\therefore$ x, y are the factors of xy.

x, yz are the factors of xyz.

yz can be further factorized into y and z.

So x, y, z are the factors of xyz and x, $(a+b)$ are the factors of $x(a+b)$.

(ii) $ab+bc$, x^2y-xy^2, $ab+bc+bd$.

Solution: In $ab+bc$, b is contained in both terms ab and bc. Therefore by dividing $ab+bc$ by b, the other factor is $(b+c)$. Hence a and $(b+c)$ are the two factors.

Similarly x, y and $(x-y)$ are the factors of $xy(x-y)$ or x^2y-xy^2 and a and $(b+c+d)$ are the factors of $ab+ac+ad$

(iii) $3x^3-9x^2$, $15a-25a^3$ and $7ab+35b^2-49b^3$.

In $3x^3-9x^2$, $3x^2$ is common to both terms $3x^3$, $9x^2$.

$\therefore$ by division $3x^2$ and $(x-3)$ are the factors of $3x^3-9x^2$.

Similarly $5a$ and $(3-5a^2)$ are the factors of $15a-25a^3$ and 7b and $(a+5b-7b^2)$ are the factors of $7ab+35b^2-49b^3$.

In other words $3x^3-9x^2=3x^2(x-3)$,

$15a-25a^3=5a(3-5a^2)$,

and $7ab+35b^2-49b^3=7b(a+5b-7b^2)$.

EXERCISE 1

Resolve the following expressions into factors:—

(1) $ab+ac$. (2) $ab+2a$. (3) $2x+4y$.

(4) a^2b+ab^2. (5) xy^2-x^2z. (6) $3x^2-9x^3$

(7) $ab+2bc+3abc$. (8) $3xy+9x^2y-27xyz$.

(9) $xyx+x^3y^2z^2+x^2y^2z$. (10) $p^3q^3+q^3r^3+p^2q^3r^3$

(11) $12m^3q^2r^3+28n^2p^2m^2-4m^4+16m^2n^3$.

(12) $-21bcd^2-7a^3b^2c-14bc^2d$.

(13) $a(b-c)+b(b-c)$.

Solution : Here $(b-c)$ is the common factor.

$\therefore$ to get the other factor of $a(b-c)$, divide $a(b-c)$ by $b-c$. The result is a.

Similarly the other factor of $b(b-c)$ is b

Hence $a(b-c)+b(b-c)=(b-c)(a+b)$.

(14) $x(p+q)+y(p+q)$.

(15) $m(l+n)-n(l+n)$.

(16) $a^2(b-c)+b^2(b-c)$.

(17) $a^2(b-c)+b^2(b-c)+c^2(b-c)$.

(18) $a(b^2-c^2)+b(b^2-c^2)+c(b^2-c^2)$.

(19) $ab(ab-cd)+a^2(ab-cd)+a^3(ab-cd)$.

(20) $6(x+y)+3(x+y)^2-9(x+y)^3$.

5. *To resolve into factors expressions like* $ab+ac+bd+cd$.

In the expression $ab+ac+bd+cd$ there are four terms ab, ac, bd, cd. In the first two terms ab and ac, a is common and the other factor is $(b+c)$ which is obtained by dividing $ab+ac$ by a. Similarly d is contained both in the third and fourth terms. The other factor of $bd+cd$ is $(b+c)$.

$\therefore\ ab+ac+bd+cd=a(b+c)+d(b+c)$.

Now in each of $a(b+c)$ and $d(b+c)$, $(b+c)$ is contained, and the other factors are a and d respectively.

$$\therefore \quad a(b+c)+d(b+c)=(b+c)(a+d)$$
$$\therefore \quad ab+ac+bd+cd=(b+c)(a+d).$$

6 *Solved examples.*

Resolve the following expressions into factors :—

(1) $2x+2y+ax+ay$.

Solution : $2x+2y+ax+ay$.
$$=(2x+2y)+(ax+ay)$$
$$=2(x+y)+a(x+y)$$
$$=(x+y)(2+a).$$

(2) $x^3+ax^2+xy^2+ay^2$.

Solution : $x^3+ax^2+xy^2+ay^2$.
$$=(x^3+ax^2)+(xy^2+ay^2)$$
$$=x^2(x+a)+y^2(x+a).$$
$$=(x+a)(x^2+y^2).$$

or, arranging in a different order,

$$x^3+ax^2+xy^2+ay^2=(x^3+xy^2)+(ax^2+ay^2)$$
$$=x(x^2+y^2)+a(x^2+y^2)$$
$$=(x^2+y^2)(x+a)$$

(3) $6ap-35bq+14bp-15aq$.

Solution : $6ap-35bq+14bp-15aq$.
$$=6ap+14bp-15aq-35bq.$$
$$=2p(3a+7b)-5q(3a+7b)$$
$$=(3a+7b)(2p-5q).$$

(4) $x^2(xy+xz)+y^2(y^2+yz)+z^2(yz+z^2)$.

Solution: $xy+xz=x(y+z)$.

$\therefore \quad x^2(xy+xz)=x^2 . x(y+z)=x^3(y+z)$.

Similarly, $y^2(y^2+yz)=y^3(y+z)$ and

$z^2(yz+z^2)=z^3(y+z)$.

$\therefore$ the expression $=x^3(y+z)+y^3(y+z)+z^3(y+z)$

$=(y+z)(x^3+y^3+z^3)$.

(5) $xy(z^2+1)+z(x^2+y^2)$

Solution : $xy(z^2+1)+z(x^2+y^2)$

$=xyz^2+xy+x^2z+y^2z$.

$=x^2z+xy+xyz^2+y^2z$ (arranging the expression in the descending powers of x)

$=x(xz+y)+yz(xz+y)$

$=(xz+y)(x+yz)$.

(6) $a-3b-apr+2c+3bpr-2cpr$.

Solution : Exp. $=a-apr-3b+3bpr+2c-2cpr$

$=a(1-pr)-3b(1-pr)+2c(1-pr)$.

$=(1-pr)(a-3b+2c)$.

EXERCISE 2

Resolve the following into simple factors :—

(1) $xy+xz+ay+az$.

(2) $3a-4b+12ab-16bp$.

(3) $6+2pr-9q-3pqr$.

(4) $x^2-x+xy-y$.

(5) x^3-x^2+x-1.

(6) $x^5+x^4-x^3-x^2$.

(7) $2a^4b-3ab^4+2a^2b^3-3a^3b^2$.

(8) $21p^3-14p^2+33p-22$.

(9) $a^2(b-c)+pb-pc$.

(10) $2a(x^2+y^2)-3b(x^2+y^2)$.

(11) $a(bp+bq)+c(dp+dq)$.

(12) $y(x+yz)+zx(yz+x)$.

(13) $xz(x^2-y^2)+x^2y(x-y)$.

[Hint : $x^2-y^2=(x+y)(x-y)$].

(14) $ax+by+az+bx+bz+ay$.

(15) $ax-by+ay-bz+az-bx.$

(16) $ab^2-a+a^2b-b+ab+b^2.$

(17) $3x^2y+6xy^2+5xyz-7x^2z+10y^2z-14xyz.$

(18) $3p^2+5abp^2+4p^2cd+5abq^2+3q^2+4cdq^2.$

(19) $xy^2+xz^2+y^2z+yz^2+2xyz.$
[Hint: $2xyz=xyz+xyz$].

(20) $3x^2-4qy^2+3z^2-4qx^2+3y^2-4qz^2.$

7. *Resolution into factors of the expressions of the form:* $a^2\pm2ab+b^2.$

We know that $(a\pm b)^2=a^2\pm2ab+b^2.$

The square of the sum or diffierence of two quantities = the sum of the squares of both the quantities $\pm$ twice their product.

Hence if an expression consists of the sum of the squares of two quantities $\pm$ twice their product, it is equal to the square of their sum or difference respectively.

For example,

(1) $x^2+8x+16=(x)^2+2.\,x.\,4+(4)^2.$
$=(x+4)^2.$

(2) $4a^2-12ab+9b^2=(2a)^2-2.\,2a.\,3b+(3b)^2.$
$=(2a-3b)^2.$

8. *Solved examples.*

Resolve into factors:

(1) $x^2+2xy+y^2$

Solution: $x^2+2xy+y^2=x^2+xy+xy+y^2.$
$=x(x+y)+y(x+y).$
$=(x+y)\,(x+y)$
$=(x+y)^2.$

(2) $4x^2-12xy+9y^2.$

Solution: $4x^2-12xy+9y^2=(2x)^2-6xy-6xy+(3y)^2$
$=2x(2x-3y)-3y(2x-3y)$
$=(2x-3y)\,(2x-3y)$
$=(2x-3y)^2.$

EXERCISE 3

Resolve into factors :—

(1) x^2+2x+1.

(2) x^2+4x+4.

(3) $4x^2-4x+1$.

(4) $9a^2+24ab+16b^2$.

(5) $25p^2-60pq+36q^2$.

(6) $25p^2+70pq+49q^2$.

(7) $(x+y)^2+2z(x+y)+z^2$.

Solution : Put $x+y=a$

$\therefore$ Exp. $=a^2+2az+z^2$

$=(a+z)^2$.

Restore the value of a

Exp. $=(x+y+z)^2$.

(8) $(a+b)^2+6c(a+b)+9c^2$.

(9) $4(x+y)^2-24(x+y)z+36z^2$.

(10) $25(p-q)^2+40(p-q)r+16r^2$.

(11) $36(a+b)^2+60(a+b)c+25c^2$.

(12) $(a+b)^2-2(a+b)(a-b)+(a-b)^2$.

Solution : Put $x=a+b, y=a-b$

Then the exp. $=x^2-2xy+y^2=(x-y)^2$.

Restore the values of x and y

The exp. $=(a+b-a+b)^2=(2b)^2=4b^2$.

(13) $(x-y)^2+2(x-y)(a+b)+(a+b)^2$.

(14) $4(a+b)^2+8(a+b)(a-b)+4(a-b)^2$.

(15) $25(l-m)^2-80(l-m)(l+m)+64(l+m)^2$.

(16) $25a^4+20a^2b^2+4b^4$.

Solution : The exp. $=(5a^2)^2+2.5a^2.2b^2+(2b^2)^2$

$=(5a^2+2b^2)^2$,

(17) $4x^4+36x^2y^2+81y^4$.

(18) $36p^6+12p^3+1$.

(19) $25a^8-60a^4b^4+36b^8$.

(20) $25(a+b)^4+30(a+b)^2(a-b)^2+9(a-b)^4$.

Find the value of :—

(21) $36x^2+84xy+49y^2$, when $x=2, y=3$.

Solution : The exp. $=(6x+7y)^2$.

Put $x=2$, $y=3$ in $(6x+7y)^2$.

$\therefore$ the exp. $=(6\times 2+7\times 3)^2=(33)^2=1089$.

(22) $25p^2+10pq+q^2$, when $p=3$, $q=4$

(23) $49x^2-28xy+4y^2$, when $x=4, y=5$.

(24) $16x^2+56xy+49y^2$, when $x=2, y=-1$.

(25) $(x+y)^2+4(x+y)(x-y)+4(x-y)^2$, when $x=4, y=2$.

(26) $4(p+q)^2-8(p+q)(p-q)+4(p-q)^2$ when $q=3$.

9. *Resolution into factors of the expressions of the form* :
$a^2+b^2+c^2+2ab+2bc+2ca$.

Solved Examples.

(1) $a^2+b^2+c^2+2ab+2bc+2ca$. Arrange the exp. according to the descending powers of a.

$$\begin{aligned}\text{Then the exp. } &=a^2+2a(b+c)+b^2+c^2+2bc\\ &=a^2+2a(b+c)+(b+c)^2\\ &=(a+b+c)^2.\end{aligned}$$

(2) $4x^2+9y^2+z^2-12xy-6yz+4zx$.

Arrange the exp. according to the descending powers of x.

$$\begin{aligned}\text{Then the exp. } &=4x^2-4x(3y-z)+9y^2+z^2-6yz\\ &=4x^2-4x(3y-z)+(3y-z)^2\\ &=(2x-3y+z)^2.\end{aligned}$$

EXERCISE 4

Resolve into simple factors:—

(1) $a^2+4b^2+9c^2+4ab+12bc+6ac$.

(2) $x^2+4y^2+25z^2-4xy-20yz+10zx$.

(3) $4x^2+9y^2+16z^2+12xy-24yz-16zx$.

(4) $p^2+9q^2+25r^2-6pq+30qr-10pr$.

10. *Resolution into factors of the expressions of the form :—*
a^2-b^2.

We know that the product of $(a+b)$ and $(a-b)$ is $a^2+ab-ab-b^2=a^2-b^2$.

Conversely, $a^2-b^2=a^2+ab-ab-b^2$

$$=a(a+b)-b(a+b)$$
$$=(a+b)(a-b).$$

First, find out the square root of each term. The first factor is the sum of both the square roots and the second factor is their difference.

11. *Solved examples.*

(1) $4a^2-25b^2$.

Solution: $4a^2-25b^2=(2a)^2-(5b)^2$

$$=(2a)^2+10ab-10ab-(5b)^2$$
$$=2a(2a+5b)-5b(2a+5b)$$
$$=(2a-5b)(2a+5b).$$

(2) x^4-y^4

$$x^4-y^4=(x^2)^2-(y^2)^2=(x^2)^2+x^2y^2-x^2y^2-(y^2)^2$$
$$=x^2(x^2+y^2)-y^2(x^2+y^2)$$
$$=(x^2-y^2)(x^2+y^2)$$
$$=(x+y)(x-y)(x^2+y^2)$$

(3) x^8-y^8

$$x^8-y^8=(x^4)^2-(y^4)^2$$
$$=(x^4+y^4)(x^4-y^4)$$
$$=(x^4+y^4)(x^2+y^2)(x^2-y^2)$$
$$=(x^4+y^4)(x^2+y^2)(x+y)(x-y)$$
$$=(x+y)(x-y)(x^2+y^2)(x^4+y^4).$$

EXERCISE 5

Resolve in factors :—

(1) x^2-4y^2

(2) $4x^2-9y^2$.

(3) $4x^2-9$.

(4) $9x^2-25y^2$.

(5) $49-a^2$

(6) a^2-b^2.

(7) a^4-b^2.

(8) a^4-b^4.

(9) $l^2x^2-p^2$.

(10) $100-p^2$.

(11) $4a^3-25ax^2$.

(12) $16x^4-1$.

(13) $64a^4-49b^4$. (14) x^2-81x^6.

(15) $98a^3x^5-128ax$. (16) $192a^9-243a^5x^4$.

(17) x^6-y^6.

Solution : $x^6-y^6=(x^3)^2-(y^3)^2$
$=(x^3+y^3)(x^3-y^3)$

(18) $4x^6-9y^6$. (19) $25a^9-64a^3b^8$.

(20) $(a+3b)^2-25c^2$.

Solution : Put $a+3b=x$.

Then $(a+3b)^2-25c^2=x^2-(5c)^2$
$=(x+5c)(x-5c)$

Restore the value of x.

Then the exp.$=(a+3b+5c)(a+3b-5c)$.

(21) $a^2-(2b-3c)^2$. (22) $(4p+5q)^2-9c^2$.

(23) $(3x^2-4y^2)^2-25z^4$.

(24) $(x+y)^2-(x-y)^2$.

Solution : Put $(x+y)=a$, $(x-y)=b$.

Then the exp.$=a^2-b^2=(a+b)\ (a-b)$

Restore the values of a and b,

Then the exp.$=(x+y+x-y)\ (x+y-x+y)$
$=2x.2y=4xy$.

(25) $(3a+5b)^2-(3a-5b)^2$

(26) $(8x+5y)^2-(8x-5y)^2$.

(27) $(a+b+c)^2-(a-b+c)^2$.

(28) $(2a+3b-5c)^2-(5a+4b+7c)^2$.

Find the value of :—

(29) 25^2-16^2.

Solution : $25^2-16^2=(25+16)(25-16)$
$=41\times 9$
$=369$.

(30) 100^2-99^2. (31) $27{\cdot}3^2-22{\cdot}7^2$.

(32) 1002×998. (33) $1{\cdot}96\times 2{\cdot}04$.

12. *Resolution into factors of the expressions of the form* :—
x^2+bx+c.

We know that

$$(x+3)\ (x+2)=x^2+5x+6.$$
$$(x-3)\ (x-2)=x^2-5x+6.$$
$$(x+3)\ (x-2)=x^2+x-6 \text{ and}$$
$$(x-3)\ (x+2)=x^2-x-6$$

From the above it is clear that when two binomial expressions are multiplied together, the result is a trinomial expression. The first term of the trinomial is the product of the first terms of the binomial expressions, the second term is the product of the algebraic sum of the second terms of the binomials and their first term and the third term is the product of the second terms of the binomials.

The factors of a trinomial are obtained by reversing the process.

The factors of a trinomial like x^2+5x+6 are two binomial quantities. The first term of each of the factors is x, the algebraic sum of the other terms is 5 and their product is 6.

Therefore find the factors of 6 such that the product of the factors is 6 and their algebraic sum is 5. These factors are 3 and 2.

$$\begin{aligned}
\text{Therefore } x^2+5x+6 &= x^2+3x+2x+6\\
&= x(x+3)+2(x+3)\\
&= (x+3)\ (x+2).\\
\text{Similarly, } x^2-5x+6 &= x^2-3x-2x+6\\
&= x(x-3)-2(x-3\\
&= (x-3)\ (x-2).\\
x^2+x-6 &= x^2+3x-2x-6\\
&= x(x+3)-2(x+3)\\
&= (x+3)\ (x-2).\\
x^2-x-6 &= x^2-3x+2x-6\\
&= x(x-3)+2(x-3).\\
&= (x-3)\ (x+2).
\end{aligned}$$

EXERCISE 6

Resolve into factors:—

(1) $z^2+10z+24$

(2) $x^2+8x+15$.

(3) $x^2+17x+66$.

(4) x^2-8x+7.

(5) $p^2+20p+96$.

(6) $x^2-10x+24$.

(7) $x^2-17x+66$.

(8) $x^2+20x-69$.

(9) $x^2+24x-112$.

(10) $x^2-24x-112$.

(11) $x^2+7xy+10y^2$.

Solution :

$$\begin{aligned} x^2+7xy+10y^2 &= x^2+5xy+2xy+10y^2. \\ &= x(x+5y)+2y(x+5y). \\ &= (x+5y)\,(x+2y). \end{aligned}$$

(12) $x^2+10xy+21y^2$

(13) $x^2-16xy+55y^2$.

(14) $x^2-8xy-240y^2$.

(15) x^4+12x^2+35

Solution :

$$\begin{aligned} x^4+12x^2+35 &= (x^2)^2+12x^2+35 \\ &= (x^2)^2+7x^2+5x^2+35 \\ &= x^2(x^2+7)+5(x^2+7) \\ &= (x^2+7)\,(x^2+5). \end{aligned}$$

(16) x^4+12x^2+11.

(17) a^4+18a^2+65.

(18) a^4+25a^2+136

(19) a^8-11a^4-80.

(20) a^6-7a^3-8.

(21) $x^{12}-6x^6+8$.

(22) $x^2+(a+b)x+ab$.

Solution :

$$\begin{aligned} x^2+(a+b)x+ab &= x^2+ax+bx+ab \\ &= x(x+a)+b(x+a) \\ &= (x+a)(x+b). \end{aligned}$$

(23) $y^2+(2a+3b)y+6ab$.

(24) $x^4+(m^2+n^2)x^2+m^2n^2$.

(25) $y^2-(2a+b)y+2ab$.

(26) $x^2-(a+5b)x+5ab$.

(27) $(a^2+2a)^2-(a^2+2a)-2$.

Solution : Put $a^2+2a=x$.

Then the exp. $=x^2-x-2=(x-2)(x+1)$.

Restore the value of x.

The exp. $=(a^2+2a-2)(a^2+2a+1)=(a^2+2a-2)(a+1)^2$

(28) $(x^2+3x)^2+3(x^2+3x)+2$.

(29) $(a^2+7a)^2-8(a^2+7a)-180$.

(30) $(x^2-2x)^2-2(x^2-2x)-3$

13. *Resolution into factors of the expressions of the form :—*

ax^2+bx+c like $2x^2+5x+2$.

We know that $(2x+1)(x+2)$

$$=2x^2+x+4x+2$$
$$=2x^2+5x+2.$$

Therefore we should reverse the above process in order to get the factors of $2x^2+5x+2$.

That is, we should put $2x^2+5x+2$

$$=2x^2+4x+x+2.$$

But why is $5x$ put equal to $4x+x$?

From many examples it is clear that we should break 4 *i. e.* 2×2 into factors whose sum must be equal to 5. These factors are 4 and 1 only. So $5x$ is put equal to $4x+x$.

The difference between this method and that in Art. 12 is that here we have to multiply the coefficient of the first term by the third term or the coefficient of the third term and find the factors of the product so obtained such that the algebraic sum of the factors is equal to the coefficient of the second term.

14. *Solved Examples.*

Resolve into factors :—

(1) $7x^2-8x+1$

Solution :
$$7x^2-8x+1=7x^2-7x-x+1$$
$$=7x(x-1)-1(x-1)$$
$$=(x-1)(7x-1).$$

(2) $2x^2+7x-49$.

Solution : $2x^2+7x-49=2x^2+14x-7x-49$
$=2x(x+7)-7(x+7)$
$=(2x-7)(x+7)$.

(3) $8x^2-22xy+15y^2$

Solution : $8x^2-22xy+15y^2=8x^2-12xy-10xy+15y^2$
$=4x(2x-3y)-5y(2x-3y)$
$=(2x-3y)(4x-5y)$.

EXERCISE 7

Resolve into factors :—

(1) $2x^2+3x+1$.
(2) $2x^2+x-1$.
(3) $2p^2+19p+9$.
(4) $3x^2+7x-6$.
(5) $6x^2+35x+44$.
(6) $2x^2-x-1$.
(7) $4p^2-8p+3$.
(8) $49a^2+21a+2$.
(9) $3-11a+6a^2$
(10) $8+x-7x^2$.
(11) $10a^2-13ab-9b^2$.
(12) $15a^2-ab-6b^2$.
(13) $27c^2-24cd+5d^2$.
(14) $6a^2+49ab-45b^2$.
(15) $12x^2-5xy-77y^2$.
(16) $12a^2+11ax-15x^2$.
(17) $3x^4-20x^2-7$.

Solution : $3x^4-20x^2-7=3x^4-21x^2+x^2-7$
$=3x^2(x^2-7)+1(x^2-7)$
$=(x^2-7)(3x^2+1)$.

(18) $6x^4-7x^2y^2-2y^4$.
(19) $32x^4y^4-84x^2y^2-135$.
(20) $21x^4-101x^2+88$.
(21) $15+x^2y^2-2x^4y^4$.
(22) $2(x+y)^2+3(x+y)-2$.

Solution : In $2(x+y)^2+3(x+y)-2$ put $a=x+y$.

Then the exp. $=2a^2+3a-2$

$=2a^2+4a-a-2$

$=2a(a+2)-1(a+2)$

$=(2a-1)(a+2)$.

Restore the value of a.

The exp. $=\{2(x+y)-1\}(x+y+2)$

$=(2x+2y-1)(x+y+2)$.

(23) $2(x^2+y^2)^2-3xy(x^2+y^2)-2x^2y^2$.

(24) $2(a^2+b^2)^2+5ab(a^2+b^2)+2a^2b^2$.

(25) $4(x^2-4xy+y^2)^2+15xy(x^2-4xy+y^2)-4x^2y^2$.

(26) $4(a^2-3)^2-5(a^2-3)-6$.

(27) $(x+7y)^2+3z(x+7y)-10z^2$.

(28) $3(2a-x)^2-46a+23x+14$.

15. *Making Complete Squares.*

We know that $(a+b)^2=a^2+2ab+b^2$

and $(a-b)^2=a^2-2ab+b^2$.

Here is a trinomial expression of which two terms are perfect squares (*e. g.* a^2, b^2 in the above examples) and the third term is twice the product of the other two terms. This third term is either added to (as in the first example) or subtracted from (as in the other example) the sum of the squares of the other two terms.

Therefore in order to make a quantity a complete square add or subtract a suitable term to make it like $(a+b)^2$ or $(a-b)^2$.

As for example, if a^2+2ab is to be made a perfect square divide $2ab$ by twice the square root of a^2, *i. e.* by 2a and then *add* the square of the quotient, *i. e.* b^2.

The quantity then becomes $a^2+2ab+b^2$ which is the square of $a+b$. Or add to a^2+2ab the square of half the coefficient of a in $2ab$, that is b^2.

16. *Solved examples.*

What should be added to :

(1) $x^2 \pm 4x$ *so that the result in each case may be a complete square ?*

Solution : Take the square of half the coefficient of x, that is, the square of 2, that is, 4 and add 4 to $x^2 \pm 4x$ to become $x^2 \pm 4x+4$. Then $x^2 \pm 4x+4$ is a complete square in each case, the square roots being $(x \pm 2)$ respectively.

(2) $9x^2 \pm 12xy$.

Solution As above add $4y^2$ and the result, $9x^2 \pm 12xy+4y^2$ is the square of $3x \pm 2y$.

(3) x^4+y^4.

Solution : Here both terms are perfect squares. So the third term will be twice the product of the square roots of both these terms, that is, $2x^2y^2$. Add or subtract $2x^2y^2$ to or from x^4+y^4 and the result, $x^4 \pm 2x^2y^2+y^4$, is the perfect square of $x^2 \pm y^2$.

EXERCISE 8

What should be added to the following so that the result may be a complete square ?

(1) x^2+4x (2) x^2-8x (3) x^2-17x

(4) $36y^2-y$ (5) $9a^2+6ab$ (6) $4x^2+4x$

(7) $25y^2+\qquad+z^2$ (8) $4x^2-\qquad+9y^2$

(9) $49x^2+\qquad+25y^2$

(10) $\frac{1}{100}+\qquad+100x^2$. (11) $(5x)^2+\qquad+(2y)^2$.

Resolve into factors by making perfect square:—

(12) $x^2-7x+12$.

Solution: $x^2-7x+12=x^2-7x+(\frac{7}{2})^2-(\frac{7}{2})^2+12$

$=x^2-7x+\frac{49}{4}-\frac{49}{4}+12$

$=x^2-7x+\frac{49}{4}-\frac{1}{4}$

$=(x-\frac{7}{2})^2-(\frac{1}{2})^2$

$=(x-\frac{7}{2}+\frac{1}{2})(x-\frac{7}{2}-\frac{1}{2})$

$=(x-3)(x-4)$.

(13) $x^2+7x-60$. (14) $16x^2+8xy-15y^2$.

(15) $4a^2+8a+3$. (16) $9x^2-3x-2$.

(17) $13p^2+41p+6$. (18) $26a^2-41a+3$.

(19) $35p^2+41pq-22q^2$. (20) x^4+4y^4.

Solution: $x^4+4y^4=(x^2)^2+4x^2y^2+(2y^2)^2-4x^2y^2$

$=(x^2+2y^2)^2-(2xy)^2$

$=(x^2+2y^2+2xy)(x^2+2y^2-2xy)$

$=(x^2+2xy+2y^2)(x^2-2xy+2y^2)$.

(21) x^4+4. (22) $4x^4+y^4$.

(23) x^4+64. (24) a^4b^4+4.

(25) $4x^4+81$. (26) x^8+64.

(27) $9x^4-28x^2+16$.

Solution: $9x^4-28x^2+16=(3x^2)^2-2.3x^2.4+16-4x^2$

$=(3x^2-4)^2-(2x)^2$.

$=(3x^2-4+2x)(3x^2-4-2x)$

$=(3x^2+2x-4)(3x^2-2x-4)$.

(28) $4x^4+27x^2y^2+49y^4$.

(29) $4x^4-37x^2+81$.

(30) $9a^4-19a^2b^2+9b^4$.

(31) $49x^4-15a^2x^2+121a^4$.

(32) $x^4-x^2y^2+16y^4$.

(33) $x^8+x^4y^4+y^8$.

Solution : $x^8+x^4y^4+y^8=x^8+2x^4y^4+y^8-x^4y^4$

$=(x^4+y^4)^2-(x^2y^2)^2$

$=(x^4+y^4+x^2y^2)\,(x^4+y^4-x^2y^2)$

$=(x^4+x^2y^2+y^4)\,(x^4-x^2y^2+y^4)$

$=\{\,x^4+2x^2y^2+y^4-x^2y^2\,\}\,(x^4-x^2y^2+y^4)$

$=\{\,(x^2+y^2)^2-(xy)^2\,\}\,(x^4-x^2y^2+y^4)$

$=(x^2+y^2+xy)\,(x^2+y^2-xy)\,(x^4-x^2y^2+y^4)$

$=(x^2+xy+y^2)\,(x^2-xy+y^2).\,(x^4-x^2y^2+y^4)$

(34) $x^8-13x^4y^4+4y^8$.

(35) a^8+a^4+1.

(36) $25-34a^4+9a^8$.

(37) $4(a+b)^4+(a-b)^4$.

[Hint—Put $a+b=x$ and $a-b=y$].

(38) $4(x+y)^4+625$.

17. *Resolution into factors of the expressions of the forms* :— a^3+b^3 **and** a^3-b^3.

We know that $(a+b)^3=a^3+3a^2b+3ab^2+b^3$

$=a^3+b^3+3ab(a+b)$

$\therefore a^3+b^3=(a+b)^3-3ab(a+b)$.

Similarly, $a^3-b^3=(a-b)^3+3ab(a-b)$.

Now to factorize a^3+b^3.

$a^3+b^3=(a+b)^3-3ab(a+b)$.

$=(a+b)\,\{\,(a+b)^2-3ab\,\}$

$=(a+b)\,(a^2+2ab+b^2-3ab)$

$=(a+b)\,(a^2-ab+b^2)$.

Similarly, to factorize a^3-b^3.

$a^3-b^3=(a-b)^3+3ab(a-b)$

$=(a-b)\,\{\,(a-b)^2+3ab\,\}$

$=(a-b)\,(a^2-2ab+b^2+3ab)$

$=(a-b)\,(a^2+ab+b^2)$.

18. *Solved Examples.*

Resolve into factors :—

(1) $27x^3+64y^3$.

Solution : $27x^3+64y^3=(3x)^3+(4y)^3$

$$=(3x+4y)^3-3.3x.4y(3x+4y)$$
$$=(3x+4y)\{(3x+4y)^2-36xy\}$$
$$=(3x+4y)(9x^2+24xy+16y^2-36xy)$$
$$=(3x+4y)(9x^2-12xy+16y^2).$$

(2) $8a^3-125b^3$

Solution : $8a^3-125b^3=(2a)^3-(5b)^3$.

$$=(2a-5b)^3+3.2a.5b(2a-5b)$$
$$=(2a-5b)\{(2a-5b)^2+30ab\}$$
$$=(2a-5b)(4a^2-20ab+25b^2+30ab)$$
$$=(2a-5b)(4a^2+10ab+25b^2).$$

II Method for finding the factors of :—

(1) a^3+b^3

Solution : Add and subtract a^2b,

$$a^3+b^3=a^3+a^2b-a^2b+b^3$$
$$=a^2(a+b)-b(a^2-b^2)$$
$$=a^2(a+b)-b(a+b)(a-b)$$

Take $a+b$ common

$$\therefore a^3+b^3=(a+b)[a^2-b(a-b)]$$
$$=(a+b)(a^2-ab+b^2).$$

Similarly, $a^3-b^3=a^3-a^2b+a^2b-b^3$

$$=a^2(a-b)+b(a^2-b^2)$$
$$=a^2(a-b)+b(a-b)(a+b)$$
$$=(a-b)\{a^2+b(a+b)\}$$
$$=(a-b)(a^2+ab+b^2).$$

EXERCISE 9

Resolve into factors :—

(1) a^3+8b^3

(2) x^3+125y^3

(3) a^3-8b^3

(4) x^3-27y^3

(5) $8p^3-27q^3$

(6) $125p^3-8q^3$

(7) $1000-64y^3$

(8) $216a^3-1000b^3$

(9) x^6+y^6

Solution : $x^6+y^6=(x^2)^3+(y^2)^3$

$$\begin{aligned}
&=(x^2)^3+x^4y^2-x^4y^2+(y^2)^3\\
&=(x^2)^2(x^2+y^2)-y^2(x^4-y^4)\\
&=x^4(x^2+y^2)-y^2(x^2+y^2)(x^2-y^2)\\
&=(x^2+y^2)\{x^4-y^2(x^2-y^2)\}\\
&=(x^2+y^2)(x^4-x^2y^2+y^4).
\end{aligned}$$

(10) x^6-y^6.

(11) x^6+125y^6.

(12) x^6-8y^3.

(13) $x^3+\frac{1}{27}$

(14) $8a^3+\dfrac{b^3}{8}$.

(15) $\dfrac{8}{27x^3}-\dfrac{27y^3}{8}$

(16) $x^3+(a+b)^3$.

[**Hint :** Put $a+b=y$].

(17) $a^3+8(x+y)^3$.

(18) $(a+b)^3-(x+y)^3$.

(19) $8(a-b)^3-27(a+b)^3$.

(20) $125(p+2q)^3-64(p-2q)^3$.

18. *Resolution into factors of the expressions of the form :* $a^3+b^3+c^3-3abc$.

Solution :—

$$a^3+b^3+c^3-3abc=(a+b)^3-3ab(a+b)+c^3-3abc$$
$$=(a+b)^3+c^3-3ab(a+b)-3abc$$
$$=(a+b+c)(a^2+2ab+b^2-ac-bc+c^2)$$
$$-3ab(a+b+c)$$
$$=(a+b+c)(a^2+2ab+b^2-ac$$
$$-bc+c^2-3ab)$$
$$=(a+b+c)(a^2+b^2+c^2-ab-bc-ca).$$

19. *Solved Examples.*

Resolve into factors :

(1) $8x^3-y^3+64z^3+24xyz.$

Solution :

$$8x^3-y^3+64z^3+24xyz$$
$$=(2x)^3-y^3+(4z)^3+24xyz$$
$$=(2x-y)^3+6xy(2x-y)+(4z)^3+24xyz$$
$$=(2x-y)^3+(4z)^3+6xy(2x-y)+24xyz$$
$$=(2x-y+4z)(4x^2-4xy+y^2-8xz+4yz+16z^2)$$
$$+6xy(2x-y+4z)$$
$$=(2x-y+4z)(4x^2-4xy+y^2-8xz+4yz+16z^2+6xy)$$
$$=(2x-y+4z)(4x^2+y^2+16z^2+2xy-8xz+4yz).$$

EXERCISE 10

Resolve into factors :—

(1) $a^3-b^3+c^3+3abc.$

(2) $a^3+b^3-c^3+3abc.$

(3) $a^3-b^3-c^3-3abc.$

(4) $8a^3+27b^3+c^3-18abc.$

(5) $8a^3+27b^3-64c^3+72abc.$

(6) $27x^3-8y^3-z^3-18xyz.$

(7) $x^3+y^3+1-3xy.$

(8) $x^3+8y^3-8+12xy$.

(9) $\dfrac{1}{x^3}+\dfrac{1}{y^3}+\dfrac{1}{z^3}-\dfrac{3}{xyz}$.

(10) $x^3+8y^3-\dfrac{z^3}{27}+2xyz$.

(11) $x^6+y^6+z^6-3x^2y^2z^2$.

(12) x^6+32x^3-64.

Solution: $x^6+32x^3-64=x^6+8x^3-64+24x^3$

$=(x^2)^3+(2x)^3-(4)^3.\ x^2\ \ 2x\ 4.$ etc.

(13) $(a+b)^3+(b+c)^3+(c+a)^3-3(a+b)(b+c)(c+a)$.

Find the quotient:—

(14) $(x^3+27-125y^3+45xy)\div(x+3-5y)$.

(15) $(8a^3-27b^3-c^3-18abc)\div(4a^2+9b^2+c^2+6ab+2ac-3bc)$.

(16) Prove that $a^3+b^3+c^3=3abc$ when

$a+b+c=0$.

20. *Resolution into factors of expressions of the form*:

$(x+a)(x+b)(x+c)(x+d)+R$

Solved Examples:

Resolve into factors:

(i) $(x+2)(x+3)(x+4)(x+5)-360$.

Solution: Exp. $=(x+2)(x+5)(x+3)(x+4)-360$

$=(x^2+7x+10)(x^2+7x+12)-360$

Put $x^2+7x=a$.

$\therefore$ Exp. $=(a+10)(a+12)-360$

$=a^2+22a+120-360$

$=a^2+22a-240$

$=(a+30)(a-8)$.

Restore the value of a,

Exp. $=(x^2+7x+30)(x^2+7x-8)$

$=(x^2+7x+30)(x-1)(x+8)$.

(2) $(x^2-8)(x+4)(x-2)-15x^2$.

$$\text{Exp.}=(x^2-8)(x^2+2x-8)-15x^2.$$

Put $x^2-8=a$,

$$\therefore \text{Exp.}=a(a+2x)-15x^2$$
$$=a^2+2ax-15x^2$$
$$=(a+5x)(a-3x).$$

Restore the value of a

$$\text{Exp.}=(x^2-8+5x)(x^2-8-3x)$$
$$=(x^2+5x-8)(x^2-3x-8).$$

EXERCISE 11

Resolve into factors :—

(1) $(x+2)(x+3)(x+4)(x+5)-24$.

(2) $(x+1)(x+3)(x+5)(x+7)-65$.

(3) $(x+1)(x+3)(x+5)(x+7)+15$

(4) $(x+1)(x+3)(x-4)(x-6)+13$

(5) $x(x-1)(x-4)(x-5)-21$.

(6) $x(2x+1)(x-2)(2x-3)-63$.

(7) $(x-3)(x+4)(x-2)(x+6)+2x^2$.

(8) $(x-2)(x+6)(x+9)(x-3)-21x^2$.

(9) Prove that if 1 be added to the product of four consecutive numbers, the result is a perfect square.

(10) Prove that if 16 be added to the product of four consecutive even numbers, the result is a perfect square.

21. *The Remainder Theorem.*

We divide x^2+3x+5 by $x-2$, x^3+3x^2+3x+1 by $x+1$ and ax^2+bx+c by $x-p$ and tabulate the result.

i.

$$
\begin{array}{r|l}
x-2 & x^2+3x+5\,(x+5 \\
 & x^2-2x \\
\hline
 & \quad 5x+5 \\
 & \quad 5x-10 \\
\hline
 & \quad\quad 15
\end{array}
$$

ii.

$$
\begin{array}{r|l}
x+1 & x^3+3x^2+3x+1\,(x^2+2x+1 \\
 & x^3+x^2 \\
\hline
 & 2x^2+3x \\
 & 2x^2+2x \\
\hline
 & \quad x+1 \\
 & \quad x+1 \\
\hline
 & \quad \times
\end{array}
$$

iii.

$$
\begin{array}{r|l}
x-p & ax^2 \quad +bx \quad +c\,(ax+(b+ap) \\
 & ax^2-apx \\
\hline
 & (b+ap)x+c \\
 & (b+ap)x-p(b+ap) \\
\hline
 & c+p(b+ap)
\end{array}
$$

The Remainder in this case is ap^2+bp+c.

Therefore, Dividend	Divisor	Remainder
x^2+3x+5	$x-2$	15
x^3+3x^2+3x+1	$x+1$	0
ax^2+bx+c	$x-p$	ap^2+bp+c

It is clear from the above examples that

(*i*) by substituting 2 for x in x^2+3x+5, the result is $2^2+3\,.\,2+5=15$.

(*ii*) by substituting -1 for x in x^3+3x^2+3x+1, the result is $(-1)^3+3(-1)^2+3(-1)+1=0$ and

(*iii*) by substituting p for x in ax^2+bx+c, the result is ap^2+bp+c.

Also these results are respectively the remainders of the first, second and third operations.

Therefore,

i. Find out the value of x by putting the divisor containing $x=0$

$$e.g.,\quad x-2=0 \qquad \therefore x=2,$$
$$x+1=0 \qquad \therefore x=-1 \quad \text{and}$$
$$x-p=0 \qquad \therefore x=p.$$

ii. Substitute this value of x in the dividend and get the result.

This result will be the *remainder*. This theorem is known as the **Remainder Theorem.**

22. With the help of this theorem we can find the remainder without actual division.

As for example :

Find the remainder of :—

$x^3-39x+18 \div x-6$

Solution : Put $x-6=0$, $\therefore x=6$

Substitute $x=6$ in $x^3-39x+18$,

$\therefore$ the remainder $=(6)^3-39\times6+18$

$=216-234+18=0.$

EXERCISE 12

Find the remainder without actual division :—

(1) $x^3-5x^2+6x+9 \div x-2.$

(2) $x^3-216 \div x-6.$

(3) $4x^3+5x^2+9x+7 \div 2x+3.$

(4) $x^3-2x^2-23x-80 \div x+5.$

(5) $2x^3-5x^2+x-7 \div 2x+1.$

(6) $x^3-3x^2+3x-1 \div x-1$.

(7) $x^4-y^4 \div x-y$.

(8) $3x^4+7x^3-3x^2+2x+1 \div x-1$.

Without actual division show that :—

(9) $x-1$ is a factor of x^3+4x^2+x-6.

Solution : Put $x-1=0$, $\therefore$ $x=1$

Substitute $x=1$ in x^3+4x^2+x-6.

The result $=1+4+1-6=0$

This result, as shown above, is the remainder, when

x^3+4x^2+x-6 is divided by $x-1$.

Therefore $x-1$ is a factor of x^3+4x^2+x-6.

(10) $x-3$ is a factor of $x^3-x^2-11x+15$.

(11) $x+1$ is a factor of x^3+2x^2+3x+2.

(12) $x-3$ is a factor of $x^4-3x^3-2x^2+8x-6$.

(13) $x-4$ is a factor of $6x^3-19x^2-26x+24$.

(14) $x+5, x+2$ are the factors of $x^3-39x-70$.

(15) $x+y$ is a factor of $x^3+3x^2y+3xy^2+y^3$.

(16) $a+b$ and $a+2b$ are the factors of $a^3+4a^2b+5ab^2+2b^3$.

(17) If $x^4-x^3-x^2+ax+6$ is divisible by $x-2$ find the value of a.

(18) If x^3+3x^2+4x+p is divisible by $x+6$, find the value of p.

(19) If $2x^3+ax^2-x-b$ is divisible by x^2+x-2, find the values of a and b.

Solution : $x^2+x-2=(x-1)(x+2)$

Put $(x-1)(x+2)=0$

$\therefore x=1$ or $x=-2$

Substitute $x=1$ and $x=-2$ in $2x^3+ax^2-x-b$.

The remainders are $a-b+1$ and $4a-b-14$ respectively.

Each of them is equal to 0.

That is, $a-b+1=0$

$4a-b-14=0$

From these equations $a=5$, $b=6$.

(20) If $x^3-kx+96$ is divisible by $x^2+2x-48$, find the value of k.

(21) If $x+2$ and $x-3$ are factors of $2x^3+3x^2+cx+d$, find the values of c and d.

(22) Find the values of a and b when $4x^3+ax^2-11x+b$ is divisible by $2x^2-5x+2$

(23) If x^2+px+q and x^2+lx+m are both divisible by $x+a$, show that $a=\frac{q-m}{p-l}$.

Solution : Substitute $x=-a$ in x^2+px+q.

The remainder $=a^2-pa+q$.

Put $x=-a$ in x^2+lx+m.

The remainder$=a^2-la+m$.

Each of these remainders$=0$.

or $a^2-pa+q=0$ and

$a^2-la+m=0$.

or $a^2-la+m=a^2-pa+q$

or $pa-la=q-m$

or $a(p-l)=q-m$

$\therefore\ a=\frac{q-m}{p-l}$.

23. *Application of the Remainder Theorem.*

This theorem is used to find the factors of expressions.

Solved Examples :

Factorize :—

(1) $x^3+x^2-16x-16$

Solution :—The last term of the expression is 16 whose factors can be 1, 2, 4, 8 and 16.

If by putting $x = \pm 1, \pm 2, \pm 4, \pm 8, \pm 16$ in the expression it vanishes, we can at once find out the factors.

Put $x=1$, $R = -30$, $\therefore (x-1)$ is not a factor.

Put $x = -1$, $R = -1 + 1 + 16 - 16 = 0$.

$\therefore x+1$ is a factor.

Put $x=4$, $R = 64+16-64-16=0$.

$\therefore x-4$ is another factor.

Similarly, the third factor is $x+4$.

$$\therefore x^3+x^2-16x-16=(x+1)(x-4)(x+4).$$

(ii) $x^3-39x-70$

Put $x=-2$ in $x^3-39x-70$.

The Remainder$=(-2)^3-39(-2)-70=-8+78-70=0$.

$\therefore x+2$ is a factor.

By rearrangement,

$$\begin{aligned} x^3-39x-70 &= x^2(x+2)-2x(x+2)-35(x+2) \\ &= (x+2)(x^2-2x-35) \\ &= (x+2)(x-7)(x+5) \end{aligned}$$

$\therefore x^3-39x-70=(x+2)(x-7)(x+5)$.

(iii) $4x^3-7x-3$.

Put $x=-1$, in $4x^3-7x-3$

The Remainder

$$=4(-1)^3-7(-1)-3=0$$

$\therefore x+1$ is a factor

The expression

$$\begin{aligned} &= 4x^2(x+1)-4x(x+1)-3(x+1) \\ &= (x+1)(4x^2-4x-3) \\ &= (x+1)\{4x^2+2x-6x-3\} \\ &= (x+1)\{2x(2x+1)-3(2x+1)\} \\ &= (x+1)(2x+1)(2x-3) \end{aligned}$$

EXERCISE 13

Resolve into factors by the Remainder Theorem :

(1) x^3-4x^2+x+2.

(2) $2x^3+5x^2-4x-3$.

(3) $3x^3+8x^2-8x-3$.

(4) $x^3-13x-12$.

(5) $x^3+5x^2-25x-125$.

(6) x^3+2x^2-x-2.

(7) a^3-3a^2+3a-9.

(8) $x^4-15x^2-10x+24$.

(9) $x^3-6xy^2+9y^3$.

Solution : Put $x=-3y$

$\therefore$ $R=-27y^3+18y^3+9y^3=27y^3-27y^3=0$.

$\therefore$ $x+3y$ is a factor.

By rearrangement,

$$x^3-6xy^2+9y^3=x^2(x+3y)-3xy(x+3y)+3y^2(x+3y)$$
$$=(x+3y)(x^2-3xy+3y^2).$$

(10) $x^3-2x^2y-5xy^2+6y^3$.

(11) x^3-ax^2-x+a.

(12) $x^3+4x^2y+xy^2-2y^3$.

(13) $2x^3+x^2y-7xy^2-6y^3$.

(14) $a^3-9ab^2-10b^3$.

23. Cyclic Order

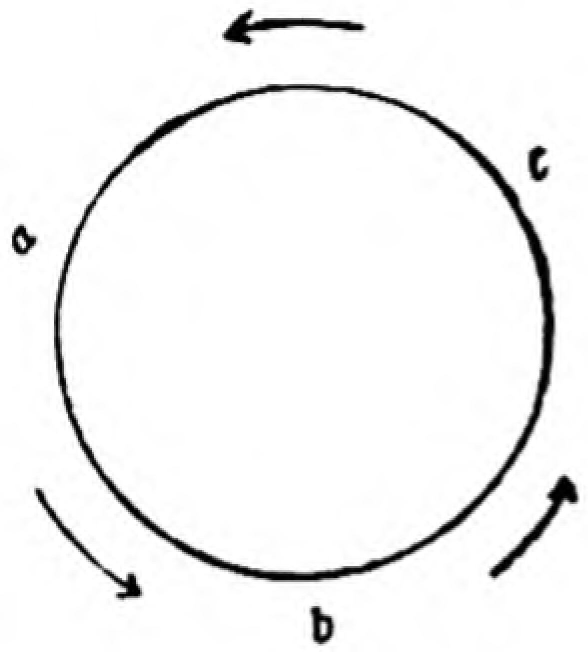
Consider the examples :

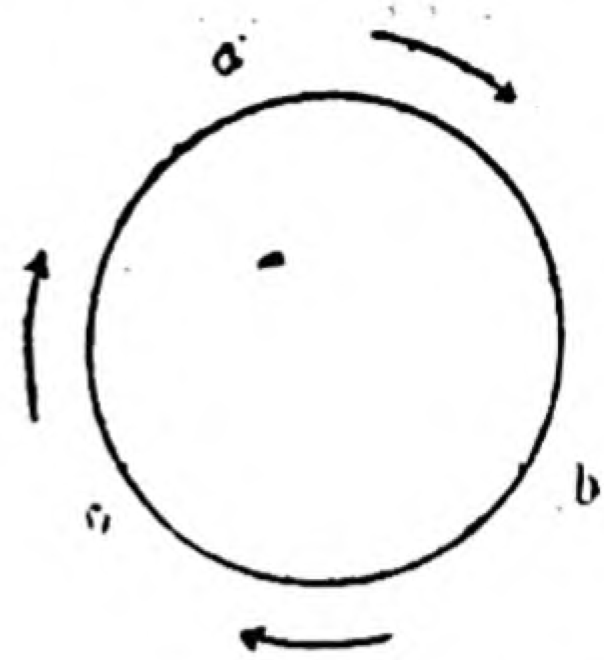
(i) $a^2(b-c)+b^2(c-a)+c^2(a-b)$

(ii) $b^2(c-a)+c^2(a-b)+a^2(b-c)$.

Both the examples have the same value. a, b, c in the first example have been respectively changed into b, c, a in the second.

Such examples are said to be in *cyclic order*.





In the first circle a, b, c are placed in order, that is, b follows a, c follows b and a follows c; a, b, c are in *cyclic order*.

Also $(a-b)$, $(b-c)$, $(c-a)$
and $(x-y)$, $(y-z)$, $(z-x)$ are in cyclic order.

24. Illustrations of expressions in *cyclic order* :—

(1) $x(y^2-z^2)+y(z^2-x^2)+z(x^2-y^2)$.

(2) $x^2(y-z)+y^2(z-x)+z^2(x-y)$.

(3) $xy(x-y)+yz(y-z)+zx(z-x)$.

(4) $x^2(y^2-z^2)+y^2(z^2-x^2)+z^2(x^2-y^2)$.

(5) $a^3(b-c)+b^3(c-a)+c^3(a-b)$ etc.

25. *Solved Examples.*

Resolve into simple factors:—

(*i*) $x(y^2-z^2)+y(z^2-x^2)+z(x^2-y^2)$.

Exp. $=x(y^2-z^2)+yz^2-yx^2+zx^2-zy^2$.

Arrange, according to the descending powers of x, the last four terms.

$$\text{Exp.}=x(y^2-z^2)-x^2y+x^2z+yz^2-zy^2$$
$$=x(y^2-z^2)-x^2(y-z)-yz(y-z)$$
$$=x(y+z)(y-z)-x^2(y-z)-yz(y-z)$$

$$=(y-z)\{x(y+z)-x^2-yz\}$$
$$=(y-z)(xy+xz-x^2-yz)$$
$$=(y-z)(xz-yz-x^2+xy)$$
$$=(y-z)\{z(x-y)-x(x-y)\}$$
$$=(y-z)(z-x)(x-y)$$
$$=(x-y)(y-z)(z-x).$$

(*ii*) $a^3(b-c)+b^3(c-a)+c^3(a-b)$.

Exp. $=a^3(b-c)+b^3c-b^3a+c^3a-bc^3$.

Arrange, in the descending powers of a, the last four terms.

Exp. $=a^3(b-c)-ab^3+c^3a+b^3c-bc^3$

$$=a^3(b-c)-a(b^3-c^3)+bc(b^2-c^2)$$
$$=a^3(b-c)-a(b-c)(b^2+bc+c^2)+bc(b+c)(b-c)$$
$$=(b-c)\{a^3-a(b^2+bc+c^2)+bc(b+c)\}$$
$$=(b-c)(a^3-ab^2-abc-ac^2+b^2c+bc^2)$$

Arrange, in descending powers of b for the second factor.

Exp. $=(b-c)(b^2c-b^2a+bc^2-abc-ac^2+a^3)$

$$=(b-c)\{(b^2c-b^2a)+(bc^2-abc)-(ac^2-a^3)\}$$
$$=(b-c)\{b^2(c-a)+bc(c-a)-a(c^2-a^2)\}$$
$$=(b-c)\{b^2(c-a)+bc(c-a)-a(c+a)(c-a)\}$$
$$=(b-c)(c-a)(b^2+bc-ca-a^2)$$

Arrange, in the descending powers of c for the third factor.

Exp. $=(b-c)(c-a)(-ac+bc-a^2+b^2)$

$$=(b-c)(c-a)\{-c(a-b)-(a-b)(a+b)\}$$
$$=(b-c)(c-a)(a-b)(-c-a-b)$$
$$=-(b-c)(c-a)(a-b)(a+b+c)$$
$$=-(a-b)(b-c)(c-a)(a+b+c).$$

(*iii*) $a^2(b+c)+b^2(c+a)+c^2(a+b)+2abc$.

Exp. $=a^2(b+c)+b^2c+ab^2+ac^2+bc^2+2abc$

Arrange in the descending power of a.

$$\begin{aligned}
\text{Exp.} &= a^2(b+c)+a(b^2+c^2+2bc)+bc(b+c) \\
&= a^2(b+c)+a(b+c)^2+bc(b+c) \\
&= (b+c)(a^2+ab+ac+bc) \\
&= (b+c)\{a(a+b)+c(a+b)\} \\
&= (b+c)(a+b)(a+c) \\
&= (a+b)(b+c)(c+a).
\end{aligned}$$

Note—In order to resolve expressions of the cyclic order into factors, they must be arranged in the descending powers of the terms, a, b, c, x, y or z. Factors like $(a-b)$, $(x-y)$ etc., can then easily be obtained.

EXERCISE 14

Resolve into factors :—

(1) $y^2(z-x)+z^2(x-y)+x^2(y-z)$.

(2) $xy(x-y)+yz(y-z)+zx(z-x)$.

(3) $bc(b+c)+ca(c+a)+ab(a+b)+3abc$.

Solution : $bc(b+c)+ca(c+a)+ab(a+b)+3abc$

$$\begin{aligned}
&= bc(b+c)+ca(c+a)+ab(a+b)+abc+abc+abc \\
&= \{bc(b+c)+abc\}+\{ca(c+a)+abc\}+\{ab(a+b)+abc\} \\
&= bc(b+c+a)+ca(c+a+b)+ab(a+b+c) \\
&= (a+b+c)(bc+ca+ab).
\end{aligned}$$

(4) $a(b+c)^2+b(c+a)^2+c(a+b)^2-3abc$.

(5) $a(b-c)^3+b(c-a)^3+c(a-b)^3$.

(6) $a^4(b-c)+b^4(c-a)+c^4(a-b)$.

(7) $xy(x+y)+yz(y+z)+zx(z+x)+2xyz$.

(8) $xy(x^2-y^2)+yz(y^2-z^2)+zx(z^2-x^2)$.

(9) $a(b^3-c^3)+b(c^3-a^3)+c(a^3-b^3)$.

(10) $(a+b+c)^3-a^3-b^3-c^3$.

Solution : Put $x=a+b$

Then $(a+b+c)^3=(x+c)^3$

$$=x^3+3cx(x+c)+c^3.$$

Restore the value of x.

Then $(a+b+c)^3=(a+b)^3+3c(a+b)(a+b+c)+c^3$

$$=a^3+b^3+3ab(a+b)+3c(a+b)(a+b+c)+c^3$$
$$=a^3+b^3+c^3+3(a+b)\{ab+c(a+b+c)\}$$
$$=a^3+b^3+c^3+3(a+b)(ab+ac+bc+c^2)$$
$$=a^3+b^3+c^3+3(a+b)\{a(b+c)+c(b+c)\}$$
$$=a^3+b^3+c^3+3(a+b)(b+c)(c+a)$$

$\therefore (a+b+c)^3-a^3-b^3-c^3$

$$=a^3+b^3+c^3+3(a+b)(b+c)(c+a)-a^3-b^3-c^3$$
$$=3(a+b)(b+c)(c+a).$$

(11) $8(a+b+c)^3-(b+c)^3-(c+a)^3-(a+b)^3$.

(12) $(a+b+c)(ab+bc+ca)-abc$.

26. Some important results

(1) $a^2(b-c)+b^2(c-a)+c^2(a-b)=-(a-b)(b-c)(c-a)$.

(2) $ab(a-b)+bc(b-c)+ca(c-a)=-(a-b)(b-c)(c-a)$.

(3) $a(b^2-c^2)+b(c^2-a^2)+c(a^2-b^2)=(a-b)(b-c)(c-a)$.

(4) $a^2(b+c)+b^2(c+a)+c^2(a+b)+2abc$.

$$=(a+b)(b+c)(c+a).$$

(5) $a^2(b+c)+b^2(c+a)+c^2(a+b)+3abc$

$$=(a+b+c)(ab+bc+ca).$$

(6) $(a+b+c)^3-a^3-b^3-c^3=3(a+b)(b+c)(c+a)$.

(7) $(b-c)+(c-a)+(a-b)=0$.

(8) $a(b-c)+b(c-a)+c(a-b)=0$.

Miscellaneous Exercises

1

Resolve into factors :—

(1) $(ay+bx)^2+(ax-by)^2$.

(2) $(2x-3y)^3+(3y-z)^3+(z-2x)^3$.

(3) $x^2-(a+{}^1/a)x+1$.

(4) $x^5+x^4+x^3+x^2$.

(5) $x^3+4x^2-11x-30$.

(6) $4x^3-21x-10$.

(7) $(x+1)(x+4)(x+7)(x+10)-360$.

(8) $a^3-3a-2a^2+4$ (9) $x^4+x^3+2x^2+x+1$.

(10) $21x^2+40xy-21y^2$. (11) $p^2+2pq+q^2-p-q$.

(12) x^6-729. (13) $x^4+x^2y^2+y^4$.

(14) $(x-y)^2-(1-xy)^2$.

(15) $6x^2-xy-12y^2-4x-11y-2$.

(16) $x^4-14x^2y^2+y^4$. (17) $4x^2-35x+24$.

(18) $xy-y^2+5y-3x-6$. (19) $(x^2-y^2)(a^2-b^2)+4abxy$

(20) $2b^2c^2+2c^2a^2+2a^2b^2-a^4-b^4-c^4$.

(21) $(x-1)(x-2)(x-3)(x-4)-120$.

(22) $x^3-y^3+3y^2-3y+1$. (23) $ab(1+c^2)-c(a^2+b^2)$.

(24) $x^3-19x-30$. (25) a^3+a^2+2a+8.

(26) x^8-y^8. (27) $x^2-xy-2y^2+x+y$.

(28) Without division prove that $x^4-4x^3+2x^2+x+6$ is divisible by x^2-5x+6 and find the quotient

Resolve into factors :—

(29) $(x^2+5x)(x^2+5x-2)-24$.
[Hint: Put $x^2+5x=a$].

(30) $2x^4-5x^3+6x^2-5x+2$.

(31) $(x-y)^3+(y-z)^3+(z-x)^3$.
[Hint: Put $a=x-y$, $b=y-z$ and $c=z-x$].

(32) $b^2(a+c)-c^2(a+b)$. (33) $(a^2+2a)^2-(a^2+2a)-6$.

(34) $15(x^2-1)-72x$. (35) $x^3-x^2y-xy^2-2y^3$.

(36) $a^2-b^2+8bc-16c^2$. (37) $x^3-8y^3+27z^3+18xyz$.

(38) $x^4+2x^3+3x^2+2x+1$. (39) x^4+8x^2+144.

(40) $6x^2-xy-y^2-6x+3y$. (41) $(x+1)^4-1$.

(42) $2x^2+xy-3y^2+x-y$.

(43) $(1-x+x^2)^2-(1+x-x^2)^2$.

(44) $2-3a+3b+a^2-2ab+b^2$.

(45) $a^3-19ab^2+30b^3$.

(46) Without actual division show that $x-1$, $x-2$, $x+3$ are three of the factors of $x^4+x^3-7x^2-x+6$.

Resolve into factors :—

(47) x^4-16x^2+36.

(48) $x^2-5xy+6y^2-3x+6y$.

(49) $x^3-17x+26$.

(50) $(a+b)^3+(b+c)^3-(a+2b+c)^3$.

(51) $(x-2y)^3+(2x-y)^3$.

(52) $x(x-1)(x-2)-3x+3$.

(53) $x^3+\dfrac{8}{y^3}-\dfrac{1}{27}+\dfrac{2x}{y}$.

(54) $81x^4+47+\dfrac{16}{x^4}$.

(55) $x^3y^2-x^2y^3+x^2y^2-x+y-1$.

CHAPTER II

HIGHEST COMMON FACTORS

27. The factors of a^2b are a, a, b, and the factors of ab^2 are a, b, b.

It is clear that a is a common factor of both a^2b and ab^2. The factors of a^2b^2 and a^2b are respectively a, a, b, b and a, a, b.

In both a^2b^2 and a^2b, a^2b is the highest common factor.

Solved Examples : Find the Highest Common Factor (H. C F.) of :–

(i) x^3y^3, x^2y^4.

Solution :

$$x^3y^3=x\times x\times x\times y\times y\times y.$$
$$x^2y^4=x\times x\times y\times y\times y\times y.$$

$\therefore$ H. C. F. $=x\times x\times y\times y\times y=x^2y^3$.

(ii) x^2y^2, x^3y^4, x^4y^3.

Solution : $x^2y^2 = x \times x \times y \times y$.

$x^3y^4 = x \times x \times x \times y \times y \times y \times y$.

$x^4y^3 = x \times x \times x \times x \times y \times y \times y$.

H. C. F. $= x \times x \times y \times y = x^2y^2$.

EXERCISE 15

Find the H. C. F. of :—

(1) ab^2, a^2b. (2) ab^3, a^3b. (3) x^2y^2, x^2y^3.

(4) a^2yz, xa^2b. (5) $5x^2y^2z^2, 15y^3z^2$.

(6) x^4, x^3, x^2. (7) $7x^2y, 14xy^3, 21xyz$.

(8) $2ab^2, a^2b, a^3b$. (9) $3a^5b^4, 6a^4b^3, 12x^3b^3$.

(10) xy, x^2y^2, x^3y^3, xy^2. (11) $a^2b^2, 2ab^3, 3a^3b^2, 4ab^2c$.

(12) $3x^3y^2z^3, 15x^3y^2z^3, 12x^2y^3z^4, 9x^3y^3z^3$.

(13) $84a^6b^8c^4, 60a^4b^4c^4, 24a^3b^2c^5, 36a^6b^6c^6$.

(14) $25x^2y^2z^2, 35x^4y^4z^4, 70x^6y^6z^6, 75x^8y^8z^8$

(15) $a+b, a^2-b^2$.

Solution : $\because a^2-b^2=(a+b)(a-b)$

$\therefore a+b$ is the H. C. F. of $a+b, a^2-b^2$.

(16) $a(a+b), a^2-b^2$.

(17) $(a+b)^2, a^2-b^2$.

(18) $(a+b)^3, (a+b)^2$.

(19) $(a+b)^3, (a+b)^2, (a+b)$.

(20) $x^2-y^2, x^4-y^4, x^6-y^6$.

(21) $a^3-y^3, a^4+a^2y^2+y^4, a^2y^4+a^3y^3+a^4y^2$.

Solution : $a^3-y^3=(a-y)(a^2+ay+y^2)$.

$a^4+a^2y^2+y^4=(a^2+ay+y^2)(a^2-ay+y^2)$.

$a^2y^4+a^3y^3+a^4y^2=a^2y^2(y^2+ay+a^2)$.

$\therefore$ H. C. F $=a^2+ay+y^2$.

(22) $3a^3-3, 3(a^2-1), 6a(a^2-2a+1)$.

(23) $x^4-y^4, 5x^4-3x^2y^2-2y^4$.

(24) $x^2+65x-134,\ x^2+100x-204.$

(25) $x^2-10x-39,\ x^2-18x+65.$

(26) $x^2+28x+195,\ x^2+10x-75.$

(27) $x^4-2ax^3-8a^2x^2,\ x^5-ax^4-6a^2x^3.$

(28) $a^3+a^2x-10ax^2+8x^3,\ a^4-16x^4.$

[Hint : Since $a-2x$ is a factor of a^4-16x^4, it may also be a factor of $a^3+a^2x-10ax^2+8x^3$].

(29) $9x^2-16,\ 3x^2-8x-16.$

(30) $15x^2-58x+43,\ 25x^2-70x+48.$

(31) $x^2+3x-10,\ x^3-x^2-14x+24.$

(32) $x^3-13x-12,\ x^3+2x^2-x-2.$

(33) $3x^3+8x^2-8x-3,\ x^3-ax^2-x+a.$

(34) $a^2-b^2,\ (a+b)^3,\ a^3+b^3.$

Solution : $a^2-b^2=(a+b)(a-b),$

$(a+b)^3=(a+b)(a+b)(a+b),$

and $a^3+b^3=(a+b)(a^2-ab+b^2).$

$\therefore$ H. C. F.$=a+b.$

(35) $x^3-y^3,\ (x-y)^2,\ x^2-y^2.$

(36) $x^4+x^2y^2+y^4,\ x^3-y^3,\ x^2+xy+y^2.$

(37) $x^3-y^3,\ (x-y)^2,\ x^4-y^4,\ 3x^4+2x^3y-5x^2y^2.$

(38) $x^3+ax^2,\ 2x^2y+2a^2y+4axy,\ x^3+3a^2x+3ax^2+a^3.$

(39) $x^3-19x+30,\ x^3-x^2-14x+24,\ x^3-10x^2+31x-30.$

(40) $x^4+2x^2+1,\ x^4-1,\ x^6+x^4-x^2-1.$

28. *To find the H. C. F. by the Division Method.*

The *Division Method* is the same for both Arithmetic and Algebra and may be summarized as under :—

Divide the quantity with the highest index of the leading letter, 'x' or 'a' etc. by the other or if they be of the same degree, either of them by the other and get the remainder. Divide the former divisor by the remainder so obtained and

get the second remainder. Proceed in this way till the division has no remainder. The last divisor is the required H. C. F.

The reason for this process depends on the fact that whatever divides each of the two quantities A and B also divides their sum or difference, the multiples of those quantities and the remainder when one is divided by the other.

Solved Examples.

Find the H. C. F. of :

$$x^3-x^2-x+1,\ x^3-3x^2+3x-1.$$

Solution :—

$$
\begin{array}{r|l|l|l}
x & x^3-x^2-x+1 & x^3-3x^2+3x-1 & 1 \\
 & x^3-2x^2+x & x^3-x^2-x+1 & \\
\hline
 & x^2-2x+1 & -2)\,-2x^2+4x-2 & \\
 & & \quad x^2-2x+1 & \\
 & & \quad x^2-2x+1 & 1 \\
\cline{3-3}
 & & \quad\quad \times &
\end{array}
$$

$\therefore$ H. C. F. $=x^2-2x+1$.

Rule—1. Arrange the expressions in the ascending or descending powers of a term.

2. Divide one expression by the other, so that the remainder contains a lower power of the term.

3. Divide the first divisor by the remainder as above.

4. Continue the operation till there is no remainder.

5. The last divisor is the H. C. F.

6. If necessary, multiply or divide the divisor or dividend by any term to avoid fraction in the remainder.

EXERCISE 16

Find the H. C. F. by the method of division:—

(1) $x^4-5x^3+4,\ x^5-11x+10.$

(2) $x^4+x^3-5x^2-3x+2,\ x^4-3x^3+x^2+3x-2.$

(3) $2x^4-9x^3+14x^2-9x+2,\ x^4-6x^3+13x^2-12x+4.$

(4) $6x^4+x^3-6x^2-5x-2,\ 2x^4+3x^3+2x^2-7x-6.$

(5) $x^6-x^3+8,\ x^5-x^2+4$

(6) $2x^3+9x^2+4x-15,\ 4x^3+8x^2+3x+20.$

(7) $3x^5-5x^3+2,\ 2x^5-5x+3.$

Solution :

Multiply by 2	$3x^5-5x^3+2$ 2	$2x^5-5x+3$ $2x^5-3x^3+x^2$	x^2	
3	$6x^5-10x^3+4$ $6x^5-15x+9$	$3x^3-x^2-5x+3$ 2		
Divide by -5 $-x$	$-5)\ -10x^3+15x-5$ $2x^3-3x+1$ $2x^3+x^2-3x$	$6x^3-2x^2-10x+6$ $6x^3-9x+3$	3	Multiply by 2
x	$-x^2+1$ $-x^2+x$	$-2x^2-x+3$ $-2x^2+2$	2	
1	$-x+1$ $-x+1$	$-x+1$		
	×			

$\therefore$ H. C. F. $= 1-x.$

(8) $x^4+9x-20,\ 5x^4+9x^3-64.$

(9) $x^5+x^2+2x+2,\ x^4+x^2+1.$

(10) $2x^5-11x^2-9,\ 4x^5+11x^4+81.$

(11) $8x^4+3x+10,\quad 10x^4+3x^3+8.$

(12) $2x^3-7x^2-46x-21,\ 2x^4+11x^3-13x^2-99x-45.$

(13) $3x^2-13x+12,\ x^2+2x-15$

(14) $x^3-3x^2+x-3,\ x^4+6x^2+5.$

(15) $x^3-4x^2+7x-6,\ 2x^3-7x^2+2x+8.$

(16) $2x^4-3x^3-3x^2+4.\ 2x^4-x^3-9x^2+4x+4.$

(17) $x^3-x^2-8x+12,\ 3x^2-2x-8$

(18) $x^4-3x^3-2x^2+12x-8,\ x^3-7x+6$

(19) $a^7-1,\ a^3-1.$

(20) $3x^4-7x^3+13x^2-7x+6.\ 2x^4-7x^3+16x^2-17x+12$

(21) $8x^3-24x^2+36x-27\ \ 16x^4+36x^2+81,\ 16x^3+54.$

(22) $x^2-3x-70,\ x^3-39x+70,\ x^3-48x+7.$

(23) $x^3+x^2-4x+2,\ x^3-x^2+x-1,\ x^3+6x^2+3x-10.$

(24) $x^3+5x^2y+7xy^2+3y^3,\ x^3+3x^2y-xy^2-3y^3,$
$x^3+x^2y-5xy^2+3y^3.$

(25) $2x^3+7x^2-5x-4\ \ x^3+8x^2+11x-20.$
$2x^3+19x^2+49x+20.$

CHAPTER III

LOWEST COMMON MULTIPLE

29. 35 is divisible by 5 and 7, 35 is a multiple of 5 and 7. Similarly a^2-b^2 is divisible by $a+b$ and $a-b$. Therefore a^2-b^2 is a multiple of $a+b$ and $a-b$. Similarly a^3-b^3 is a multiple of $a-b$ and a^2+ab+b^2.

Definition:—When an expression is divisible by two or more expressions, it is the common multiple of each of the latter expressions. If this multiple be the least, it is the lowest common multiple (L. C. M.). For instance, a^2-b^2 is a multiple of $a+b$ and $a-b$. Also, a^3-b^3 is a multiple of $a-b$ and a^2+ab+b^2.

Therefore the L. C M. of $a+b$, $a-b$, a^2+ab+b^2 is $(a^2-b^2)(a^2+ab+b^2)$ or $(a+b)(a^3-b^3)$, which is separately divisible by $a+b$, $a-b$ and a^2+ab+b^2.

30. *Solved Examples* :

Find the L. C. M. of :

(1) x^2y^2, a^2by, x^2y^3.

Solution : $x^2y^2 = x\times x\times y\times y$.

$a^2by = a\times a\times b\times y$.

$x^2y^3 = x\times x\times y\times y\times y$.

In the first term, y is contained twice, in the second it is contained once and in the third, it is contained three times. So in the common multiple y must be contained three times. Similarly the common multiple must contain a^2, x^2, b.

$\therefore$ L. C. M. $=a^2bx^2y^3$.

EXERCISE 17

Find the L. C M. of :—

(1) a^2y, by^2.

(2) $2x^2y$, $3x^3y^3$.

(3) $4x^2y^4$, $5abxy^3$.

(4) x^4y^2, $ab^2x^2y^2$

(5) $3p^3q^2$, $4a^2pb^2q$.

(6) $7x^4y^3z$, $21x^5y^4z^3$.

(7) $3a^2b^3c^2$, $5a^2b^2c^3$.

(8) $5a^3b^2xy$, $15ab^3x^3y^2$, $25a^3b^2x^3y^4$.

(9) a^3-b^3, $a^4+a^2b^2+b^4$, a^2+ab+b^2.

Solution : $a^3-b^3=(a-b)(a^2+ab+b^2)$,

$a^4+a^2b^2+b^4=(a^2+ab+b^2)(a^2-ab+b^2)$,

$a^2+ab+b^2=a^2+ab+b^2$.

$\therefore$ L. C. M. $=(a-b)(a^2+ab+b^2)(a^2-ab+b^2)$,

$=(a-b)(a^4+a^2b^2+b^4)$.

(10) a^2-b^2, a^3-b^3, $(a-b)^3$.

(11) a^2+ab+b^2, $(a-b)^2$, $(a-b)(a^2-b^2)$.

(12) $x^2y^2(a-b)^2$, $xyz^2(a+b)^3$, $xy(a^2-b^2)$

(13) $x^2+7x+12$, x^2-x-20.

Solution :

$x^2+7x+12 \qquad =(x+3)(x+4)$.

$x^2-x-20 \qquad =(x-5)(x+4)$.

$\therefore$ L. C. M. $=(x+3)(x+4)(x-5)$.

(14) x^2+x-2, x^2-2x+1.

(15) $2x^2-7x+5$, $2x^2+x-15$

(16) x^2-9, x^2-x-6, x^2-6x+9.

(17) $9x^2-4$, $6x^2+19x+10$, $6x^2+7x+2$.

(18) $x^2+4x-21$, $x^2+9x+20$, $x^2+2x-15$.

(19) $9a^2-21ax+6x^2$, $6a^2+10ax-4x^2$ and $9a^2-6ax+x^2$

(20) $2x^2+3x-2$, $3x^2+7x+2$, $6x^2-x-1$.

(21) $6a^2-5ab-6b^2$, $3a^2-ab-2b^2$ $8a^3-27b^3$.

(22) $3x^3+8x^2-5x-6$, $3x^3+14x^2+17x+6$.

(23) $9a^2b-ab^2-b^3+9a^3$, $3a^3+b^3-3ab^2-a^2b$.

(24) x^3+2x^2-1, x^3-2x+1.

(25) $6x^2-5x-6$, $16x^4+36x^2+81$, $8x^3+27$.

31. To prove that the product of two expressions is equal to the product of their L. C. M and H. C. F.

Proof :— Suppose the expressions are A and B, whose H. C. F. is h and L. C. M., l.

$\therefore$ $A=ah$ and $B=bh$ where a and b have no common factor, since all the common factors of A and B are contained in h.

$$\therefore \quad l=abh=\frac{ah\times bh}{h}=\frac{AB}{h}\ldots\ldots\ldots\ldots(1).$$

$\therefore$ $lh=AB$.

Example : a^2-b^2 and $(a-b)^2$ are two expressions. Their L. C. M. $=(a+b)(a-b)^2$ and H. C. F. $=a-b$.

$\therefore$ the product of their L. C. M. and H C. F.

$$=(a+b)\ (a-b)^2(a-b)$$
$$=(a^2-b^2)\ (a-b)^2$$

which is also the product of the given expressions.

By this method the L. C. M. of expressions which cannot be factorized easily is obtained since

$$\text{L. C. M.} = \frac{\text{Product of the two expressions}}{\text{Their H. C. F}}$$

EXERCISE 18

(1) $2a^2-9a+4$, $3a^2-7a-20$.

(2) $4x^3+16x^2-3x-45$, $10x^3+63x^2+119x+60$.

(3) x^3+x^2-x+2, x^3-4x^2+4x-3.

(4) $a^3+2a^2b-ab^2-2b^3$, $a^3-2a^2b-ab^2+2b^3$.

(5) $6a^3-11a^2+5a-3$, $9a^3-9a^2+5a-2$.

(6) $9x^4-x^2-2x$, $3x^3-10x^2-7x-4$.

(7) $2x^2+6px-10qx-30pq$, $3x^2-9px-15qx+45pq$.

(8) $4x^3-10x^2-18x+45$, $6x^3+8x^2-27x-36$.

(9) $5x^3+8x^2-7x-6$, $5x^3+28x^2+45x+18$:

(10) The H C. F. and L. C. M. of two expressions of second degree are $x+3$ and x^3-7x+6 respectively. Find each.

CHAPTER IV

SQUARE ROOT

32. The square of $3a$ or $-3a$ is $9a^2$.

Therefore the square root of $9a^2$ is $3a$ or $-3a$.

Similarly $\sqrt{25x^4} = \pm 5x^2$.

and $\sqrt{(a+b)^2} = \pm(a+b)$

Solved examples:—

Find the square root of :

(1) $9a^4b^4$

Solution : $9a^4b^4 = 3\times3\times a\times a\times a\times a, \times b\times b\times b\times b,$

$\therefore \sqrt{9a^4b^4} = \pm 3a^2b^2$

EXERCISE 19

Find the square root of :

(1) x^2y^2, $16x^2y^2$, $25x^4y^2$, $36x^6y^4$, $9x^8y^2$.

(2) $49a^2b^2c^2$, $64a^4b^2c^6$, $81a^8b^6c^4$, $a^6b^8p^2q^4$.

(3) $\cdot01a^4b^2$, $\cdot36x^8y^{10}$, $\dfrac{64a^{10}}{49b^8}$, $\dfrac{196x^4y^6}{225x^4y^4}$.

(4) $a^2+2ab+b^2$,

Solution : $a^2+2ab+b^2=(a+b)^2$

$\therefore \sqrt{(a+b)^2} = \pm(a+b)$

(5) $x^2+10x+25$

(6) x^2-6x+9

(7) $4a^2+12ab+9b^2$.

(8) $36x^4+120x^2+100$.

(9) $49a^6-70a^3+25$.

(10) $49x^4+84x^2y^2+36y^4$.

(11) $\frac{1}{4}a^2 - \dfrac{ab}{3} + \frac{1}{9}b^2$.

Solution : $\frac{1}{4}a^2 - \frac{1}{3}ab + \frac{1}{9}b^2$

$$= (\tfrac{1}{2}a)^2 - 2.\frac{a}{2}\cdot\frac{b}{3} + (\tfrac{1}{3}b)^2$$

$$= (\tfrac{1}{2}a - \tfrac{1}{3}b)^2$$

$$\therefore \sqrt{(\tfrac{1}{2}a - \tfrac{1}{3}b)^2} = \pm(\tfrac{1}{2}a - \tfrac{1}{3}b).$$

(12) $\dfrac{x^2y^2}{9} + \dfrac{xy^2z}{9} + \dfrac{y^2z^2}{36}$.

(13) $\dfrac{a^4b^6}{25} - \dfrac{a^2b^3xy^2}{25} + \dfrac{x^2y^4}{100}$.

(14) $a^2+b^2+c^2+2ab+2bc+2ca$.

Solution Since $a^2+b^2+c^2+2ab+2bc+2ca=(a+b+c)^2$,

$\therefore \sqrt{a^2+b^2+c^2+2ab+2bc+2ca} = \pm(a+b+c)$.

(15) $4x^2+9y^2+25z^2+12xy+30yz+20zx$.

(16). $9a^2+16b^2+25c^2-24ab-40bc+30ca$.

(17) $x^4+y^4+z^4-2x^2y^2+2y^2z^2-2z^2x^2$.

(18) $\frac{a^2}{4}+\frac{b^2}{9}+\frac{c^2}{25}+\frac{ab}{3}+\frac{2bc}{15}+\frac{ca}{5}$.

33. *Method of Indeterminate Coefficients.*

Solved Example :

Extract the square root of :

$36x^4-36x^3+33x^2-12x+4$

Solution : Since the expression is of fourth degree, its, square root is of second degree, whose first term is $6x$.

Suppose $6x^2+px+q$ is the required square root where p and q are unknown quantities.

$\therefore (6x^2+px+q)^2 \equiv 36x^4-36x^3+33x^2-12x+4$.

Now $36x^4+p^2x^2+q^2+12px^3+2pqx+12qx^2$

$=36x^4+12px^3+x^2(p^2+12q)+2pqx+q^2$

$\equiv 36x^4-36x^3+33x^2 \quad -12x \quad +4$,

Equate the coefficients of the same powers of x in both expressions.

$12p = \quad -36$ ....(i)

$p^2+12q=33$ ....(ii)

$2pq \quad = -12$............(iii)

From (i) $p=-3$.

From (ii) $9+12q=33$, or $q=2$

Substitute the values of p and q in $6x^2+px+q$.

The reqd. sq. root $=6x^2-3x+2$.

EXERCISE 20

Find the square root of :

(1) $4x^4-12x^3+5x^2+6x+1$.

(2) $4x^4-8x^3y^3+4xy^6+y^9$.

(3) $4x^4-12x^3a-11x^2a^2+30xa^3+25a^4$.

(4) $x^4-4x^3+10x^2-12x+9$.

(5) $x^4-4x^3+6x^2-4x+1$.

(6) $4x^4+12x^3-11x^2-30x+25$.

(7) $16x^4-24x^3+25x^2-12x+4$.

(8) $x^4-2ax^3+5a^2x^2-4a^3x+4a^4$.

(9) $4x^4+2x^3-\dfrac{31x^2}{4}-2x+4$.

(10) $9x^4-30x^3-29x^2+90x+81$

34. *To find the square root by factors :*

Solved Example :

Find the square root of :

$(x+1)(x+2)(x+3)(x+4)+1$.

Solution : $(x+1)(x+2)(x+3)(x+4)+1$

$=(x+1)(x+4)(x+2)(x+3)+1$

$=(x^2+5x+4)(x^2+5x+6)+1$

Put $x^2+5x=y$

Then the exp. $=(y+4)(y+6)+1$

$=y^2+10y+24+1$

$=y^2+10y+25$

Now $\sqrt{y^2+10y+25} = \pm(y+5)$

Restore the value of y.

The reqd. sq. root $= \pm(x^2+5x+5)$.

EXERCISE 21

Find the square root of :

(1) $x(x+1)(x+2)(x+3)+1$

(2) $(x+2)(x+3)(x+4)(x+5)+1$

(3) $(x+3)(x+4)(x+5)(x+6)+1$

(4) $(2x+1)(2x-1)(2x+3)(2x+5)+16$

(5) What should be added to $(x+1)(x+3)(x+5)(x+7)$ to make it a perfect square ?

(6) What should be added to $(2x-1)(2x-3)(2x-5)(2x-7)$ in order to make it a perfect square ?

Find the square root of :—

(7) $(x^2+5x+6)(x^2+4x+3)(x^2+3x+2)$

Solution : $x^2+5x+6=(x+3)\ (x+2)$

$x^2+4x+3=(x+1)\ (x+3).$

$x^2+3x+2=(x+2)\ (x+1).$

$\therefore (x^2+5x+6)\ (x^2+4x+3)\ (x^2+3x+2)$

$=(x+3)\ (x+2)\ (x+1)\ (x+3)\ (x+2)\ (x+1).$

$\therefore$ The reqd. sq. root$=\pm(x+1)\ (x+2)\ (x+3).$

(8) $(2x^2-x-1)\ (x^2+2x-3)\ (2x^2+7x+3)$

(9) $(2x^2+3x-9)\ (2x^2+7x+3)\ (4x^2-4x-3).$

(10) $(x^2+ax-2a^2)\ (x^2+3ax+2a^2)\ (x^2-a^2).$

(11) $x^2+\frac{1}{x^2}-4\left(x+\frac{1}{x}\right)+6.$

Solution :

$$\left(x+\frac{1}{x}\right)^2=x^2+2.x.\frac{1}{x}+\frac{1}{x^2}$$

$$=x^2+2+\frac{1}{x^2}.$$

$$\therefore\ x^2+\frac{1}{x^2}=\left(x-\frac{1}{x}\right)^2-2.$$

$$\therefore\ \text{Exp.}=\left(x+\frac{1}{x}\right)^2-2-4\left(x+\frac{1}{x}\right)+6$$

$$=\left(x+\frac{1}{x}\right)^2-4\left(x+\frac{1}{4}\right)+4$$

$$=\left(x+\frac{1}{x}-2\right)^2$$

$\therefore$ The reqd. sq. root $=\pm\left(x+\dfrac{1}{x}-2\right)$.

(12) $x^2+\dfrac{1}{x^2}+4\left(x+\dfrac{1}{x}\right)+6.$

(13) $\left(x+\dfrac{1}{x}\right)^2-5\left(x-\dfrac{1}{x}\right)+\frac{9}{4}.$

(14) $\left(x-\dfrac{1}{x}\right)^2+6\left(x+\dfrac{1}{x}\right)+13.$

(15) $\left(x-\dfrac{1}{2x}\right)^2-14\left(x+1\dfrac{1}{2x}\right)+51$

(16) $\left(x^2-\dfrac{1}{x^2}\right)^2-4\left(x^2+\dfrac{1}{x^2}\right)+8$

35. *Common Method to find the square root.*

Solved Examples :

Find the square root of :

(1) $4x^4+12x^3+11x^2-30x+25.$

Solution :

$$
\begin{array}{r|l}
 & \pm(2x^2+3x-5) \\
2x^2 & 4x^4+12x^3-11x^2-30x+25. \\
 & 4x^4 \\
\hline
4x^2+3x & 12x^3-11x^2 \\
 & 12x^3+9x^2 \\
\hline
4x^2+6x-5 & -20x^2-30x+25 \\
 & -20x^2-30x+25 \\
\hline
 & \times
\end{array}
$$

$\therefore$ The reqd. sq. root $=\pm(2x^2+3x-5)$.

(2) $x^4 - \frac{6x^3}{y} + \frac{17x^2}{y^2} - \frac{24x}{y^3} + \frac{16}{y^4}$.

Solution :

$$
\begin{array}{l}
\qquad\qquad \pm\left(x^2 - \frac{3x}{y} + \frac{4}{y^2}\right) \\
x^2 \,\big)\, x^4 - \frac{6x^3}{y} + \frac{17x^2}{y^2} - \frac{24x}{y^3} + \frac{16}{y^4} \\
\qquad\quad x^4 \\
2x^2 - \frac{3x}{y} \,\big)\, -\frac{6x^3}{y} + \frac{17x^2}{y^2} \\
\qquad\qquad\quad \underline{-\frac{6x^3}{y} + \frac{9x^2}{y^2}} \\
2x^2 - \frac{6x}{y} + \frac{4}{y^2} \,\big)\, \frac{8x^2}{y^2} - \frac{24x}{y^3} + \frac{16}{y^4} \\
\qquad\qquad\qquad\quad \underline{\frac{8x^2}{y^2} - \frac{24x}{y^3} + \frac{16}{y^4}} \\
\qquad\qquad\qquad\qquad\qquad \times
\end{array}
$$

$\therefore$ the reqd. sq. root $= \pm\left(x^2 - \frac{3x}{y} + \frac{4}{y^2}\right)$

36. *Rule* :— (1) Arrange the expression according to the ascending or descending powers of 'x' or 'a', etc.

(2) Find the square root of the first term.

(3) Divide the first term of the expression by this square root.

(4) Take down the remainder.

(5) Multiply the first term of the required square root by 2 and divide the first term of the remainder by the product and add the quotient to twice the first term of the required

square root. Divide the remainder by this new divisor and take down the first remainder

(6) Proceed similarly till there is no remainder.

(7) The quotient is the required sq. root.

EXERCISE 22

Extract the square root of :

(1) $x^2+y^2+z^2+2xy+2yz+2zx$.

(2) $4a^2+9b^2+16c^2-12ab-24bc+16ca$.

(3) $4x^2+25y^2+1-20xy-10y+4x$.

(4) $x^4+2x^3+3x^2+2x+1$.

(5) $16a^4-24a^3-31a^2+30a+25$.

(6) $4x^4+12x^3+29x^2+30x+25$.

(7) $x^4+4x^3+10x^2+12x+9$.

(8) $4x^4+76x^3+369x^2+76x+4$.

(9) What must be subtracted from $x^4-4x^3+10x^2-14x+17$ to make it a perfect square ?

Solution :

$$
\begin{array}{r|l|l}
x^2 & x^4-4x^3+10x^2-14x+7 & x^2-2x+3 \\
 & x^4 & \\
\hline
2x^2-2x & -4x^3+10x^2 & \\
 & -4x^3+4x^2 & \\
\hline
2x^2-4x+3 & 6x^2-14x+7 & \\
 & 6x^2-12x+9 & \\
\hline
 & \quad -2x-2 &
\end{array}
$$

$\therefore -2x-2$ must be subtracted from $x^4-4x^3+10x^2-14x+7$ to make it a perfect square.

(10) Find the value of x in $4x^6+4x^5+9x^4+16x^3$ $+10x^2+13x+7$ to make it a perfect square.

(11) For what value of x will $x^4-2x^3-x^2+x-4$ be a perfect square ?

Find the square root of :

(12) $9x^4+12x^3+10x^2+4x+1$.

(13) $9x^6-12x^5+34x^4-62x^3+53x^2-70x+49$.

(14) $25x^6+30x^5+9x^4-20x^3-12x^2+4$.

(15) $x^6-2x^4+2x^3+x^2-2x+1$.

(16) $x^6-2x^5+5x^4+2x^3-2x^2+12x+9$.

(17) $\frac{a^2}{b^2}-\frac{3a}{b}+\frac{17}{4}-\frac{3b}{a}+\frac{b^2}{a^2}$.

(18) $\frac{4a^2-12ab-6bc+4ac+9b^2+c^2}{4a^2+9c^2-12ac}$

[Hint : Extract the square roots of the numerator and denominator separately. Divide the former by the latter.]

(19) $x^2+\frac{a^2}{9}-bx+\frac{b^2}{4}-\frac{ab}{3}+\frac{2ax}{3}$.

(20) $\frac{x^4}{4}+4x^2+\frac{ax^2}{3}+\frac{a^2}{9}-2x^3-\frac{4ax}{3}$.

(21) $x^4+\frac{x^2}{2}-\frac{x}{2}+\frac{1}{16}-2x^3$.

(22) $x^2-6x+5+\frac{12}{x}+\frac{4}{x^2}$.

(23) What must be added to $\frac{a^2}{b^2}+\frac{b^2}{a^2}+\frac{2a}{b}+\frac{2b}{a}$ to make it a perfect square ?

(24) What must be added to $x^4+x^3+\frac{x^2}{4}-2x+3+\frac{1}{x^2}$ to make it a perfect square ?

(25) What must be added to a^4+2a^3-a to make it a perfect square ?

CHAPTER V

FRACTIONS

37. In arithmetic $\frac{75}{105}=\frac{5\times5\times3}{5\times7\times3}=\frac{5}{7}$

So in algebra $\frac{a^3b^2}{ab}=\frac{a\times a\times a\times b\times b}{a\times b}=a\times a\times b=a^2b.$

In order to simplify an expression we have to divide its numerator and denominator by their H. C. F.

Solved Examples :

Simplify :—

(1) $\frac{49x^3y^3z^3}{7x^2y^4z^6}$

Solution : The H. C. F. of the numerator and denominator is $7x^2y^3z^3$.

$$\therefore \text{ Fraction } =\frac{49x^3y^3z^3\div7x^2y^3z^3}{7x^2y^4z^6\div7x^2y^3z^3}$$

$$=\frac{7x}{yz^3}$$

(2) $\frac{c(a-b)}{a^2-b^2}$

Solution : $\frac{a(a-b)}{a^2-b^2}=\frac{a(a-b)}{(a+b)\ (a-b)}=\frac{a}{a+b}.$

(3) $\frac{x^3-y^3}{x^4+x^2y^2+y^4}$

Solution : $\frac{x^3-y^3}{x^4+x^2y^2+y^4}=\frac{(x-y)(x^2+xy+y^2)}{(x^2+xy+y^2)\ (x^2-xy+y^2)}$

$$=\frac{x-y}{x^2-xy+y^2}$$

EXERCISE 23

Simplify the following :—

(1) $\dfrac{4a^3b^2}{16ab^2}$.

(2) $\dfrac{x^2y^2z^2}{x^3yz^3}$.

(3) $\dfrac{4x^2y^3z^3}{16xy^4z^3}$.

(4) $\dfrac{5xyz}{20x^2y^3z^2}$.

(5) $\dfrac{15a^3b^2c^4d^3}{75a^4bc^5d^3}$

(6) $\dfrac{8x^3y^2za^2}{16ab^2x^2}$.

(7) $\dfrac{10a^3b^2c^4d^2}{50a^6b^4c^3d^4}$.

(8) $-\dfrac{13x^3y^2a^4b^3}{39x^4y^2a^3b^4}$.

(9) $-\dfrac{14p^2q^2r^3z}{49p^3q^3r^3z^3}$.

(10) $-\dfrac{3a^2b^2c^2d^2}{9abcd}$.

(11) $\dfrac{ab+ac}{a^2+ab}$.

(12) $\dfrac{ab-ac}{b^2-c^2}$

(13) $\dfrac{a^2b^2(a+b)}{ab(a^2-b^2)}$.

(14) $\dfrac{(a-b)^2}{a^3-b^3}$.

(15) $\dfrac{5a(x+y)^2}{15ab(x^3+y^3)}$.

(16) $\dfrac{x^3+y^3}{(x+y)^3}$.

(17) $\dfrac{xyz(x^2-y^2)}{3x^2y^2z^2(x+y)^2}$.

(18) $\dfrac{x^3+y^3}{x^4+x^2y^2+y^4}$.

(19) $\dfrac{x^4-4a^4}{yz(x^2+2a^2)}$.

(20) $\dfrac{x^4-y^4}{x^6-y^6}$.

(21) $\dfrac{a^2b^3(a^2-b^2)}{3ab^2(a^3+b^3)}$.

(22) $\dfrac{(x+3)(x+4)}{(x+1)(x+3)}$.

(23) $\dfrac{(x+1)(x^2+x+1)}{x^3-1}$

(24) $\dfrac{(x-y)(x^2-xy+y^2)}{x^3-y^3}$.

(25) $\dfrac{(x+3)(x+4)}{5(x^2-16)}$.

(26) $\dfrac{(a+b)^2-(x+y)^2}{(a+x)^2-(b+y)^2}$.

(27) $\dfrac{(x^2-y^2)(x^3+y^3)}{(x^2+xy+y^2)(x+y)}$.

(28) $\dfrac{x^2+14x+49}{x^2-49}$.

(29) $\dfrac{x^2-3x+2}{x^2+x-6}$.

(30) $\dfrac{4x^2+16xy+15y^2}{4x^2-25y^2}$.

(31) $\dfrac{a^2+b^2-c^2+2ab}{a^2-b^2-c^2-2bc}$.

(32) $\dfrac{2a^2+3a+1}{2a^2-a-1}$.

(33) $\dfrac{4x^4+7x^2+3}{4x^4-x^2-3}$.

(34) $\dfrac{x^2y^2}{ab} \times \dfrac{a^2b^2}{x^3y^3}$.

(35) $\dfrac{5a^2x^2}{3b^2y} \times \dfrac{9b^3y^2}{15ax}$.

(36) $\dfrac{2a^2b^2c^2}{x^2y^2z^2} \times \dfrac{5xyz}{4abc}$.

(37) $\dfrac{xy}{yz} \div \dfrac{x^2y^2}{y^2z^2}$.

(38) $\dfrac{2x}{3y} \div \dfrac{a}{b} \times \dfrac{3a}{2b}$.

(39) $\dfrac{a^2}{bc} \times \dfrac{c^2}{ab} \div \dfrac{c}{a^2b^2}$.

(40) $\dfrac{2x^2}{y^2} \times \dfrac{y^4}{2x} \times \dfrac{x}{3x^2}$.

(41) $\dfrac{x+1}{x-1} \times \dfrac{x-1}{x+2} \times \dfrac{x+2}{x+1}$.

(42) $\dfrac{x+y}{x^2-y^2} \div \dfrac{x+y}{x-y}$.

(43) $\dfrac{x+1}{x^3-1} \div \dfrac{x^2+2x+1}{x^2-1}$.

(44) $\dfrac{x+3}{x+5} \times \dfrac{x^2-5x+6}{x^2-25} \div \dfrac{x^2-9}{x^2-10x+25}$.

(45) $\dfrac{x^3+y^3+z^3-3xyz}{x^2+y^2+z^2-xy-yz-zx}$.

(46) $\dfrac{a^2-(b-c)^2}{(a-b)^2-c^2} \times \dfrac{(c-a)^2-b^2}{(a+b)^2-c^2} \div \dfrac{a-b-c}{a-b+c}$.

(47) $\dfrac{a^4+a^2b^2+b^4}{a^3-b^3}\times\dfrac{a^2+ab+b^2}{a^3+b^3}\div\dfrac{a+b}{a-b}$

(48) $\dfrac{a^6+b^6}{a^6-b^6}\div\dfrac{a^4-a^2b^2+b^4}{a^4+a^2b^2+b^4}\times\dfrac{a+b}{a-b}$.

(49) $\dfrac{27a^3-8b^3}{81a^4-64b^4}\div\dfrac{9a^2+6ab+4b^2}{9a^2-8b^2}\times\dfrac{9a^2-8b^2}{3a-2b}$.

(50) $\dfrac{x^3+y^3+1-3xy}{x^6+y^6}\times\dfrac{x^2+y^2}{x^2+y^2+1-xy-y-x}\div\dfrac{x+y+1}{x^4-x^2y^2+y^4}$

(51) $\dfrac{x^3+8}{x^2-x-6}\times\dfrac{x^2-5x+6}{x^4-4x^2+16}\div\dfrac{x^2-2x}{x^3+2x^2+4x}$.

(52) $\dfrac{x^3-1}{x^2-1}\times\dfrac{x^2+3x+2}{x^4+x^2+1}\div\dfrac{x^2+x-2}{x^4+x}$.

(53) $\dfrac{x^3-8y^3}{x^2-xy}\times\dfrac{(x-y)^2+xy}{(x+2y)^2-2xy}\div\dfrac{x^3+y^3}{x^3-xy^2}$.

(54) $\dfrac{37{\cdot}2\times37{\cdot}2\times37{\cdot}2\times1000000}{4\times4\times4\times(12{\cdot}4)^3}$.

(55) $\dfrac{x^3-2x^2-x+2}{x^3-x^2-4x+4}$

Solution :

$$
\begin{array}{r|l|l|l}
 & x^3-x^2-4x+4 & x^3-2x^2-x+2 & 1 \\
-x & x^3-3x^2+2x & x^3-x^2-4x+4 & \\
\hline
-2 & 2x^2-6x+4 & -x^2+3x-2 & \\
 & 2x^2-6x+4 & & \\
\hline
 & \times & &
\end{array}
$$

$\therefore$ H. C. F. $=x^2-3x+2$

$\therefore$ Fr. $=\dfrac{x(x^2-3x+2)+1(x^2-3x+2)}{x(x^2-3x+2)+2(x^2-3x+2)}$.

$=\dfrac{(x+1)\,(x^2-3x+2)}{(x+2)\,(x^2-3x+2)}=\dfrac{x+1}{x+2}$.

(56) $\dfrac{x^4+x^2+25}{x^4-9x^2+30x-25}$. (57) $\dfrac{7-10x-11x^2-6x^3}{14+x+4x^2-3x^3}$.

(58) $\dfrac{x^3-7x-6}{x^3+x^2-22x-40}$. (59) $\dfrac{6x^3-11x^2+5x-3}{9x^3-9x^2+5x-2}$.

(60) $\dfrac{20x^4+x^3-1}{25x^4+5x^3-x-1}$. (61) $\dfrac{3x^3-27ax^2+78a^2x-72a^3}{2x^3+10ax^2-4a^2x-48a^3}$.

(62) $\dfrac{2x^4-x^3-9x^2+13x-5}{7x^3-19x^2+17x-5}$.

(63) $\dfrac{7x^3-2x^2y-63xy^2+18y^3}{5x^4-3x^3y-43x^2y^2+27xy^3-18y^4}$.

(64) $\dfrac{x^3-2x^2-x+2}{x^3-x^2-4x+4}$. (65) $\dfrac{x^4-5x^3+x^2+21x-18}{x^3-2x^2-5x+6}$.

38. *Reduction to the lowest denominator* :

Solved Examples:

Reduce the following to the lowest denominator :

(1) $\dfrac{x}{ab}$, $\dfrac{y}{bc}$, $\dfrac{z}{ca}$

Solution : The L. C. M. of the denominators is abc.

$$\therefore \frac{x}{ab}=\frac{cx}{abc}\ ;\ \frac{y}{bc}=\frac{ay}{abc}\ ;\ \frac{z}{ca}=\frac{bz}{abc}.$$

(ii) $\dfrac{a}{a-b}$, $\dfrac{b}{a+b}$, $\dfrac{c}{a^2-b^2}$

Solution : The L. C. M. of the denominators is a^2-b^2.

$$\therefore \frac{a}{a-b}=\frac{a(a+b)}{a^2-b^2}\ ;\ \frac{b}{a+b}=\frac{b(a-b)}{a^2-b^2}$$

$$\frac{c}{a^2-b^2}=\frac{c}{a^2-b^2}$$

(iii) $\dfrac{x+3}{x^2-x-12}, \dfrac{x+2}{x^2-x-6}, \dfrac{x+4}{x^2+7x+12}$

Solution : $x^2-x-12=(x-4)(x+3)$

$x^2-x-6=(x-3)(x+2)$

$x^2+7x+12=(x+3)(x+4)$

The fractions are :

$$\frac{x+3}{(x-4)(x+3)}, \quad \frac{x+2}{(x-3)(x+2)}, \quad \frac{x+4}{(x+3)(x+4)}.$$

$$= \frac{1}{x-4}, \quad \frac{1}{x-3}, \quad \frac{1}{x+3}.$$

The L. C. M. of the fractions $=(x-4)(x-3)(x+3)$

Hence

$$\frac{x+3}{x^2-x+12}=\frac{1}{x-4}=\frac{(x-3)(x+3)}{(x-4)(x-3)(x+3)}$$

$$\frac{x+2}{x^2-x-6}=\frac{1}{x-3}=\frac{(x-4)(x+3)}{(x-4)(x-3)(x+3)}$$

$$\frac{x+4}{x^2+7x+12}=\frac{1}{x+3}=\frac{(x-4)(x-3)}{(x-4)(x-3)(x+3)}$$

EXERCISE 24

Reduce to the lowest denominator:—

(1) $\dfrac{a}{5}, \dfrac{b}{6}, \dfrac{c}{8}.$

(2) $\dfrac{a}{ab}, \dfrac{b}{bc}, \dfrac{c}{ca}.$

(3) $\dfrac{a}{x+1}, \dfrac{b}{x-1}.$

(4) $\dfrac{1}{x^2+3x+2}, \dfrac{1}{x^2-7x-18}.$

(5) $\dfrac{5}{4(5y-4x)}, \dfrac{4}{5(5y+4x)}.$

(6) $\dfrac{ab}{5ab+6b^2}, \dfrac{ab}{5a^2-6ab}.$

(7) $\dfrac{a-b}{a+b}, \dfrac{a+b}{a-b}.$

(8) $\dfrac{x+5}{x^2-x-12}, \dfrac{x-4}{x^2+x-20}, \dfrac{x+3}{x^2+8x+15}.$

(9) $\dfrac{x-a}{x+a}$, $\dfrac{x+a}{x-a}$, $\dfrac{1}{x^2+a^2}$, $\dfrac{1}{x^2-a^2}$

(10) $\dfrac{1}{(1+x)^2}$, $\dfrac{2}{1-x^2}$, $\dfrac{1}{(1-x)^2}$.

39. *Additicn and subtraction of fractions.*

Solved Examples:—

Simplify :

(1) $\dfrac{x}{2}+\dfrac{y}{2}$.

Solution : $\dfrac{x}{2}+\dfrac{y}{2}=\dfrac{x+y}{2}$.

(ii) $\dfrac{x}{3}+\dfrac{y}{4}$.

Solution : $\dfrac{x}{3}+\dfrac{y}{4}=\dfrac{4x+3y}{12}$.

(iii) $\dfrac{x}{y}+\dfrac{y}{z}$.

Solution : $\dfrac{x}{y}+\dfrac{y}{z}=\dfrac{zx+y^2}{yz}$.

(iv) $\dfrac{3}{x+2}+\dfrac{2}{x+3}$.

Solution : $\dfrac{3}{x+2}+\dfrac{2}{x+3}=\dfrac{3(x+3)+2(x+2)}{(x+2)(x+3)}$

$$=\frac{5x+13}{(x+2)(x+3)}.$$

(v) $\dfrac{a+b}{a-b}-\dfrac{a-b}{a+b}$.

Solution : $\frac{a+b}{a-b}-\frac{a-b}{a+b}=\frac{(a+b)^2-(a-b)^2}{a^2-b^2}$

$$=\frac{(a+b+a-b)(a+b-a+b)}{a^2-b^2}$$

$$=\frac{2a\times 2b}{a^2-b}$$

$$=\frac{4ab}{a^2-b^2}$$

EXERCISE 25

Simplify :—

(1) $\frac{1}{2x}+\frac{1}{x}+\frac{1}{3x}$. (2) $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$.

(3) $\frac{a}{bc}+\frac{b}{ac}+\frac{c}{ab}$. (4) $\frac{2x}{a}-\frac{3y}{a}+\frac{4z}{a}$.

(5) $\frac{3x}{5y}+\frac{4x}{7z}+\frac{5z}{9z}$. (6) $\frac{x}{yz}+\frac{y}{xz}+\frac{z}{xy}$.

(7) $\frac{x}{a}+\frac{a}{x}+\frac{1}{2x^2}$. (8) $\frac{a}{2bx}+\frac{b}{cx^2}+\frac{c}{3ax}$.

(9) $\frac{2x-1}{3}-\frac{4x-8}{6}$. (10) $\frac{x+1}{2}+\frac{x+2}{3}+\frac{x-4}{4}$

(11) $\frac{2x+1}{3x}-\frac{3x+2}{5x}+\frac{1}{7}$. (12) $\frac{y-2z}{2y}-\frac{y-5z}{4y}+\frac{y+yz}{8y}$.

(13) $\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}-3$. (14) $\frac{a-b}{ab}-\frac{a-c}{ac}+\frac{b-c}{bc}$

(15) $\frac{1}{a+b}+\frac{1}{a-b}$. (16) $\frac{a}{a-b}+\frac{b}{a-b}$.

(17) $\dfrac{x+4}{x+5}-\dfrac{x+2}{x+3}$. (18) $\dfrac{y^2}{y-y^3}-\dfrac{y}{1+y^2}$.

(19) $\dfrac{3}{x^2-4}+\dfrac{1}{(x-2)^2}$ (20) $\dfrac{4}{x-4}-\dfrac{16-3x}{x^2-16}$.

(21) $\dfrac{1}{a(a-b)}+\dfrac{1}{b(a+b)}$. (22) $\dfrac{x}{x^2-a^2}+\dfrac{a}{x^2-x^2}$.

Solution : $\dfrac{x}{x^2-a^2}+\dfrac{a}{a^2-x^2}=\dfrac{x}{x^2-a^2}-\dfrac{a}{x^2-a^2}$.

$$=\frac{x-a}{x^2-a^2}=\frac{x-a}{(x-a)(x+a)}=\frac{1}{x+a}.$$

(23) $\dfrac{2x^3}{x^2-y^2}-\dfrac{2x^2}{x^2+y^2}$. (24) $\dfrac{x-y}{xy}+\dfrac{y-z}{yz}+\dfrac{z-x}{zx}$.

(25) $\dfrac{2x-y}{xy}-\dfrac{y-2x}{yz}$. (26) $\dfrac{x+a}{x-a}+\dfrac{x^2-a^2}{ax-a^2}$.

(27) $\dfrac{a+3b}{a-2b}-\dfrac{2a+6b}{2a+5b}$. (28) $\dfrac{a+b}{a-b}+\dfrac{4ab}{a^2-b^2}+\dfrac{a-b}{a+b}$.

(29) $\dfrac{x+y}{x-y}-\dfrac{x-y}{x+y}+\dfrac{x^2y^2}{x^2-y^2}$. (30) $\dfrac{2x-7}{(x-3)^2}-\dfrac{2(x+2)}{x^2-9}$.

(31) $\dfrac{a+b}{a-b}-\dfrac{a-b}{a+b}-\dfrac{4ab}{a^2-b^2}$. (32) $\dfrac{4x^2+9y^2}{4x^2-9y^2}\ \dfrac{2x-3y}{2x+3y}$.

(33) $\dfrac{x^2}{x^2-1}-\dfrac{x+1}{x-1}+\dfrac{x-1}{x+1}$.

(34) $\dfrac{3(x-1)}{x+1}-\dfrac{5(x+1)}{x-1}+\dfrac{2(x^2+1}{x^2-1}$.

(35) $\dfrac{a^2+ac}{a^2c-c^3}-\dfrac{a-c}{c(a+c)}-\dfrac{2c}{a^2-c^2}$

(36) $\dfrac{b}{a+b}-\dfrac{a+b}{2b}+\dfrac{a^2+b^2}{2b(a+b)}$.

(37) $\dfrac{1+x^2}{1-x^2}+\dfrac{4x^2}{1+x^4}-\dfrac{1-x^2}{1+x^2}$.

(38) $\dfrac{a+2}{a}-\dfrac{a}{a+2}-\dfrac{a^3-2a^2}{2a^2-8}$.

(39) $\dfrac{1}{x^4+2x^3}+\dfrac{1}{x^4-2x^3}+\dfrac{2}{x^4+4x^2}$.

(40) $\dfrac{1+x^2}{1-x^2}-\dfrac{4x^2}{1-x^4}-\dfrac{1-x^2}{1+x^2}$.

(41) $\dfrac{1}{x^2-9}-\dfrac{1}{x^2+x-6}$.

Solution : $\dfrac{1}{x^2-9}-\dfrac{1}{x^2+x-6}=\dfrac{1}{(x+3)(x-3)}-\dfrac{1}{(x+3)(x-2)}$

$$=\frac{x-2-x+3}{(x+3)(x-3)(x-2)}$$

$$=\frac{1}{(x+3)(x-3)(x-2)}$$

$$=\frac{1}{(x^2-9)(x-2)}.$$

(42) $\dfrac{2}{(x-3)(x-2)}-\dfrac{1}{(x-2)(x+3)}$.

(43) $\dfrac{1}{(a-b)(a-c)}+\dfrac{1}{(a-c)(b-c)}$.

(44) $\dfrac{1}{x^2+7x+10}+\dfrac{1}{x^2+13x+40}$.

(45) $\dfrac{2}{x^2-4x+3}-\dfrac{1}{x^2-3x+2}$.

(46) $\dfrac{1}{2x^2-3ax-2a^2}-\dfrac{1}{2x^2-5ax+2a^2}$.

(47) $\dfrac{7}{x^2+13x+30}+\dfrac{1}{x^2+5x+6}$.

(48) $\dfrac{8}{x^2+10x+9}+\dfrac{5}{x^2-3x-4}$.

(49) $\dfrac{1}{x^2+7x+10}-\dfrac{1}{x^2+13x+40}$.

(50) $\dfrac{x-2}{x^2-x-2}+\dfrac{x-4}{x^2-5x-4}$.

(51) $\dfrac{x+4}{x^2-3x-28}-\dfrac{x-5}{x^2+2x-35}$.

(52) $\dfrac{1}{x^2+x+1}+\dfrac{2x}{x^4+x^2+1}$.

(53) $\dfrac{1}{(x-1)\,(x-3)}+\dfrac{2}{(3-x)\,(x-5)}-\dfrac{3}{(5-x)\,(x-1)}$.

Solution : Exp. $=\dfrac{1}{(x-1)\,(x-3)}-\dfrac{2}{(x-3)\,(x-5)}+\dfrac{3}{(x-5)\,(x-1)}$

$$=\frac{x-5-2x+2+3x-9}{(x-1)(x-3)(x-5}$$

$$=\frac{2x-12}{(x-1)(x-3)(x-5)}$$

$$=\frac{2(x-6)}{(x-1)(x-3)(x-5)}.$$

(54) $\dfrac{1}{(x-2)(x-3)}+\dfrac{1}{(x-3)(x-1)}+\dfrac{1}{(x-1)(x-2)}$.

(55) $\frac{1}{(x-4)(x+5)}-\frac{2}{(x-3)(x+5)}+\frac{1}{(x-3)(x-4)}$.

(56) $\frac{1}{x^2-8x+15}+\frac{1}{x^2-4x+3}-\frac{2}{x^2-6x+5}$.

(57) $\frac{2}{x^2-1}+\frac{3}{x^2+x-2}-\frac{2}{x^2+3x+2}$.

(58) $\frac{1}{x^2-7x+12}-\frac{3}{x^2-x-6}+\frac{2}{x^2-2x-8}$.

(59) $\frac{2}{a^2-ab}+\frac{3}{ab(a+b)}-\frac{3a-2b}{ab(a+b)}$.

(60) $\frac{2(x-3)}{(x-4)(x-5)}-\frac{x-1}{(x-3)(x-4)}-\frac{x-2}{(x-5)(x-3)}$.

(61) $\frac{1}{1+x+x^2}-\frac{1}{1-x+x^2}+\frac{2x}{1-x^2+x^4}$.

(62) $\frac{x+27}{x+6x^2-15}-\frac{2x+3}{4x^2+3-8x}+\frac{x+19}{5-7x-6x^2}$.

(63) $\frac{a+x}{a^2+ax+x^2}+\frac{a-x}{a^2-ax+x^2}+\frac{2x^3}{x^4+x^2a^2+a^4}$.

(64) $\frac{1}{x^3-x^2+x-1}+\frac{3}{2x^2-x-1}+\frac{1-3x}{2x^3+x^2+2x+1}$.

(65) $\frac{3}{x^3-3x+2}-\frac{1}{x^3+2x^2-x-2}-\frac{1}{x^3+x^2-x-1}$.

(66) $\frac{1}{x}+\frac{1}{x-1}+\frac{1}{x+1}-\frac{x}{x^2-1}+\frac{3}{x(x^2-1)}$.

Solution :

$$\text{Fr.}=\frac{1}{x}+\frac{x+1+x-1}{(x-1)(x+1)}-\frac{x}{x^2-1}+\frac{3}{x(x^2-1)}$$

$$=\frac{1}{x}+\frac{2x}{x^2-1}-\frac{x}{x^2-1}+\frac{3}{x(x^2-1)}$$

$$=\frac{1}{x}+\frac{2x-x}{x^2-1}+\frac{3}{x(x^2-1)}$$

$$=\frac{1}{x}+\frac{x}{x^2-1}+\frac{3}{x(x^2-1)}$$

$$=\frac{x^2-1+x^2+3}{x(x^2-1)}$$

$$=\frac{2x^2+2}{x(x^2-1)}=\frac{2(x^2+1)}{x(x^2-1)}.$$

(67) $\dfrac{1}{x-2y}+\dfrac{1}{x+2y}+\dfrac{2x}{x^2+4y^2}+\dfrac{4x^3}{x^4+16y^4}.$

(68) $\dfrac{1}{(x+1)^2(x+2)^2}-\dfrac{1}{(x+1)^2}+\dfrac{2}{x+1}-\dfrac{2}{x+2}.$

(69) $\dfrac{2}{x-a}+\dfrac{2}{x+a}-\dfrac{4x}{x^2+a^2}+\dfrac{8x^3}{x^4-a^4}.$

(70) $\dfrac{1}{x+a}+\dfrac{1}{x-a}+\dfrac{2x}{x^2+a^2}+\dfrac{4x^3}{x^4+a^4}.$

(71) $\dfrac{1+x}{1-x}+\dfrac{4x}{1+x^2}+\dfrac{8x}{1+x^4}-\dfrac{1-x}{1+x}-\dfrac{16x}{1-x^8}.$

(72) $\dfrac{1}{(a-b)(a-c)}+\dfrac{1}{(b-a)(b-c)}+\dfrac{1}{(c-a)(c-b)}$

Solution : Fr. $=-\dfrac{1}{(a-b)(c-a)}-\dfrac{1}{(a-b)(b-c)}$

$$-\frac{1}{(c-a)(b-c)}$$

$$=-\left[\frac{1}{(a-b)(c-a)}+\frac{1}{(a-b)(b-c)}+\frac{1}{(c-a)(b-c)}\right]$$

$$=-\left[\frac{b-c+c-a+a-b}{(a-b)(b-c)(c-a)}\right]$$

$$=-\left[\frac{0}{(a-b)(b-c)(c-a)}\right]$$

$$=0.$$

(73) $\dfrac{ax}{(a-b)(a-c)}+\dfrac{bx}{(b-a)(b-c)}+\dfrac{cx}{(c-a)(c-b)}$.

(74) $\dfrac{a+x}{(a-b)(a-c)}+\dfrac{b+x}{(b-c)(b-a)}+\dfrac{c+x}{(c-a)(c-b)}$.

(75) $\dfrac{a+1}{(a-b)(c-a)}+\dfrac{b+1}{(b-c)(a-b)}+\dfrac{c+1}{(b-c)(c-a)}$.

(76) $\dfrac{a^2(b-c)}{(a+b)(a+c)}+\dfrac{b^2(c-a)}{(b+c)(b+a)}+\dfrac{c^2(a-b)}{(c+a)(c+b)}$.

(77) $\dfrac{p+q}{(q-r)(r-p)}+\dfrac{q+r}{(r-p)(p-q)}+\dfrac{r+p}{(p-q)(q-r)}$.

(78) $\dfrac{a^2(b+c)}{(a-b)(a-c)}+\dfrac{b^2(c+a)}{(b-a)(b-c)}+\dfrac{c^2(a+b)}{(c-a)(c-b)}$.

(79) $\dfrac{a^2}{(a-b)(a-c)}+\dfrac{b^2}{(b-a)(b-c)}+\dfrac{c^2}{(c-a)(c-b)}$.

Solution : Fr. $=-\left[\dfrac{a^2}{(a-b)(c-a)}+\dfrac{b^2}{(a-b)(b-c)}\right.$

$$\left.+\frac{c^2}{(c-a)(b-c)}\right]$$

$$=-\left[\frac{a^2(b-c)+b^2(c-a)+c^2(a-b)}{(a-b)(b-c)(c-a)}\right]$$

$$=-\left[\frac{-(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)}\right]$$

$$=1.$$

(80) $\dfrac{bc}{(b-a)(c-a)}+\dfrac{ca}{(c-b)(a-b)}+\dfrac{ab}{(a-c)(b-c)}$.

(81) $\dfrac{a(b+c)}{(a-b)(a-c)}+\dfrac{b(c+a)}{(b-c)(b-a)}+\dfrac{c(a+b)}{(c-a)(c-b)}$.

(82) $\dfrac{1}{a(a-b)(a-c)}+\dfrac{1}{b(b-c)(b-a)}+\dfrac{1}{c(c-a)(c-b)}$

(83) $\dfrac{a^3}{(a-b)(a-c)}+\dfrac{b^3}{(b-a)(b-c)}+\dfrac{c^3}{(c-a)(c-b)}$.

[Hint : Remember that $a^3(b-c)+b^3(c-a)+c^3(a-b)$
$=-(a-b)(b-c)(c-a)(a+b+c)$

[See solved Example 2 § 25 cyclic order.]

(84) $\dfrac{ab+ac}{(a-b)(a-c)}+\dfrac{bc+ba}{(b-c)(b-a)}+\dfrac{ca+bc}{(c-a)(c-b)}$.

(85) $\dfrac{bc(x-a)}{(a-b)(a-c)}+\dfrac{ca(x-b)}{(b-a)(b-c)}+\dfrac{ab(x-c)}{(c-a)(c-b)}$.

(86) $\dfrac{a^2+a+1}{(a-b)(a-c)}+\dfrac{b^2+b+1}{(b-c)(b-a)}+\dfrac{c^2+c+1}{(c-a)(c-b)}$.

(87) $\dfrac{b^2+c^2-2a^2}{(a-b)(a-c)}+\dfrac{c^2+a^2-2b^2}{(b-c)(b-a)}+\dfrac{a^2+b^2-2c^2}{(c-a)(c-b)}$.

(88) $\dfrac{(a+1)^2}{(a-b)(a-c)}+\dfrac{(b+1)^2}{(b-c)(b-a)}+\dfrac{(c+1)^3}{(c-a)(c-b)}$.

(89) $\dfrac{1}{(a-b)(a-c)(x-a)}+\dfrac{1}{(b-c)(b-a)(x-b)}$
$+\dfrac{1}{(c-a)(c-b)(x-c)}$

(90) $\dfrac{a^2}{(a-b)(a-c)(x+a)}+\dfrac{b^2}{(b-c)(b-a)(x+b)}$
$+\dfrac{c^2}{(c-a)(c-b)(x+c)}$.

(91) $\dfrac{1}{a^2+2bc-b^2-c^2}+\dfrac{1}{b^2+2ca-c^2-a^2}$

$$+\frac{1}{c^2+2ab-a^2-b^2}.$$

Solution :

$$\begin{aligned} a^2+2bc-b^2-c^2 &= a^2-(b^2-2bc+c^2) \\ &= a^2-(b-c)^2 \\ &= (a+b-c)(a-b+c). \\ b^2+2ca-c^2-a^2 &= b^2-(a^2-2ac+c^2) \\ &= b^2-(a-c)^2 \\ &= (b+a-c)\,(b-a+c). \\ c^2+2ab-a^2-b^2 &= c^2-(a^2-2ab+b^2) \\ &= c^2-(a-b)^2 \\ &= (c+a-b)(c-a+b). \end{aligned}$$

$$\therefore \text{Fr.} = \frac{1}{(a+b-c)(a-b+c)}+\frac{1}{(a+b-c)(b+c-a)}$$

$$+\frac{1}{(a+c-b)(b+c-a)}.$$

$$=\frac{b+c-a+a-b+c+a+b-c}{(a+b-c)(a-b+c)(b+c-a)}$$

$$=\frac{a+b+c}{(a+b-c)(a-b+c)(b+c-a)}.$$

(92) $\dfrac{(2x-3y)^2-x^2}{4x^2-(3y+x)^2}+\dfrac{4x^2-(3y-x)^2}{9(x^2-y^2)}+\dfrac{9y^2-x^2}{(2x+3y)^2-x^2}$

(93) $\dfrac{9x^2-(y-z)^2}{(3x+z)^2-y^2}+\dfrac{y^2-(z-3x)^2}{(3x+y)^2-z^2}+\dfrac{z^2-(3x-y)^2}{(y+z)^2-9x^2}.$

(94) $\dfrac{x^2-(y-z)^2}{(x+z)^2-y^2}+\dfrac{y^2-(z-x)^2}{(x+y)^2-z^2}+\dfrac{z^2-(x-y)^2}{(y+z)^2-x^2}.$

(95) $\dfrac{a^2-(b-c)^2}{(c+a)^2-b^2}+\dfrac{b^2-(c-a)^2}{(a+b)^2-c^2}+\dfrac{c^2-(a-b)^2}{(b+c)^2-a^2}.$

FRACTIONS—*(contd.)*

40. *Solved Examples* :

Simplify :

(1) $$\frac{\dfrac{1}{a+b}+\dfrac{1}{a-b}}{\dfrac{1}{a+b}-\dfrac{1}{a-b}}$$

Solution : $$\text{Fr.}=\frac{\dfrac{a-b+a+b}{a^2-b^2}}{\dfrac{a-b-a-b}{a^2-b^2}}$$

$$=\frac{\dfrac{2a}{a^2-b^2}}{\dfrac{-2b}{a^2-b^2}}$$

$$=-\frac{a}{b}.$$

(2) $$\frac{\dfrac{x+y}{x-y}-\dfrac{x-y}{x+y}}{\dfrac{x+y}{x-y}+\dfrac{x-y}{x+y}}.$$

Solution : $$\text{Fr}=\frac{\dfrac{(x+y)^2-(x-y)^2}{x^2-y^2}}{\dfrac{(x+y)^2+(x-y)^2}{x^2-y^2}}$$

$$=\frac{4xy}{2(x^2+y^2)}$$

$$=\frac{2xy}{x^2+y^2}.$$

(3) $1+\dfrac{1}{1+\dfrac{1}{x}}$

Solution : Fr. $=1+\dfrac{1}{\dfrac{x+1}{x}}$

$$=1+\frac{x}{1+x}.$$

$$=\frac{1+x+x}{1+x}.$$

$$=\frac{2x+1}{x+1}.$$

EXERCISE 26

Simplify :

(1) $\dfrac{\dfrac{1}{1+x}}{1-\dfrac{1}{1+x}}$.

(2) $\dfrac{\dfrac{a}{a+b}+\dfrac{b}{a-b}}{\dfrac{a}{a-b}-\dfrac{b}{a+b}}$.

(3) $\dfrac{\dfrac{1}{a}}{1-\dfrac{1}{a}}$.

(4) $\dfrac{1}{1+\dfrac{a}{b}}+\dfrac{1}{1+\dfrac{b}{a}}$

(5) $\dfrac{1-\left(\dfrac{x-y}{x+y}\right)^2}{1+\left(\dfrac{x-y}{x+y}\right)^2}$

(6) $\dfrac{\dfrac{c}{a+b}-\dfrac{a}{b+c}}{\dfrac{a}{b+c}-\dfrac{b}{c+a}}$

(7) $a+\dfrac{1}{a+\dfrac{1}{a+\dfrac{1}{a}}}$

(8) $x-\dfrac{1}{x+\dfrac{1}{x-\dfrac{1}{x}}}$

(9) $\dfrac{x}{x-a}-\dfrac{x}{x+a}-\dfrac{\dfrac{x+a}{x-a}-\dfrac{x-a}{x+a}}{\dfrac{x+a}{x-a}+\dfrac{x-a}{x+a}}$

(10) $\dfrac{\dfrac{x^2+y^2}{y}-x}{\dfrac{1}{y}-\dfrac{1}{x}}\times\dfrac{x^2-y^2}{x^3+y^3}$

(11) $\dfrac{\dfrac{a}{a-b}-\dfrac{a}{a+b}}{\dfrac{b}{a-b}-\dfrac{b}{a+b}}\div\dfrac{\dfrac{a+b}{a-b}+\dfrac{a-b}{a+b}}{\dfrac{a+b}{a-b}-\dfrac{a-b}{a+b}}\times\dfrac{a^2}{a^2+b^2}$

(12) $\dfrac{1+\dfrac{a-b}{a+b}}{1-\dfrac{a-b}{a+b}}\div\dfrac{1+\dfrac{a^2-b^2}{a^2+b^2}}{1-\dfrac{a^2-b^2}{a^2+b^2}}$

(13) $\dfrac{\dfrac{1}{a}+\dfrac{1}{b+c}}{\dfrac{1}{a}-\dfrac{1}{b+c}}\left\{1+\dfrac{b^2+c^2-a^2}{2bc}\right\}$

(14) $\dfrac{a}{b-\dfrac{a}{b-\dfrac{a}{b}}}.$

(15) $\frac{3}{4}-\dfrac{3}{4+\dfrac{1}{x+6}}.$

(16) $\dfrac{\dfrac{a^3}{b^3}-\dfrac{b^3}{a^3}}{\dfrac{a^2}{b^2}+1+\dfrac{b^2}{a^2}}\div\dfrac{\dfrac{a}{b}-\dfrac{b}{a}}{\dfrac{1}{b}+\dfrac{1}{a}}$

17) $\left(\dfrac{x+y}{x-y}-\dfrac{x-y}{x+y}\right)\div\left\{\dfrac{1}{\dfrac{x}{y}-\dfrac{y}{x}}-\dfrac{1}{\dfrac{x}{y}+\dfrac{y}{x}}\right\}.$

(18) $$\frac{\dfrac{x^3-y^3}{x-y}-\dfrac{x^3+y^3}{x+y}}{\dfrac{x+y}{x-y}-\dfrac{x-y}{x+y}}.$$

(19) $$\frac{\dfrac{x+y}{x-y}+\dfrac{x^2+y^2}{x^2-y^2}}{\dfrac{x-y}{x+y}-\dfrac{x^3-y^3}{x^3+y^3}}\div\frac{x^4+x^2y^2+y^4}{(y-x)^2}$$

(20) $$\frac{\dfrac{x+y}{x-y}-\dfrac{x^2+y^2}{x^2-y^2}}{\dfrac{x+y}{x-y}+\dfrac{x^2+y^2}{x^2-y^2}}\div\frac{\dfrac{1}{y}-\dfrac{1}{x}}{x^3-y^3}$$

Miscellaneous Exercises

II

Resolve into factors :—

(1) $81x^8-7x^4y^4+y^8$.

(2) $ab(x^2+1)-x(a^2+b^2)$.

(3) $x^4+x^3y+xz^3+yz^3$.

(4) $(2a-3b)^3+(3b-5c)^3+(5c-2a)^3$.

(5) $(x+4)(x+3)(2x+9)(2x+7)-2$.

(6) $x(y^3-z^3)+y(z^3-x^3)+z(x^3-y^3)$.

(7) xy^5-yx^5.

(8) $81-x^4$.

(9) $32(x+y)^2-2x-2y$.

(10) $a^3+b^3+c^3-3abc$.

(11) a^6-b^6.

(12) $a(b+c)^2-b(c+a)^2$.

(13) $1-8x^3+y^3+6xy$.

(14) $x^{12}+x^6-2$.

(15) $yz(y^2-z^2)+zx(z^2-x^2)+xy(x^2-y^2)$.

(16) $a^4+2a^3b-2ab^3-b^4$.

(17) $x(x-1)(x-2)-3x+3$

(18) $x^2+3y^2-1-4xy+2y$

(19) $(x-1)^3+9(x^2-1)$.

(20) $(2b-a)^3+(2a-b)^3-(a+b)^3$.

Find the H. C. F. of :—

(21) $6x^3+7x^2-29x+12$. $12x^3-22x^2+23x-20$

(22) $2x^4-2x^3+x^2+3x-6$, $4x^4-2x^3+3x-9$.

(23) $3x^3+10x^2+7x-2$, $3x^3+13x^2+17x+6$.

(24) $8x^5-8x^4+16x^3-10x^2+4x+1$, $6x^4-x^2+x+1$.

(25) $12x^2+7xy-10y^2$, $15x^2+2xy-8y^2$, $15x^2+5xy-10y^2$

(26) x^3-x^2-x+1, $3x^2-2x-1$, x^3-x^2+x-1.

(27) a^3+4a^2-5, a^3-3a+2, a^3+4a^2-8a+3

(28) $x^3+6x^2+11x+6$, $x^3+9x^2+27x+27$.

(29) $2x^5-11x^2-9$ and $4x^5+11x^4+81$

(30) $4x^3-8x^2+3x-6$, $12x^3+4x^2+9x+3$.

Find the L. C. M. of :—

(31) x^2-x-6, x^2-4x+3.

(32) x^2-1, x^2+1, $(x-1)^2$, $(x+1)^2$, x^3-1, x^3+1.

(33) a^2-ab, b^2+ab, a^2-b^2, a^3-b^3, a^3+b^3.

(34) $a^2+2ab+b^2-c^2$, $a^2+2ac-b^2+c^2$.

(35) x^2-1, x^3-1, x^3+1, x^4+x^2+1.

(36) $a^4-x^4, a^3-a^2x-ax^2+x^3$.

(37) $3x^2-10ax+7a^2$, $x^3-5ax^2+7a^2x-3a^3$.

(38) $6x^3+7x^2-9x+2$, $8x^4+6x^3-15x^2+9x-2$.

(39) The H. C. F. of two expressions is x^2+3x+2 and their L. C. M. is $x^4+5x^3-7x^2-41x-30$. One of them is $x^3+8x^2+17x+10$. Find the other.

(40) The product of two quadratic expressions is $(x+1)^2(x+3)(x-5)$ and their H. C. F. is $x+1$ Find the expressions.

(41) The product of two expressions of fourth degree is $(12x^2-35x-33)^4$ and their L. C M. is $(12x^2-35x-33)^3$. Find the expressions and their H. C. F.

Find the H. C F and L. C. M. of :—

(42) $4x^3-x^2-4x+1$, $3x^3-3x^2+x-1$.

(43) x^4+x, x^4-x^2, x^5-x^2, x^5+x^3+x

(44) $a^3+p^3+a^2p+ap^2$, $3a^3+3a^2p-5ap^2-5p^3$.

(45) If the H. C. F. of two expressions x and y is h and their L. C. M is l, and $h+l=x+y$, prove that $h^3+l^3=x^3+y^3$.

Find the square root of :—

(46) $\dfrac{x^4}{4}+4x^2+\dfrac{ax^2}{3}+\dfrac{a^2}{9}-2x^3-\dfrac{4ax}{3}$.

(47) $\dfrac{x^4}{4}-\dfrac{2x^3}{3}-\dfrac{11x^2}{36}+x+\dfrac{9}{16}$.

(48) $x^4-2x^2-\dfrac{2}{x^2}+\dfrac{1}{x^4}+3$.

(49) $x^6+\dfrac{1}{x^6}-4x^4+4\left(x^2-\dfrac{1}{x^2}\right)+2$

(50) For what value of p is $4x^2-12x+29-\frac{30}{x}+\frac{p}{x^2}$ a perfect square ?

(51) For what value of x is $x^4+6x^3+11x^2+3x+31$ a perfect square ?

(52) For what values of a, b, c is $x^4-2x^3+ax^2+bx+4$ is the square of x^2-x+c ?

Simplify :—

(53) $\frac{1}{x(x-y)(x-z)}+\frac{1}{y(y-z)(y-x)}+\frac{1}{z(z-x)(z-y)}$.

(54) $\frac{x^3+8}{x^2-x-6}+\frac{x^2-5x+6}{x^4+4x^2+16}\div\frac{x^2-2x}{x^3+2x^2+4x}$.

(55) $\frac{a^2-(b+c)^2}{(a+b)^2-c^2}+\frac{b^2-(c+a)^2}{(b-c)^2-a^2}+\frac{c^2-(a-b)^2}{(c-a)^2-b^2}$.

(56) $\frac{x^6}{x^2-1}-\frac{x^4}{x^2+1}-\frac{1}{x^2-1}+\frac{1}{x^2+1}$.

(57) $\frac{(a-b)^2}{(a-c)(b-c)}+\frac{(b-c)^2}{(c-a)(b-a)}+\frac{(c-a)^2}{(a-b)(c-b)}$.

(58) $\frac{(a+b)^2-c^2}{(a+b+c)^2}\div\left[\frac{(a-c)^2-b^2}{a^2+ab+ac}\times\frac{a^2-ab+ac}{(a-b)^2-c^2}\right]$.

(59) $\frac{(2a-3b)^2-(3a-2b)^2}{(2a+3b)^2-(3a+2b)^2}$.

(60) $\frac{5x-4}{x^2-3x-10}-\frac{2x+3}{x^2+5x+6}-\frac{3}{x+2}$.

(61) $\frac{1}{1-x}+\frac{1}{1+x}+\frac{2}{1+x^2}+\frac{4}{1+x^4}$.

(62) $\left(\frac{x}{x-1}-\frac{x+1}{x}\right)\div\left(\frac{x}{x+1}-\frac{x-1}{x}\right)$.

(63) $\frac{1}{a^2-(b-c)^2}+\frac{1}{b^2-(c-a)^2}+\frac{1}{c^2-(a-b)^2}$

(64) $\frac{a}{1-2a}+\frac{a}{1+2a}-\frac{2a}{1+4a^2}+\frac{8a^3}{1+16a^4}$.

(65) $\frac{1}{x-\frac{3}{x-2}}-\frac{1}{x+\frac{2}{x+3}}$.

CHAPTER VI

QUADRATIC EQUATIONS

41. In simple equation the highest power of the unknown quantity is one. In quadratic equation the highest power is 2. As for example (i) $3x^2=27$, (ii) $x^2+3x+2=0$. In the (i) example the highest power of x is 2 and there is no term of the first power of x. Such equations are called *Pure Quadratic Equations.* In the (ii) example the powers of x are one and two both. Such equations are called *Adfected Quadratic Equations.*

42. Solved Examples :—

Solve :

(i) $3x^2=27$

Solution : $3x^2=27$

or, $x^2=9$

Take the square root of both sides.

Then $x= +3, -3$

$\therefore$ the roots of $3x^2=27$ are ± 3.

(ii) $ax^2-bc=0$

Solution : $ax^2-bc=0$

or, $ax^2=bc$

or, $x^2=\dfrac{bc}{a}$

$\therefore x=\pm\sqrt{\dfrac{bc}{a}}$.

(iii) $3x(x+6)-2x(x+9)-16=0$

Solution : $3x(x+6)-2x(x+9)-16=0$

or, $3x^2+18x-2x^2-18x-16=0$

or, $x^2-16=0$

or, $x^2=16$

$\therefore x=\pm 4$

(iv) $x^2+2ax+b^2=a(a+2x)$.

Solution : $x^2+2ax+b^2=a(a+2x)$

or $x^2+2ax+b^2=a^2+2ax$

or $x^2=a^2-b^2$

$\therefore x=\pm\sqrt{a^2-b^2}$.

EXERCISE 27

Solve the equations : —

(1) $x^2=4$.

(2) $6x^2=96$.

(3) $4x^2-100=0$.

(4) $a^2x^2=b^2$.

(5) $\frac{1}{5}x^2=20$.

(6) $3(x^2+4)=2x^2+13$.

(7) $3x^2-2(x+1)+2x=25$

(8) $6x(x-1)+3(2x-5)=9$

(9) $2(x^2-7)+x(3x+2)=2(x+3)$

(10) $2x(3x+5)-5x(x+2)=36$

43. *Second Methcd* : Solution by factors :–

Solved Examples :

Solve :—

(1) $6x^2+13x-5=0$.

Solution : $6x^2+13x-5=0$.

or $(3x-1)(2x+5)=0$.

$\therefore (3x-1)=0$, or $3x=1, \therefore x=\frac{1}{3}$.

or $2x+5=0$. or $2x=-5, \therefore x=-\frac{5}{2}$.

(ii) $(x+3)(x-3)+(x+5)(x-5)=(x+2)(x-2)+3(3x+2)$.

Solution :

$$(x+3)(x-3)+(x+5)(x-5)=(x+2)(x-2)+3(3x+2).$$

or $x^2-9+x^2-25=x^2-4+9x+6$

or $x^2-9x-36=0$.

or $(x-12)(x+3)=0$.

Either $x-12=0$, or $x=12$.

or $x+3=0$, or $x=-3$.

$\therefore$ $x=12$, or -3.

EXERCISE 28

Solve the equations :—

(1) $x^2+4x-5=0$.

(2) $13x^2-6x-7=0$.

(3) $6x^2+5x-4=0$.

(4) $6x^2-29x+9=0$.

(5) $22x^2-93x-27=0$.

(6) $\dfrac{x+7}{x+4}=\dfrac{2x-1}{x}$.

(7) $a^2x^2+2abx+b^2=0$.

(8) $3x(3x-4)=6x^2$.

(9) $(x+3)^2+2x+6=0$.

(10) $(x-3)^2+12x=0$.

(11) $16(x-4)^2=9(x+3)^2$.

(12) $x^2+7x+\dfrac{49}{4}=\frac{1}{4}$.

(13) $3(\frac{1}{2}x+1)^2=27$.

(14) $(x+7)(9-x)=(x+9)(x-7)+76.$

(15) $(2x-1)(3x+1)-(3x-2)^2=(x-2)^2.$

(16) $x(2x-3)-\frac{3}{2}(2x-3)=0.$

(17) $(x-3)^2-2(x-3)+1=0.$

(18) $(2x-5)(3x-7)-(x-1)(4x-5)=x^2-3(x+14)$

(19) $(x-5)(\frac{1}{5}+5x)=0.$

(20) $x^2-\dfrac{23x}{60}-\frac{1}{3}=0$ (21) $x=\dfrac{1}{x}.$

Solution : $x=\dfrac{1}{x}$

By cross multiplication—

$$\text{or } x^2=1$$

$$\therefore x=\pm 1.$$

(22) $4\left(x+\dfrac{1}{x}\right)=17.$ (23) $\dfrac{9-x}{1-9x}=\dfrac{1-8x}{8-x}.$

(24) $\dfrac{x+10}{x-5}-\dfrac{10}{x}=\frac{11}{6}.$ (25) $\dfrac{4}{x-1}+\dfrac{3}{x+1}=\frac{5}{12}.$

(26) $\dfrac{x-1}{x+1}+\dfrac{x+4}{x-4}-2=1-\dfrac{6x+5}{6x-9}.$

Solution : $\dfrac{x-1}{x+1}+\dfrac{x+4}{x-4}-2=1-\dfrac{6x+5}{6x-9}.$

or $\dfrac{(x+1)-2}{x+1}+\dfrac{(x-4)+8}{x-4}-2=1-\dfrac{(6x-9)+14}{6x-9}$

or $1-\dfrac{2}{x+1}+1+\dfrac{8}{x-4}-2=1-\left[1+\dfrac{14}{6x-9}\right]$

or $-\dfrac{2}{x+1}+\dfrac{8}{x-4}=1-1-\dfrac{14}{6x-9}$

or $\dfrac{8}{x-4}+\dfrac{14}{6x-9}=\dfrac{2}{x+1}$

or $\frac{4}{x-4}+\frac{7}{6x-9}=\frac{1}{x+1}$

or $\frac{24x-36+7x-28}{(x-4)(6x-9)}=\frac{1}{x+1}$

or $\frac{31x-64}{6x^2-33x+36}=\frac{1}{x+1}$

or $(31x-64)(x+1)=6x^2-33x+36$

or $31x^2-33x-64=6x^2-33x+36$

or $25x^2=100$

or $x^2=4$

$\therefore\ x=\pm 2.$

(27) $\frac{x+2}{x+1}+\frac{x+1}{x+2}=2\frac{1}{6}$.

(28) $\frac{x-1}{x+1}+\frac{x+3}{x-3}=2\left(\frac{x+2}{x-2}\right)$.

(29) $\frac{x+1}{x-1}+\frac{x+2}{x-2}=\frac{2x+13}{x+1}$.

(30) $\frac{x-1}{x+2}+\frac{2}{8}=\frac{3-x}{4+x}$.

44. *Third Method*: By making perfect square.

Solved Examples:—

Solve :—

(i) $x^2+6x-40=0$.

Solution: $x^2+6x-40=0$

or $x^2+6x+9-40-9=0$

or $x^2+6x+9-49=0$

or $(x+3)^2-7^2=0$

or $(x+3+7)(x+3-7)=0.$

or $(x+10)(x-4)=0.$

$\therefore x=-10, 4.$

(ii) $6x^2-x-2=0.$

Divide both sides by 6.

Then $x^2-\frac{x}{6}-\frac{1}{3}=0.$

or $x^2-\frac{x}{6}+\frac{1}{144}-\frac{1}{3}-\frac{1}{144}=0$

or $\left(x-\frac{1}{12}\right)^2-\frac{49}{144}=0$

or $\left(x-\frac{1}{12}\right)^2=\frac{49}{144}$

or $x-\frac{1}{12}=\pm\frac{7}{12}$

or $x=\frac{1}{12}\pm\frac{7}{12}$

or $x=\frac{8}{12}, -\frac{6}{12}.$

$\therefore x=\frac{2}{3}, -\frac{1}{2}.$

(iii) $ax^2+bx+c=0.$

Solution : Divide both sides by a.

Then $x^2+\frac{bx}{a}+\frac{c}{a}=0.$

Transpose $\frac{c}{a}.$

$x^2+\frac{bx}{a}=-\frac{c}{a}$

Add $\frac{b^2}{4a^2}$ to both sides in order to make the L H. S. a perfect square.

$$x^2+\frac{bx}{a}+\frac{b^2}{4a^2}=\frac{b^2}{4a^2}-\frac{c}{a}.$$

$$\text{or}\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

Take the square root of both sides.

Then $x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$

Transpose $\frac{b}{2a}$

$$\therefore x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

(iv) $2x^2+3x-2=0.$

Solution Compare this with the above example

$a=2,\ b=3,\ c=-2.$

$$\therefore x=\frac{-3\pm\sqrt{9+16}}{4}=\frac{-3\pm5}{4}=\tfrac{1}{2},-2.$$

45. *To find the equation when roots are given.*

Solved Examples :

(i) Find the equation whose roots are :

0, 5; 1,—2; 3,—$\frac{1}{2}$.

Solution : (i) The equation whose root is 5 is $x=5$.

or $x-5=0$.

The equation whose root is 0 is $x-0=0$.

Therefore the equation whose roots are 5, 0 is
$x(x-5)=0$, or $x^2-5x=0$.

Similarly, the equation whose roots are $1, -2$ is $(x-1)(x+2)=0$, or $x^2+x-2=0$.

Also, the equation whose roots are $3, -\frac{1}{2}$ is
$(x-3)(x+\frac{1}{2})=0$.

or $x^2-\dfrac{5x}{2}-\frac{3}{2}=0$.

or $2x^2-5x-3=0$

EXERCISE 29

Solve the following equations :—

(1) $5x^2+2x-7=0$. (2) $8x=3(4x^2-5)$

(3) $27x^2-2=3x$. (4) $4x^2+x-3=0$

(5) $4x^2+126=65x$. (6) $x^2-ax=20a^2$

(7) $36x^2-35b^2=12bx$. (8) $\dfrac{2}{x}+\dfrac{1}{3-x}=2$.

(9) $\dfrac{x+3}{x-3}+\dfrac{x+4}{x-4}=2\left(\dfrac{x-2}{x-5}\right)$.

(10) For what value of x are $(x+1)(x+2)(x+3)$ and $(x-1)(x-2)(x-3)+120$ equal ?

(11) Construct the equations whose roots are :—

(i) $0, -3$; (ii) $1, \frac{2}{3}$; (iii) $\frac{3}{7}, -\frac{1}{6}$, (iv) 2, 2, (v) 2, 3.

CHAPTER VII

PROBLEMS LEADING TO QUADRATIC EQUATIONS

46. Every Quadratic Equation has two roots both of which satisfy the equation. But in certain problems only one root satisfies the conditions of the problem and the other is inadmissible. The former value only should therefore be given.

Solved Examples :

(1) The sum of a number and its square is 182. Find the number.

Solution : Suppose the number is x.

By the condition of the problem,

$x^2+x=182$

or $x^2+x-182=0$

or $(x+14)(x-13)=0$

$\therefore x=-14$ or $x=13$.

Therefore the number is -14 or 13.

[Verification : $-14+(14)^2=-14+196=182$

Also $13+(13)^2=13+169=182$].

(ii) The sum of a number and its reciprocal is $2\frac{1}{2}$. Find the number.

Solution : Suppose the number is x.

$\therefore$ its reciprocal is $\frac{1}{x}$.

By the condition of the problem

$$x+\frac{1}{x}=\frac{5}{2}$$

or $$\frac{x^2+1}{x}=\frac{5}{2}$$

or $2x^2+2=5x$, multiplying across.

or $2x^2-5x+2=0$

or $(2x-1)(x-2)=0$.

$\therefore\ x=\frac{1}{2}$ or $x=2$.

Therefore the number is $\frac{1}{2}$ or 2.

[Verification : $\frac{1}{2}+2=2\frac{1}{2}$ or $2+\frac{1}{2}=2\frac{1}{2}$].

EXERCISE 30

(1) Find the number to which if 50 be added, the sum is less than the square of the number by 6

(2) Find the number whose square is greater than twice that number by 1443.

(3) Find two numbers whose difference is 2 and the sum of whose squares is 74.

(4) Find two numbers such that their sum is 80 and the sum of whose squares is 3208.

(5) Find two numbers whose difference is -3 and the sum of whose squares is 369.

(6) Find two consecutive odd numbers the sum of whose squares is 394.

Solution: Suppose the numbers are $2x+1$, $2x+3$.

Therefore by the condition of the problem,

$(2x+1)^2+(2x+3)^2=394$

or $4x^2+4x+1+4x^2+12x+9=394$

or $8x^2+16x+10=394$

or $8x^2+16x-384=0$

or $x^2+2x-48=0$

or $(x+8)(x-6)=0$.

$\therefore x=-8$, or 6.

$\therefore$ the numbers are $2\times-8+1$, $2\times-8+3$

that is -15, -13.

or $2\times6+1$, $2\times6+3$.

that is, 13, 15.

[Verification. $(-15)^2+(-13)^2=225+169=394$

or $(15)^2+(13)^2=225+169=394$].

(7) Find two consecutive odd numbers the sum of whose square is 290.

(8) Find two consecutive even numbers, the sum of whose squares is 340.

(9) Find two consecutive numbers the sum of whose squares is 421.

(10) Find two consecutive even numbers, the difference of whose squares is 12.

(11) Find the value of a dozen eggs when if one egg more is bought for annas 6, the price of a dozen eggs is reduced by anna 1.

Solution: Suppose the price of one dozen eggs is x annas. Then $72/x$ eggs are purchased for 6 annas.

Therefore, by the condition of the problem,

the reduced price of $\frac{72}{x}+1$ eggs is annas 6

or " " $\frac{72+x}{x}$ " " 6

$\therefore$ the price of a dozen eggs $=\frac{12\times6x}{72+x}=\frac{72x}{72+x}$

Again by the given condition this value is annas $x-1$.

$$\therefore \frac{72x}{72+x}=x-1.$$

Multiply across.

$$72x=(72+x)(x-1)$$

$$\text{or } 72x=72x+x^2-72-x$$

$$\text{or } x^2-x-72=0$$

$$\text{or } (x-9)(x+8)=0$$

$\therefore x=9$ or $x=\ -8$ (inadmissible).

Therefore the price of one dozen eggs is annas 9.

[Verification : The price of 12 eggs is annas 9.

$\therefore$ the price of 8 eggs is annas 6.

By the given condition the price of 9 eggs is annas 6

$\therefore$ the price of 12 eggs is as. $\frac{61\times 2}{9}$,

That is, the price of 12 eggs =annas 8]

(12) A customer purchased eggs for 3s. Had he purchased one dozen eggs less for 3s. the price of one dozen eggs would have risen by 3d. How many eggs did he buy ?

(13) The price of oranges is such that if a reduction of $\frac{1}{2}$d is made in the price of a dozen, then the number of oranges purchased for 1s, is increased by 4. Find the former price of 12 oranges.

(14) A dealer sold a horse for £ 27 and his loss per cent. was $\frac{1}{3}$ the figure which denoted the price of the horse in £. Find the cost price of the horse.

Solution : Suppose the price of the horse is £ x.

$\therefore$ Loss per cent $= \frac{x}{3}$

$\therefore$ Loss on £ $x =$ £ $\frac{x^2}{300}$

$\therefore$ Selling price $=$ £ $\left(x - \frac{x^2}{300}\right)$

By the given condition,

$$x - \frac{x^2}{300} = 27$$

or $300x - x^2 = 8100$

or $x^2 - 300x + 8100 = 0$

or $(x-30)(x-270) = 0$

$\therefore x = 30$ or $x = 270$

Therefore the price of the horse is £ 30 or £ 270.

[Verification : when the C. P. is £ 30 the actual loss is £

$\therefore$ S. P. = £ 27.

When the C. P. is £ 270, the actual loss is £ 243

$\therefore$ S P. = £ (270−243) = £ 27.

(15) A dealer sold a horse for £ 39 and got so much gain per cent as was the price of the horse. Find the price of the horse.

(16) A merchant sells an article for Rs. 24. He suffers so much loss per cent as was the pice of the article. Find the price of the article.

(17) A dealer bought a number of horses for £ 360. He gave away 5 horses to his friends. Then he sold every horse at a price higher than the cost price by £ 3 and gained £ 176 as profit. How many horses did he buy ?

(18) Divide 21 into two parts such that the ratio of their squares is 16 : 9.

Solution : Suppose one part is x ; the other is therefore $12-x$

By the given condition, $\dfrac{x^2}{(21-x)^2}=\dfrac{16}{9}$

or $16(21-x)^2=9x^2$

or $4(21-x)=\pm 3x$

or $84-4x=\pm 3x$

$\therefore$ either $7x=84$ $\therefore$ $x=12$

or $x=84$ (inadmissible)

$\therefore$ the numbers are 12 and 9.

$\left[\text{Verification : } \dfrac{x^2}{(21-x)^2}=\dfrac{(12)^2}{(9)^2}=\dfrac{144}{81}=\dfrac{16}{9}\right]$

(19) Divide 16 into such parts that the square of one part may be equal to nine times the square of the other.

(20) Divide 12 into two such parts that three time the square of the smaller part may be greater than the square of the other by 26.

(21) A man and a woman together do a piece of work in 15 days. Two women together complete it in 4 days less than a man does. Find in how many days will (i) a man (ii) a woman do it ?

Solution : Suppose a man does it in x days. Then by the given condition, 2 women do it in $x-4$ days.

$\therefore$ 1 woman does it in $2(x-4)$ days.

Now one man finishes $\dfrac{1}{x}$ of it in 1 day

and one woman finishes $\dfrac{1}{2(x-4)}$ of it in 1 day

But one man and one woman together do

$\frac{1}{x}+\frac{1}{2(x-4)}$ of it in one day.

$\therefore \frac{1}{x}+\frac{1}{2(x-4)}=\frac{1}{15}$

Simplify the equation.

$\therefore\ 2x^2-53x+120=0$

or $(x-24)(2x-5)=0$

$\therefore\ x=24$ or $x=\frac{5}{2}$ (inadmissible)

$\therefore$ One man finishes it in 24 days and one woman in 40 days.

[Verification : $\frac{1}{24}+\frac{1}{40}=\frac{5+3}{120}=\frac{8}{120}$ of the work is done in 1 day.

$\therefore$ the whole work is done in $\frac{120}{8}=15$ days.]

(22) Two pipes fill a cistern in $2\frac{2}{3}$ minutes. The larger pipe fills it in 4 minutes less. In how many minutes does each of them fill it ?

(23) Two pipes fill a cistern in $6\frac{2}{3}$ minutes If one of them takes 3 minutes more to fill it and the other 3 minutes less, both would fill it in 6 minutes, In how many minutes does each fill it ?

(24) If the length of a rectangle is increased by 8″ and its breadth by 12″, its area becomes 3 times its former area. If the difference between its length and breadth is 4″, find them.

Solution : Suppose the length is x'', the breadth is $(x-4)''$.

$\therefore$ Area $= x(x-4)$ sq. in.

By the given condition the new area$=(x+8)\ (x-4+12)$ sq. inches.

$\therefore\ (x+8)\ (x+8)=3x(x-4)$.

Simplifying the equation. we get $x=-2''$ (inadmissible) or $x=16''$.

$\therefore$ length$=16''$ and breadth$=12''$.

Verification : area $=16\times 12=192$ sq. in.
Other area $=(16+8)(12+12)=24\times 24=576$ sq in
Now $576=3\times 192$].

(25) A garden is 160′ long and 120′ broad. In it there is a tank whose sides are at equal distances from the sides of the garden and whose area is half the area of the garden. Find the length and breadth of the tank.

(26) On a rectangular table $9'\times 6'$ is spread a table cloth. The ends of the cloth hang equally down the four sides of the table. If the area of the cloth is twice the area of the table, find how much is the cloth hanging from one side.

(27) A number is made of two digits whose product is 30. If the digits are interchanged the number so formed is greater than the original number by 9. Find the number.

Solution : Suppose the digit in the unit's place is x.

$\therefore$ the digit in the ten's place $=\dfrac{30}{x}$.

$\therefore$ original number $=\dfrac{300}{x}+x$.

New number $=10x+\dfrac{30}{x}$

$\therefore$ By the given condition,

$$10x+\frac{30}{x}=\frac{300}{x}+x+9$$

or $x^2-x-30=0$.

$\therefore$ $(x-6)(x+5)=0$.

or $x=6$, or $x=-5$ (inadmissible).

$\therefore$ the number is 56.

[Verification : The difference between 65 and $56=9$ and the product of 5 and $6=30$].

(28) The sum of two digits of a number is 15 and their product is less than the number by 22. Find the number.

(29) A number is formed of two digits. One digit is the square of the other. If the digits are interchanged, the new number is less than the original number by 54. Find the original number.

(30) A train travels 240 miles at a uniform speed If she had run 4 miles less in an hour she would have taken 2 hours more to complete the journey. Find her speed per hour.

Solution : Suppose she goes x miles per hour.

$\therefore$ She goes 240 miles in $\frac{240}{x}$ hours.

With slow speed she goes 240 miles in $\frac{240}{x-4}$ hours.

By the given condition,

$$\frac{240}{x} = \frac{240}{x-4} - 2.$$

or Simplifying the equation,

$$x^2 - 4x - 480 = 0$$

or $\quad (x-24)(x+20) = 0$

$\therefore\ x=24 \quad$ or $\quad x=-20$ (inadmissible).

$\therefore$ the train travels at the rate of 24 miles per hour.

[*Verification* : She travels 240 miles in 10 hours at 24 miles per hour and with 20 m. p. h. speed, she goes 240 miles in 12 hours. The difference is 2 hours.]

(31) A cyclist rides 63 miles at a uniform speed. If he reduces his speed by 2 m. p. h. he takes 2 hours more to finish his journey. Find his speed.

(32) A train does a journey of 209 miles with a uniform speed. If her speed is increased by 1 mile per hour she takes 16 minutes less to finish her journey. Find her speed.

CHAPTER VIII

SIMULTANEOUS QUADRATIC EQUATIONS

47. *Solved Examples* :

Solve : (i) $x^2+y^2=74$, *and* $x+y=12$

Solution : $x^2+y^2=74$.........(i)

$x+y = 12$.........(ii)

From (ii) $x=12-y$

Substitute $x=12-y$ in (i)

$\therefore\ (12-y)^2+y^2=74$

or $144-24y+y^2+y^2=74$.

or Simplify the equation :

$y^2-12y+35=0$

or $(y-7)(y-5)=0$

$\therefore\ y=7$ or $y=5$

$\therefore\ x=5$ or $x=7$

$\therefore\ \left.\begin{array}{l} x=5 \\ y=7 \end{array}\right\}$ or $\left.\begin{array}{l} x=7 \\ y=5 \end{array}\right\}$

48. *Second Method* :

Multiply (i) by 2

$\therefore\ 2x^2+2y^2=148$

Square (ii)

$\therefore\ x^2+2xy+y^2=144$

Subtract the second result from the first

$\therefore\ x^2-2xy+y^2=4$

or $(x-y)^2=4$

$\therefore\ x-y=\pm 2$

Now $x+y=12$

and $x-y=2$

$\therefore\ x=7,\qquad y=5.$

also $x+y=12$

$x-y=-2$

$\therefore \quad x=5, y=7$

Hence $x=5, \quad 7$

and $y=7, \quad 5.$

(ii) $x+y=10$

$xy=21$

Solution : $x+y=10$.........(i)

$xy=21$.........(ii)

From (i) $\quad x=10-y$

From (ii) $\quad (10-y)y=21$

Solve the equation :

$$y^2-10y+21=0$$

or $(y-3)(y-7)=0$

$\therefore \quad y=3 \quad$ or $\quad y=7$

and $x=7 \quad$ or $\quad x=3$

$\therefore \quad x=7, \quad 3$

and $y=3, \quad 7.$

49. *Second Method* :

Square the (i)

$x^2+2xy+y^2=100$

Multiply (ii) by 4

$4xy=84.$

Subtract the second result from the first result.

$x^2-2xy+y^2=16$

or $(x-y)^2=16$

$\therefore x-y=\pm 4.$

Proceed as in solved ex. (i) Method second.

Hence $\quad x=7, \quad 3.$

and $\quad y=3, \quad 7.$

EXERCISE 31

Solve :

(1) $x^2+y^2=661$, $\qquad x^2-y^2=589$.

(2) $x+y=14$, $\qquad x^2+y^2=130$.

(3) $x+y=\frac{19}{20}$, $\qquad x^2+y^2=\frac{241}{400}$.

(4) $3x+y=9$, $\qquad 9x^2+y^2=45$.

(5) $5x+4y=28$, $\qquad xy=8$.

(6) $xy=\frac{1}{6}$, $\qquad \dfrac{1}{x}-\dfrac{1}{y}=1$.

Solution :

$$\frac{1}{x}-\frac{1}{y}=1 \text{.........(i)}$$

$$xy=\tfrac{1}{6} \text{.........(ii)}$$

From (i) $\dfrac{y-x}{xy}=1$

or $y-x=xy$

$\therefore\ y-x=\frac{1}{6}$, from (ii).

or $y=x+\frac{1}{6}$

or $x(x+\frac{1}{6})=\frac{1}{6}$, from (ii).

Simplify and solve the above equation.

$(2x+1)(3x-1)=0$

$\therefore\ x=-\frac{1}{2},\ x=\frac{1}{3}$.

and $y=-\frac{1}{3},\ y=\frac{1}{2}$ from (i).

(7) $xy=-1$, $\dfrac{2}{x}+\dfrac{1}{y}=1$.

(8) $x-y=2$, $\qquad \dfrac{1}{x}-\dfrac{1}{y}=-\dfrac{2}{35}$.

(9) $x+y=1+\dfrac{4}{x}=1+\dfrac{2}{y}$.

(10) $2x-y=1$, $2x^2-y^2+xy=8$.

(11) $3x-y=9$, $9x^2+y^2=81$.

(12) $9x^2+xy+4y^2=91$, $3x-2y=13$.

Miscellaneous Exercises III

I

(1) Resolve into factors :

(*a*) $625-x^4$.

(*b*) $12a^6b+11a^4b^2-15a^2b^4$.

(*c*) $5a^2-7ab-6b^2$.

(2) Find the H. C. F. of : x^5-x^3+8, x^5-x^2+4.

(3) Find the L. C. M. of :

$x^4-2x^3+2x^2-2x+1$, x^4-2x^3+2x-1, x^3-x^2-x+1 and $(x-1)^3$.

(4) Find the square root of :

$(3x+1)(3x+4)(3x+7)(3x+10)+81$.

(5) Simplify :—

$$\frac{x^2-(y-z)^2}{(x+z)^2-y^2}+\frac{y^2-(x-z)^2}{(x+y)^2-z^2}+\frac{z^2-(x-y)^2}{(y+z)^2-x^2}.$$

(6) Solve the equations :

(i) $6x+5y-11=5x-3y-45=0$.

(ii) $\left(\dfrac{x^2-2}{x^2-6}\right)^2=\dfrac{x^2-4}{x^2-12}$.

(7) Find two factors of 243 such that one is three times the other.

II

(1) Resolve into factors :

(i) x^8-112x^4+256.

(ii) $(1-x+x^2)^2-(1+x+x^2)^2$.

(iii) $ac+bd-bc-ad$.

(2) Find the H. C F. of :

$4x^3+8x^2+3x+20, \qquad 2x^3+9x^2+4x-15.$

(3) Find the L. C. M. of :

$3x^2-5x+2. \qquad 4x^3-4x^2-x+1.$

(4) Find the square root of :

$$4x^2+4x-11-\frac{10}{x}+\frac{7}{x^2}+\frac{6}{x^3}+\frac{1}{x^4}$$

(5) Simplify :

$$\frac{x-1}{x+1}-\frac{x+2}{x+3}.$$

(6) Solve : $3x^2+x-14=0.$

(7) Divide 16 into two parts such that the square of one may be equal to nine times the square of the other.

III

(1) Resolve into factors :

(i) $x^3+4x^2+x-6.$

(ii) $(ab+1)^4-4ab(ab+1)^2-(a^2-b^2)^2.$

(iii) $(x+1)(x+3)(x+5)(x+7)+15.$

(2) Find the H. C. F. of :

$abc(x^2-y^2-2zx+2yz)$ and $abx^2-ay^2.$

(3) Find the L. C. M. of :

$a^4+4, a^3-2a-4, a^3-2a+4.$

(4) Find the square root of :

$$x^4-x^3-\frac{7x^2}{4}+x+1.$$

(5) Simplify :

$$\frac{1}{2x+3y}+\frac{1}{2x-3y}+\frac{4x}{4x^2+9y^2}.$$

(6) Solve the equation :

$$20x^2-18x-14=0$$

(7) Find the number which when added to its square makes 462.

IV

(1) Resolve into factors :—

(i) $x^3y(a+b+c)-2ab(a+b+c)+3bc(a+b+c)$.

(ii) $91\,abcd-7\,abc-21\,acd-56\,dcae$.

(2) Find the H. C. F. of :—

$x^3+3x^2y-6xy^2-8y^3$ and $x^3-2x^2y-xy^2+2y^3$.

(3) Find the L. C. M. of :—

$38st^2u^3v^4,\ 95s^5t^6u^4v^3,\ 190s^2t^4u^5v^3$.

(4) Find the square root of :—

(i) $x^2+4xy+4y^2$.

(ii) $25x^2+10ax+a^2$.

(5) Simplify :

$$x+1-\cfrac{x}{x+2-\cfrac{x+1}{x+\cfrac{1}{x+2}}}.$$

(6) Solve :— (i) $3x^2+5=53$.

(ii) $9x^2+15x-14=0$.

(7) The difference of two numbers is 8 and their product is 308. Find the numbers.

V

(1) Resolve into factors :—

(i) $343x^3+1$.

(ii) $4\left(m-\frac{1}{m}\right)^2-9$.

(2) Find the H. C. F. of :—

$12x^3-22x^2+23x-20,\ 6x^3+7x^2-29x+12.$

(3) Find the L. C. M. of :—

$(x+z)(x-z)-y(2z+y),\ (x+z)(x-z)+y(2x+y).$

(4) If $N=4x^4+12x^3-7x^2-24x+16$, prove that N is a perfect square and find the square root of $4x^4+12x^3-7x^2-24x+16$.

(5) Simplify :—

$$\frac{x+1}{x-1}\times\frac{x^2+x-6}{x^4-13x^2+36}\div\frac{x^2+2x+1}{x^2-x-6}.$$

(6) Solve the equations :

(i) $x^2-2x-3=0$ (ii) $3x^2=6\frac{3}{4}$.

(7) Find two numbers such that their sum is 20 and the sum of their cubes is 2060.

VI

(1) Resolve into factors :—

(i) $(2a-3)^3-(a-1)^3-(a-2)^3$.

(ii) $p^3-q^3+s^3+3pqs$.

(2) Find the H. C. F. of :

$3x^2-5x+2,\ 2x^2-9x+7.$

(3) Find the L. C. M. of :

$25(a^3+b^3)(a^4-b^4),\ 35ab(a^2+b^2),\ 63a(a^3-b^3).$

(4) Find the square root of :

$$4x^4+4x+\frac{6}{x^3}+\frac{1}{x^4}+\frac{7}{x^2}-\frac{10}{x}-11.$$

(5) Simplify : —

$$\frac{a^2-b^2}{a^2-2ab+b^2}+\frac{a-b}{(a+b)}.$$

(6) Solve the equations :—

(i) $6x^2-x-15=0$. (ii) $3x^2-7x-6=0$.

(7) A number is greater than another by 7 and their product is 60. Find them.

VII

(1) Prove that $(a+b+c)^3-a^3-b^3-c^3=3(b+c)(c+a)(a+b)$.

(2) Resolve into factors :

(i) $8(x+y+z)^3-(y+z)^3-(z+x)^3-(x+y)^3$.

(ii) $(a+b+c)\ (bc+ca+ab)-abc$.

(3) Find the L. C. M. of

a^3-3a-2 and a^3-a^2-5a-3.

(4) Find the square root of :

$25x^4+40x^3-14x^2-24x+9$.

(5) Simplify :—

$$\frac{y^2-z^2}{y^2z^2}+\frac{z^2-x^2}{z^2x^2}+\frac{x^2-y^2}{x^2y^2}.$$

(6) Solve the equations :—

(i) $33x^2+7x-10=0$.

(ii) $6x^2+7x-20=0$.

(iii) $2x^2-11x-21=0$.

(7) The ratio between two numbers is 4: 7 and the sum of their squares is 585. Find the numbers.

VIII

(1) Find the value of $p^3+q^3+s^3-3pqs$ when $p=337, q=345$ and $s=-682$

(2) Resolve into factors :

(i) $\frac{x^3}{8}+8y^3+1-3xy$

(ii) $a^6-b^6-8c^6-6a^2b^2c^2$

(3) Find the H. C. F. or :

$3x^3-17x^2-62x-14;\quad 7x^3-52x^2-46x-8.$

4) Find the value of p when $x^2-4px+16$ is a perfect square.

(5) Simplify :

$$\frac{a+2}{2a}-\frac{a^2-1}{a^2}+\frac{1}{2}.$$

(6) Solve the equations ;

(i) $6x^2-x-2=0.$

(ii) $40-x^2=6x.$

(7) Divide 23 into two parts such that the difference of their cubes be 161.

IX

(1) Resolve into factors :

(i) $(x-4a)^3-(2x-3a)^3+(x+a)^3.$

(ii) $1-x^2(y^4+1)+y^2(x^4+1)-x^2y^2.$

(2) Find the H. C. F. of :

$x^4-x^2+4x-4.\ x^3-x^2-2x+8.$

(3) Find the L. C. M. of :

$4x^3-8x^2+3x-6$ and $12x^3+4x^2+9x+3.$

(4) Find the square root of :

$36x^4-36x^3+33x^2-12x+4$

(5) Solve the equations :

(i) $7x=4-2x^2.$

(ii) $14=18x^2+9x.$

(6) Simplify :

$$\frac{x}{x-2a}+\frac{x}{x+2a}+\frac{2x^2}{x^2+4a^2}.$$

(7) Find a number such that if its square be added to three times the number, the sum may be 154

X

(1) Resolve into factors :—

(i) $(y^2-z^2)^3+(z^2-x^2)^3+(x^2-y^2)^3$.

(ii) $(3x-1)^3+(x-4)^2+(5-4x)^3$.

(2) Find the value of $a^3-b^3+c^3+3abc$ when $a=\cdot737$, $b=1\cdot379$ and $c=\cdot642$.

(3) Find the H. C. F. of :

x^3-7x+6 and $x^4-3x^3-2x^2+12x-8$.

(4) Find the value of a when x^2-2x+a is a perfect square.

(5) Simplify :

$$\frac{2x+3}{x^2+4x+16}-\frac{2}{x-4}+\frac{13x+44}{x^3-64}.$$

(6) The length of a room is greater than its breadth by 2ft., and its area is 195 sq. ft. Find the breadth of the room.

(7) Solve the equation :

$6x^2-2x-20=0$

XI

(1) Resolve into factors :

(i) x^4+8x^2+144

(ii) $3y+6x^2-xy-y^2-6x$.

(iii) $a^2b^2-15ab+44$.

(2) If $a^2+b^2+c^2=100$ and $ab+bc+ca=48$, find the value of $a+b+c$.

(3) If $x+y=6$ and $x^3+y^3=72$, find the value of xy.

(4) Find the square root of :

$x^4-3x^3+3\frac{1}{4}x^2-1\frac{1}{2}x+\frac{1}{4}$.

(5) Simplify :

$$\frac{x^3-2x^2-x+2}{x^3-x^2-4x+4}.$$

(6) Solve :

(i) $(x-a)(x+2a)=0$

(ii) $(3x+7)(2x-5)=0$

(7) Find the two numbers such that their difference is 7 and the sum of their squares is 137

XII

(1) Without actual division, prove that $x^3-39x+70$ is divisible by $x+7$.

(2) Find the H. C. F. of :

a^3-2a^2+2a-1 and a^3-3a^2+3a-2.

(3) If h and l be the H C. F and L. C. M. of x and y respectively and if $x+y=h+l$, prove that $h^3+l^3=x^3+y^3$.

(4) Simplify :–

$$\left(1-\frac{y^2}{y^2+a^2}\right)\div\left(1-\frac{a^2}{a^2+y^2}\right).$$

(5) Find the square root of :

$4x^4-8x^3+4x+1$.

(6) Solve:–

$(x+3)(x+7)=(2x+3)(3x+7)$.

(7) Find two numbers whose sum is 16 and whose product is 60.

XIII

(1) Divide $x^4+x^2y^2+y^4$ by x^2+xy+y^2 and verify your result by putting $x=2$, $y=3$.

(2) Find the H. C. F. of :

x^2-2x-3 and x^3-2x^2-2x-3.

(3) Find the H. C. F. of :

x^3+a^3, x^3-a^3, $x^4+a^2x^2+a^4$, x^2-ax+a^2.

(4) Find the square root of :

$9x^4+12x^3+10x^2+4x+1$.

(5) Simplify :

$$\frac{x+2y}{x-2y}+\frac{x-2y}{x+2y}.$$

(6) If $x+\frac{1}{x}=4$, find the values of $x^2+\frac{1}{x^2}$, $x^3+\frac{1}{x^3}$ and $x^4+\frac{1}{x^4}$.

(7) Solve :—

(i) $x^2+5x+4=0$.

(ii) $x^2-10x+21=0$.

(iii) $4x^2+17x+4=0$.

XIV

(1) **Prove that $x-1$, $x-2$, $x-3$, are the factors of $x^4+x^3-7x^2-x+6$.**

(2) Simplify :— $\frac{(7x^2-23x+6)(6x^2-35x+11)}{3x^2-10x+3}$.

(3) If $9x^4-12x^3+mx^2-4x+1$ is a perfect square, find the value of m.

(4) Solve :—

(i) $72x^2-25x-77=0$.

(ii) $56x^2-5x-99=0$.

(5) If $x^4+4x^3+3x^2+mx+n$ is divisible by x^2+x-2, find the value of m and n,

(6) Simplify :—

$$\frac{x^2-4}{x^3+8}\times\frac{x^2-5x+6}{x^2+x-6}\div\frac{x^2-4x+3}{x^2-2x+4}.$$

(7) Find the expression which when multiplied by itself gives $25x^4-20x^3-6x^2+4x+1$.

XV

(1) Find the value of x in the following:—

$$\frac{1}{x-2}+\frac{1}{x-10}=\frac{1}{x-5}+\frac{1}{x-7}.$$

(2) Resolve into factors :—

(i) $(x+1)^4-1$.

(ii) $2x^2+xy-3y^2+x-y$.

(3) If $a+b+c=0$, show that $a^3+b^3+c^3=3abc$.

(4) Solve :—

$$\frac{x+b}{a}-\frac{x-a}{b}=2.$$

(5) Find the square root of :—

$$\left(x^2-\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}\right)+8.$$

(6) Find the L. C. M. of :—

x^3-2x+1, x^4-x^3-x+1.

(7) Find the H. C. F. of :—

a^4+a^3-a-1 and a^3+a^2+a+1.

CHAPTER IX

DIFFICULT EQUATIONS OF FIRST DEGREE

50. *Solved examples.*

Solve:—

(i) $75-\frac{2}{3}(2x-7)=5x+\frac{x-4}{10}-\frac{3x-2}{4}$.

Solution :

$$75-\frac{2}{3}(2x-7)=5x+\frac{x-4}{10}-\frac{3x-2}{4}$$

Simplify both sides.

$$75-\frac{4x}{3}+\frac{14}{3}=5x+\frac{x}{10}-\frac{4}{10}-\frac{3x}{4}+\frac{2}{4}.$$

Transpose

$$-\frac{4x}{3}-5x-\frac{x}{10}+\frac{3x}{4}=-75-\frac{14}{3}-\frac{4}{10}+\frac{2}{4}.$$

Simplify both sides

$$\frac{-80x-300x-6x+45x}{60}=\frac{-4500-280-24+30}{60}.$$

or $\qquad -341x = -4774$

or $\qquad x = \frac{4774}{341}$

$\qquad \therefore x = 14.$

EXERCISE 32

Solve :

(1) $7\frac{3}{8}+\frac{3x-1}{4}-\frac{7x+3}{16}=\frac{8x+16}{8}.$

(2) $\frac{1}{7}(8x-13)-\frac{3}{5}(2x-7)=\frac{4}{7}(5x-2)-14.$

(3) $x-\left(3x-\frac{2x+5}{10}\right)=\frac{2x+67}{6}+\frac{5+x}{3}.$

(4) $\frac{7}{8}(5x-3)-\frac{3}{7}(6x-13)=\frac{3}{8}(3x-5)+\frac{13}{17}(4x-26).$

(5) $(x+3)(\cdot5x-1)-(x-1)(\cdot375x+\cdot75)$
$=(\cdot5x-3\cdot5)(\cdot25x+1\cdot25)$

Solution : $(x+3)(\cdot5x-1)-(x-1)(\cdot375x+\cdot75)$
$=(\cdot5x-3\cdot5)(\cdot25x+1\cdot25)$

or $\cdot5x^2+\cdot5x-3-(\cdot375x^2+\cdot375x-\cdot75)$
$=\cdot125x^2-\cdot25x-4\cdot375$

or $\cdot5x^2+\cdot5x-3-\cdot375x^2-\cdot375x+\cdot75$
$=\cdot125x^2-\cdot25x-4\cdot375$

or $\cdot5x^2-\cdot375x^2-\cdot125x^2+\cdot5x-\cdot375x$
$+\cdot25x=3-\cdot75-4\cdot375$

or $\cdot375x=-2\cdot125$

$$\therefore\ x=-\frac{2\cdot125}{\cdot375}=-5\tfrac{2}{3}.$$

(6) $\dfrac{\cdot25x-\cdot075}{\cdot125}=\dfrac{2x-\cdot45}{1\cdot25}+\cdot6.$

(7) $\cdot4(x+5)-\cdot6(x-10)=\cdot3(2x-5)+17\cdot5.$

(8) $\dfrac{\cdot08x-\cdot04}{\cdot1}=\cdot6x+2-\dfrac{\cdot07x-1\cdot03}{\cdot6}.$

(9) $(\cdot4x-2)(\cdot2x-1)+\frac{1}{6}(\cdot2x-1)(\cdot3x-2)$
$=(\cdot3x-2)(\cdot3x-1)$

(10) $\dfrac{1}{x}+\dfrac{1}{2x}+\dfrac{1}{3x}+\dfrac{1}{4x}=\dfrac{1}{6}$

Solution : Multiply both sides by $12x$.

$12+6+4+3=2x$

or $25=2x$ $\therefore\ x=12\cdot5$

(11) $\dfrac{1}{x}+\dfrac{1}{2x}-\dfrac{3}{4x}-\dfrac{5}{12}=\dfrac{7}{24}$ (12) $\dfrac{0\cdot4}{x}=\dfrac{0\cdot5}{x}\left(1-\dfrac{0\cdot1}{x}\right)$

(13) $\dfrac{0\cdot4}{x}=0\cdot25-\dfrac{0\cdot625}{x}.$ (14) $\dfrac{2x}{x-1}-\dfrac{x-1}{2x}=\dfrac{3}{2}.$

Solution : Multiply both sides by $2x\ (x-1)$.

$4x^2-(x-1)^2=3x(x-1)$

or $4x^2-x^2+2x-1=3x^2-3x$

or $5x=1$ $\therefore\ x=\frac{1}{5}$

(15) $x+\dfrac{a}{b-a}=\dfrac{bx}{a+b}$. (16) $\dfrac{1}{2x-3}+\dfrac{x}{3x-2}=\dfrac{1}{\ }$.

(17) $\dfrac{2}{5(3x+4)}+\dfrac{4}{2x+3}=\dfrac{6}{3x+4}$.

(18) $\dfrac{1}{x-1}+\dfrac{2}{x-2}=\dfrac{3}{x-3}$.

(19) $\dfrac{1}{2x+b}-\dfrac{1}{2x+a}=\dfrac{a-b}{4x^2+2ab}$.

(20) $\dfrac{2x-3}{x-4}+\dfrac{5(x-4)}{x-3}=\dfrac{2x+1}{x-2}+\dfrac{5(x-2)}{x-1}$.

Solution : Transpose—

$$\frac{2x-3}{x-4}-\frac{2x+1}{x-2}=\frac{5(x-2)}{x-1}-\frac{5(x-4)}{x-3}.$$

$$=5\left\{\frac{x-2}{x-1}-\frac{x-4}{x-3}\right\}$$

or $\dfrac{(2x-3)(x-2)-(2x+1)(x-4)}{(x-4)(x-2)}$

$$=5\left\{\frac{(x-2)(x-3)-(x-4)(x-1)}{(x-1)(x-3)}\right\}$$

or $\dfrac{(2x^2-7x+6)-(2x^2-7x-4)}{(x-4)(x-2)}$

$$=5\left\{\frac{(x^2-5x+6)-(x^2-5x+4)}{(x-1)(x-3)}\right\}$$

or $\dfrac{10}{x^2-6x+8}=\dfrac{10}{x^2-4x+3}$

or $x^2-6x+8=x^2-4x+3$ or $x=\frac{5}{2}$

2nd Method :

Since $\frac{2x-3}{x-4}=\frac{2(x-4)+5}{x-4}=\frac{2(x-4)}{x-4}+\frac{5}{x-4}=2+\frac{5}{x-4}$,

$$\frac{x-4}{x-3}=\frac{(x-3)-1}{x-3}=\frac{x-3}{x-3}-\frac{1}{x-3}=1-\frac{1}{x-3}.$$

$$\frac{2x+1}{x-2}=\frac{2(x-2)+5}{x-2}=2+\frac{5}{x-2}.$$

$$\frac{x-2}{x-1}=\frac{(x-1)-1}{x-1}=1-\frac{1}{x-1}.$$

Now $\frac{2x-3}{x-4}+\frac{5(x-4)}{x-3}=\frac{2x+1}{x-2}+\frac{5(x-2)}{x-1}$

$$\therefore \left(2+\frac{5}{x-4}\right)+5\left(1-\frac{1}{x-3}\right)=\left(2+\frac{5}{x-2}\right)+5\left(1-\frac{1}{x-1}\right)$$

or $7+\frac{5}{x-4}-\frac{5}{x-3}=7+\frac{5}{x-2}-\frac{5}{x-1}$

Subtract 7 from both sides and divide both sides by 5.

$$\frac{1}{x-4}-\frac{1}{x-3}=\frac{1}{x-2}-\frac{1}{x-1}$$

or $\frac{x-3-x+4}{(x-3)(x-4)}=\frac{x-1-x+2}{(x-1)\ (x-2)}$

or $\frac{1}{(x-3)(x-4)}=\frac{1}{(x-1)(x-2)}$

or $(x-2)\ (x-1)=(x-3)\ (x-4)$

or $x^2-3x+2=x^2-7x+12$

or $4x=10$

$\therefore\ x=2{\cdot}5.$

(21) $\dfrac{x-4}{x-3}-\dfrac{x-6}{x-5}=\dfrac{x-8}{x-7}-\dfrac{x-10}{x-9}.$

(22) $\dfrac{x}{x-1}-\dfrac{2x+1}{2x-1}-\dfrac{x-4}{x-3}+\dfrac{2x-9}{2x-7}=0.$

(23) $\dfrac{2x-3}{x-2}+\dfrac{3x-20}{x-7}=\dfrac{x-3}{x-4}+\dfrac{4x-19}{x-5}.$

(24) $\dfrac{x-1}{x-2}-\dfrac{x-2}{x-3}-\dfrac{x-3}{x-4}+\dfrac{x-4}{x-5}=0.$

(25) $\dfrac{x-1}{2-x}+2\left\{\dfrac{x-2}{1-x}\right\}+3=0.$

(26) $\dfrac{x+3}{x+1}-\dfrac{x+4}{x+2}+\dfrac{x-6}{x-4}=\dfrac{x-5}{x-3}.$

(27) $\dfrac{2x+1}{x+3}-\dfrac{2x+3}{x+4}=4\left\{\dfrac{x+2}{2x-1}-\dfrac{x+1}{2x-3}\right\}.$

(28) $\dfrac{x-1}{x-2}+\dfrac{x-5}{x-6}=\dfrac{x-2}{x-3}+\dfrac{x-4}{x-5}.$

(29) $\dfrac{2x+3}{2(x+1)}+\dfrac{2x+11}{2(x+5)}=\dfrac{2x+5}{2(x+2)}+\dfrac{2x+9}{2(x+4)}.$

(30) $\dfrac{4x+17}{x+4}-\dfrac{5x+36}{x+7}=\dfrac{2x+7}{x+3}-\dfrac{3x+19}{x+6}.$

(31) $\dfrac{6}{2x+3}+\dfrac{12}{3x-4}=\dfrac{21}{3x-1}$

Solution : $\dfrac{21}{3x-1}=\dfrac{9}{3x-1}+\dfrac{12}{3x-1}$

or, $\dfrac{6}{2x+3}+\dfrac{12}{3x-4}=\dfrac{9}{3x-1}+\dfrac{12}{3x-1}$

or, $\dfrac{6}{2x+3}-\dfrac{9}{3x-1}=\dfrac{12}{3x-1}-\dfrac{12}{3x-4}$

Simplify each side separately.

$$\frac{6(3x-1)-9(2x+3)}{(2x+3)(3x-1)}=\frac{12(3x-4)-12(3x-1)}{(3x-1)(3x-4)}$$

or, $$\frac{-33}{(2x+3)(3x-1)}=\frac{-36}{(3x-1)(3x-4)}$$

Divide both sides by $\dfrac{-3}{3x-1}$

or, $$\frac{11}{2x+3}=\frac{12}{3x-4}$$

Multiply across.

$33x-44=24x+36$

or, $9x=80$ $\quad\therefore x=8\frac{8}{9}$.

(32) $\dfrac{1}{x-1}+\dfrac{1}{x-2}=\dfrac{2}{x-3}$.

(33) $\dfrac{1}{x+1}+\dfrac{2}{x+3}+\dfrac{5}{x+5}=\dfrac{8}{x+4}$.

(34) $\dfrac{8}{2x-1}+\dfrac{9}{3x-1}=\dfrac{7}{x+1}$.

Hint : $\dfrac{7}{x+1}=\dfrac{3}{x+1}+\dfrac{4}{x+1}$, etc.

(35) $\dfrac{x^2+3}{x-1}+\dfrac{x^2-x+1}{x-2}=\dfrac{2x^2-4x+1}{x-3}$.

(36) $\dfrac{x^2-4x+4}{x-1}+\dfrac{x^2-3x-1}{x-2}=2\left[\dfrac{x^2-5x+5}{x-3}\right]$.

(37) $\dfrac{(x-2)(x-4)}{(x+2)(x+1)}=\dfrac{x-6}{x+3}$.

Solution : Multiply both sides by $x+2$.

$$\frac{(x-2)(x-4)}{x+1}=\frac{(x-6)(x+2)}{x+3}$$

or, $\dfrac{x^2-6x+8}{x+1}=\dfrac{x^2-4x-12}{x+3}$

or, $x-7+\dfrac{15}{x+1}=x-7+\dfrac{9}{x+3}$

or, $\dfrac{5}{x+1}=\dfrac{3}{x+3}$

or, $5(x+3)=3(x+1)$ $\qquad\therefore\ x=-6.$

(38) $\dfrac{(x+7)\,(x+12)}{(x+6)\,(x+11)}=\dfrac{(x+4)\,(x+15)}{(x+3)\,(x+14)}$.

(39) $\dfrac{(x+3)\,(x+4)}{(x+1)\,(x-3)}=\dfrac{x+7}{x-2}$.

(40) $\dfrac{x+19}{x+10}=\left(\dfrac{2x+33}{2x+24}\right)^2$.

Solution : $\dfrac{x+19}{x+10}=\left(\dfrac{2x+33}{2x+24}\right)^2$

or $1+\dfrac{9}{x+10}=\dfrac{4x^2+132x+1089}{4x^2+96x+576}$

or $1+\dfrac{9}{x+10}=1+\dfrac{36x+513}{4x^2+96x+576}$

or $\dfrac{1}{x+10}=\dfrac{4x+57}{4x^2+96x+576}$

or $4x^2+96x+576=(x+10)\,(4x+57)$

or $4x^2+96x+576=4x^2+97x+570$ $\qquad\therefore\ x=6.$

(41) $\left(\dfrac{x+11}{x-12}\right)^2=\dfrac{(x+9)\,(x+13)}{(x-10)\,(x-14)}$

(42) $\left(\dfrac{x-5}{x-3}\right)^3=\dfrac{x-7}{x-1}$.

CHAPTER X

51. PROBLEMS LEADING TO SIMPLE EQUATIONS

EXERCISE 33

(1) A sum of 100 is divided between A and B such that when each of them had spent £ 6, A had three times the amount of B Find their shares.

Solution :— Suppose A had £ x. Then B had £ $100-x$.

After spending £ 6, A had £$x-6$ and B, £ $100-x-6$ $=£94-x$.

By the given condition,

$$x-6=3(94-x)$$
$$=282-3x$$

or $4x=288$ $\therefore x=72.$

Hence A had £ 72 and B, £ 28.

(2) A sum of money is divided among A. B and C such that A gets half the total amount, A and B together get Rs. 76 and C and A together Rs. 62. How much money does each get ?

(3) Divide 112 into two parts such that $\frac{1}{11}$ of one's share is equal to $\frac{1}{5}$ of the other's.

(4) Four thieves divided their property in such a way that the third got Rs. 9 more than the fourth, the second, Rs. 12, more than the third, and the first, Rs. 18, more than the second. The whole amount was greater than seven times the fourth's share by Rs. 6 Find the share of each.

(5) When I give $1a.$ $8p.$ to each of some boys I have 11 as. less. When I give each of them $1a.$ $5p.$ I save $3a.$ $3p.$ Find the number of boys.

(6) The ratio between my and my brother's ages is $\frac{2}{3}$. After 4 years this ratio will be $\frac{5}{7}$. Find the age of each.

Solution :—Suppose my age is x years. Then the age of my brother is $\frac{3x}{2}$.

After 4 years my age will be $x+4$ years and my brother's, $\frac{3x}{2}+4$ years.

By the given condition,

$$\frac{x+4}{\frac{3x}{2}+4}=\frac{5}{7} \qquad \text{or} \quad \frac{2(x+4)}{3x+8}=\frac{5}{7}$$

or $15x+40=14x+56$ $\qquad \therefore x=16$

Hence my age is 16 years and my brother's 24 years.

(7) A says to B, "The sum of our ages is 63 years. When I was so old as you are now, my age was twice yours." Find the age of each.

(8) The sum of the ages of a boy and his father is 80 years. If the boy's age is doubled, it will be 10 years greater than the father's. Find the age of each.

(9) The age of a man is four times that of his younger son. After 6 years the age of the man will be twice the present age of his elder son. The age of the elder son is 8 years greater than that of his younger son. Find their present ages.

(10) The age of a man is 40 years. His son's age is 9 years. When will he be twice as old as his son?

(11) A labourer was kept for 40 days on the condition that he will receive Rs. 1-4-0 each day he works and will be fined 3*a*, 6*p*. for the day of absence. After the appointed time he got Rs. 38-4-0 For how many days was he absent ?

Solution : Suppose he remained absent for x days.

$\therefore$ he got Rs. $\frac{5}{4}(40-x)$ as wages for $40-x$ days.

But he was fined Rs. $\dfrac{7x}{32}$ for absence.

$$\therefore \text{ his net receipt} = \text{Rs. } \frac{5}{4}(40-x) - \frac{7x}{32}.$$

By the given condition,

$$\frac{5}{4}(40-x) - \frac{7x}{32} = \frac{153}{4},$$

Solving the equation, we get $x=8$.

Hence he remained absent for 8 days,

(12) A mason was employed for 40 days on the condition that he will get Rs. 2-8-0 for each day he works. But he will be fined 10 as. for each day of absence. After 40 days he got nothing How many days did he work ?

(13) A labourer was employed for 40 days on the condition that he will receive Re. 1 each day he works but will be fined. 4 as. for each day of absence. He received Rs. 27-8-0 after the appointed time. How many days did he remain absent ?

(14) A labourer was employed for 30 days on the condition that he will receive 2s. 6d. each day he works, but will be fined 1 s. for each day of absence. He got £ 2 7s. only after the appointed time. For how many days did he work ?

(15) A number consists of two digits, whose sum is 8. If 18 be added to the number, the digits are reversed. Find the number.

Solution : Suppose the digit in unit's place is x

$\therefore$ the digit in the ten's place is $8-x$

$\therefore$ the number $=10(8-x)+x$

and the number with digits reversed $=10x+8-x$.

By the given condition,

$10(8-x)+x+18=10x+8-x$

Solving the equation, we get $x=5$

Hence the number is 35.

(16) A number is composed of two digits whose sum is 12. The difference between the number and the one with reversed digits is 36. Find the number.

(17) A number consists of three digits which increase successively by 1 from left to right. When the number is divided by the sum of the digits, the quotient is 26. Find the number.

(18) A number is made of two digits. The digit in the unit's place is four times the digit in the ten's. If the digits are reversed then the new number added to 2 is equal to three times the old number. Find the numbers.

(19) A number consists of two digits. The digit in the unit's place is 5 less than that in the ten's place. If five times the sum of the digits is subtracted from the number, the digits are reversed. Find the number.

(20) A number is made of 2 digits. The ten's digit is twice the unit's digit. When the digits are reversed the new number is 27 less than the old number. Find the number.

(21) A grocer bought some oranges at 3 an anna and double the quantity at 4 an anna. He sold the mixture at 7 for two annas, thus gaining only as 2. How many oranges of each kind did he buy?

Solution : Suppose he bought x oranges of superior kind Then he bought $2x$ oranges of the other kind.

Cost price of x oranges $= \frac{x}{3}$. as.

Cost price of $2x$ oranges $= \frac{2x}{4}$. as.

$\therefore$ he spent as. $\left(\frac{x}{3}+\frac{x}{2}\right)=\frac{5x}{6}$. as.

Selling price of $3x$ oranges at 7 per two annas

$$=\text{as. } \frac{6x}{7}.$$

By the given condition,

$$\frac{5x}{6}+2=\frac{6x}{7}.$$

Solving the equation, we get $x=84$.

Hence he bought 84 oranges at 3 per anna and 168 oranges at 4 per anna.

(22) A man sold an article at 6 per cent profit. If he had bought it at 4 per cent less and sold it for Rs. 1-3-0 more, he would have gained 12 per cent. Find the cost price of the article.

(23) A trader marks his goods at a certain percentage higher than its cost price and allows 10 per cent discount on the marked price He wishes to gain $12\frac{1}{2}$ per cent. What should be the marked price of the goods whose actual price is one guinea ?

(24) A man sold an article at 20 per cent profit. Had he bought it at 10 per cent less and sold it for 10s. less, he would have gained 25 per cent. Find the cost price of the article.

(25) A person buys some oranges, 12 for four annas. 50

are spoiled. He sells the rest at 9 per four annas and gains Rs. 6-4-0 How many oranges does he buy ?

(26) The denominator of a fraction is greater than its numerator by 11. If 8 is added to the numerator or 9 is subtracted from the denominator, the resulting fractions are equal Find the original fraction.

(27) Two trains start from two stations, A and B towards each other at 45 miles and $27\frac{1}{2}$ miles per hour respectively. When they meet, one has travelled 28 miles longer than the other. Find the distance between A and B

(28) A boy swims at $3\frac{1}{4}$ miles an hour against the current and at $4\frac{1}{2}$ miles an hour with the current He takes 2 hours and 4 minutes to go and come back a certain distance. How far does he go ?

(29) Two persons started simultaneously from a place A. One rode at $7\frac{1}{2}$ miles per hour and reached another station, B 30 minutes later than the other man who travelled the same distance by train at 30 miles per hour Find the distance between A and B.

(30) Two persons started simultaneously from two stations A and B, 561 miles apart, to meet each other. One travelled 24 miles and the other 27 miles per day. In how many days will they meet together ?

(31) A does half the work which B does, B does half the work which C does They together complete a work in 42 days. How many days will each take separately to do it ?

(32) A, B and C complete a work in a certain time. A alone can finish it in 6 hours more, B alone can finish it in 15 hours

more and C alone can finish it in twice the time. Find the time when they all together can finish it.

(33) In a square courtyard, square pieces of stone are fixed If the length and the breadth of the courtyard be increased by 5 ft. and 2 ft. respectively, 164 pieces more will be required. Find the length of the court yard.

(34) A person bought some pictures for £ 16 15 s. at 35 s. each picture and some books at 16 s. per book. the number of books was greater than the number of the pictures by 5. Find the number of each.

(35 Queen Victoria was 67 years in 1886 and Prince of Wales, Edward VII, 45 years. In which year was the Queen's age double that of the Prince ?

(36) The fourth and fifth parts of a sum are together equal to £ 2-8 s. less than half the sum. Find the sum.

(37) A sum of £ 1000 was divided among five brothers such that each brother got £ 20 more than his next younger brother. Find the youngest brother's share.

(38) How much tea worth Rs. 1-4-0 per lb. be mixed with tea worth 12 annas per lb so that the mixture may be sold at 14 annas per lb. ?

(39) Two persons A and B start simultaneously from two places, 94 miles apart at 6 o'clock A walks 13 miles in three hours and B, 7 miles in 2 hours. How long will each have walked before they meet ?

CHAPTER XI

52. DIFFICULT SIMULTANEOUS EQUATIONS OF TWO UNKNOWN QUANTITIES

Solved Examples :

Solve : (i) $\frac{x}{a}+\frac{y}{b}=1, \frac{x}{3a}+\frac{y}{6b}=\frac{2}{3}.$

Solution :

$$\frac{x}{a}+\frac{y}{b}=1 \ldots\ldots\ldots\ldots\ldots(i)$$

$$\frac{x}{3a}+\frac{y}{6b}=\frac{2}{3} \ldots\ldots\ldots\ldots\ldots(ii)$$

Multiplying (ii) by 3 we get

$$\frac{x}{a}+\frac{y}{2b}=2.$$

Subtracting this result from (i), we get

$$\frac{y}{2b}=-1, \qquad \text{Or } y=-2b.$$

Substitute $y=-2b$ in (i)

$$\frac{x}{a}-2=1. \qquad \text{or, } \frac{x}{a}=3. \qquad \therefore x=3a.$$

Hence $x=3a$, $y=-2b$.

(ii) $\frac{x+y}{8}+\frac{x-y}{6}=5.$

$$\frac{x+y}{4}-\frac{x-y}{3}=10.$$

Put $x+y=a$, $x-y=b$,

$\therefore$ the first eqn. $= \frac{a}{8} + \frac{b}{6} = 5.$

and the second eqn. $= \frac{a}{4} - \frac{b}{3} = 10.$

Solving as above, we get

$a = 40; \quad b = 0.$

or $x + y = 40,$

and $x - y = 0.$

$\therefore \qquad x = 20 \qquad\qquad$ and $y = 20.$

EXERCISE 34

Solve :—

(1) $\frac{x}{9} - \frac{y}{5} = 3, \qquad \frac{x}{5} + \frac{y}{2} = 14.$

(2) $\frac{x}{\cdot 6} + \frac{y}{\cdot 8} = 5\cdot 5, \qquad \frac{x}{\cdot 2} + \frac{y}{\cdot 5} = 12\cdot 3.$

(3) $\frac{x-y}{3} = \frac{y-1}{4}, \quad \frac{4x-5y}{7} = x - 7.$

(4) $x + y - \frac{2}{3}(x - y) = 6.$

$x - y + \frac{3}{4}(x + y) = 9.$

(5) $\frac{2x-y}{3} + 2y = \frac{1}{2}, \quad \frac{4x+5y}{40} = x - y.$

(6) $\frac{2x+y}{5} - 3 = \frac{3x-5y}{2}, \quad \frac{x}{2} + \frac{y}{3} = \frac{y}{2} - \frac{x}{4} + 8.$

(7) $\frac{m}{x} + \frac{n}{y} = a, \frac{n}{x} + \frac{m}{y} = b.$

Solution : Multiply the first equation by n and the second by m.

Then $\dfrac{mn}{x}+\dfrac{n^2}{y}=an$ .... (i)

and $\dfrac{mn}{x}+\dfrac{m^2}{y}=bn$ ....(ii).

Subtracting (ii) from (i), we get

$\dfrac{n^2-m^2}{y}=an-bm.$

or, $y=\dfrac{n^2-m^2}{an-bm}$. Similarly $x=\dfrac{m^2-n^2}{am-bn}$.

(8) $\dfrac{9}{x}-\dfrac{1}{y}=2\tfrac{3}{4}$, $\dfrac{6}{x}+\dfrac{4}{y}=3.$

(9) $\dfrac{2}{x}+\dfrac{3}{y}=2$, $\dfrac{5}{x}+\dfrac{10}{y}=5\tfrac{5}{6}.$

(10) $\dfrac{2}{x}+\dfrac{5}{y}=\tfrac{5}{6}$, $\dfrac{3}{x}+\dfrac{4}{y}=\dfrac{9}{10}.$

(11) $\dfrac{2}{x}+3y=15$, $\dfrac{5}{x}-4y=3.$

(12) $\dfrac{1}{x}+\dfrac{1}{y}=\tfrac{5}{6}$, $3x+2y=2xy.$

(13) $\dfrac{1}{5x}+\dfrac{y}{9}=5$, $\dfrac{1}{3x}+\dfrac{y}{2}=14.$

(14) $2xy+3=4y$, $3xy+2=5y.$

(15) $\dfrac{2}{x-y}+\dfrac{3}{x+y}=\tfrac{7}{4}$, $\dfrac{3}{x-y}+\dfrac{4}{x+y}=\tfrac{5}{2}.$

(16) $\dfrac{a_1}{x}+\dfrac{b_1}{y}=c_1$, $\dfrac{a_2}{x}+\dfrac{b_2}{y}=c_2.$

(17) $10x+9y=8xy$, $15y-4x=3xy.$

(18) $ax+by=Pxy$, $bx+ay=Qxy.$

(19) $\frac{x}{l}+\frac{y}{m}=1, \quad \frac{m}{x}+\frac{l}{y}=-\frac{l^3}{mxy}$

(20) $(x-3)(y-4)=xy+1.$

$(x+4)(y-3)=xy+11.$

CHAPTER XII

53. SIMULTANEOUS EQUATIONS OF THREE UNKNOWN QUANTITIES

Solved Examples :

Solve :

(1) $5x+4y-3z=32$..............(i).

$4x+5y+7z=78$..............(ii).

$7x+9y-8z=36$..............(iii).

Multiply (i) by 4 and (ii) by 5,

$20x+16y-12z=128.$

$20x+25y+35z=390.$

Subtract the second result from the first.

$-9y-47z=-262$

or, $9y+47z=262$.........(a)

Multiply (ii) by 7 and (iii) by 4,

$28x+35y+49z=546$

$28x+36y-32z=144.$

Subtract the second product from the first.

$-y+81z=402$............(b).

Solve (a) and (b).

$\therefore y=3, z=5.$

Substitute these values in (i).

$5x+12-15=32.$

or $\quad 5x=35. \qquad \therefore \quad x=7.$

Hence $x=7, y=3$ and $z=5$.

(2) $\dfrac{1}{z}+\dfrac{1}{x}=9, \dfrac{1}{y}+\dfrac{1}{z}=12, \quad \dfrac{1}{x}+\dfrac{1}{y}=7.$

Solution : Add the three equations,

$$\frac{2}{x}+\frac{2}{y}+\frac{2}{z}=28$$

$$\text{or } \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=14\text{.........(i)}$$

Subtract the given equations respectively from (i).

$$\therefore \quad \frac{1}{y}=5, \frac{1}{x}=2, \frac{1}{z}=7.$$

$$\therefore \quad x=\tfrac{1}{2}, y=\tfrac{1}{5}, \text{ and } z=\tfrac{1}{7}.$$

(3) $5x+6y+8z=0$.........(i).

$3x+4y+6z=0$.........(ii).

$x+5y+16z=3$.........(iii).

Use the method of cross multiplication which is as follows :—

(1) The denominator of $x=$ [the coefficient of y in equation (i) $\times$ the co-efficient of z in equation (ii)] *minus* [the co-efficient of y in equation (ii) $\times$ the co-efficient of z in equation (i)].

(2) The denominator of $y=$[the co-efficient of z in equation (i) $\times$ the co-efficient of x in equation (ii)] *minus* [the co-efficient of z in equation (ii) $\times$ the co-efficient of x in equation (i)].

(3) The denominator of z=[the co-efficient of x in equation (i) $\times$ the co-efficient of y in equation (ii) *minus* [the co-efficient of x in equation (ii) $\times$ the co-efficient of y in equation (i)].

(4) Put each of these respective fractions equal to a constant quantity, k (suppose), find the values of x, y, z in terms of k. Substitute these values in (iii) and solve it.

Solution : From (i) and (ii)

$$\frac{x}{36-32}=\frac{y}{24-30}=\frac{z}{20-18}.$$

or $\frac{x}{4}=-\frac{y}{6}=\frac{z}{2}$.

or $\frac{x}{2}=-\frac{y}{3}=z=k$ (suppose).

$\therefore$ $x=2k$, $y=-3k$, $z=k$.

Substitute these values in (iii)

$\therefore$ $2k-15k+16k=3$. or $k=1$

$\therefore$ $x=2, y=-3, z=1$.

EXERCISE 35

Solve :—

(1) $6x+7y-3z=23$
$7x+5y-4z=17$
$8x+3y-5z=11$

(2) $5x-2y+z=27$
$3x+18y-3z=15$
$4x+3y+2z=44$.

(3) $x-y-z=-15$
$y+x+2z=40$
$4z-5x-6y=-150$.

(4) $3x+4y-11=0$
$5y-6z=-8$
$7z-8x-13=0$.

(5)
$$2y+z=11$$
$$2z+x=12$$
$$2x+y=13.$$

(6)
$$8x-5y=23$$
$$9y-7z=12$$
$$8z-9x=21.$$

(7)
$$cy+bz=bc$$
$$az+cx=ca$$
$$bx+ay=ab.$$

(8)
$$\frac{1}{x}+\frac{2}{y}=10$$
$$\frac{4}{y}+\frac{3}{z}=18$$
$$\frac{2}{z}+\frac{3}{x}=16$$

(9)
$$\frac{2}{x}-\frac{3}{y}-\frac{4}{z}=3.$$
$$\frac{5}{x}-\frac{9}{y}+\frac{2}{z}=5$$
$$\frac{6}{y}+\frac{7}{x}+\frac{11}{z}=-\frac{5}{4}$$

(10)
$$\frac{1}{2x}+\frac{1}{3y}+\frac{1}{6z}=12$$
$$\frac{1}{2y}+\frac{1}{3z}-\frac{1}{6x}=8$$
$$\frac{1}{3z}+\frac{1}{2x}=10$$

(11)
$$\frac{1}{x}+\frac{1}{y}=3$$
$$y+z=5yz$$
$$z+x=4zx.$$

(12)
$$x+y+z=a+b+c$$
$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=3$$
$$ax+by+cx=a^2+b^2+c^2$$

(13)
$$x+2y+z=0$$
$$8x+10y+3z=0$$
$$6x-3y+8z=87.$$

(14)
$$3x-3y+z=0$$
$$2z+4x-5y=0$$
$$x^2+y^2+z^2=14.$$

(15)
$$2x-4y+2z=0$$
$$5x-6y+2z=0$$
$$2x^2+3y^2+4z^2=99.$$

(16)
$$\frac{xy}{x+y}=\frac{1}{5}$$
$$\frac{xz}{x+z}=\frac{1}{6}$$
$$\frac{yz}{y+z}=\frac{1}{7}$$

CHAPTER XIII

PROBLEMS LEADING TO SIMULTANEOUS EQUATIONS

54. *Solved Examples* :

(1) There are two numbers. Twice the greater is greater than three times the smaller by 28 and their sum is greater than three times their difference by 2. Find the numbers

Solution : Suppose the greater number is x and the other is y.

$\therefore$ by the first condition,

$$2x-3y=28\ldots\ldots\ldots\ldots\ldots\text{(i)},$$

and by the second,

$$(x+y)-3(x-y)=2$$

or, $-2x+4y=2.$

or, $\quad x-2y=-1\ldots\ldots\ldots\ldots\text{(ii)}$

Solve (i) and (ii).

$\therefore x=59, \qquad y=30.$

[Verification : $2\times59=118$, $3\times30=90$, $118-90=28$; also $59+30=89$, $59-30=29$, $3\times29+2=89$.]

(2) If the numerator of a fraction be doubled and 1 be added to its denominator, the fraction becomes $\frac{1}{2}$. If its denominotor be doubled and 1 be added to its numerator, the fraction becomes $\frac{1}{6}$. Find the fraction.

Suppose the fraction is $\dfrac{x}{y}$.

By the first condition,

$$\frac{2x}{y+1}=\tfrac{1}{2}.$$

or, $4x = y + 1$.

or, $4x - y = 1$..................(i).

By the second condition,

$$\frac{x+1}{2y} = \frac{1}{6}$$

or, $6x + 6 = 2y$.

or, $6x - 2y = -6$.

or, $3x - y = -3$...........(ii).

Solve (i) and (ii).

$\therefore x = 4, \quad y = 15$.

$\therefore$ Fr. $= \frac{4}{15}$

[*Verification* : Fr. $= \frac{4}{15}$, $\frac{2 \times 4}{15+1} = \frac{1}{2}$; also $\frac{4+1}{2 \times 15} = \frac{1}{6}$]

(3) The cost price of a mixture of 12 lb of tea and 14 lb of coffee is Rs 27-4 as. The cost price of another mixture of 10 lb. of tea and 9 lb. of coffee is Rs. 20-6 as. Find the cost price of 1 lb. of each article.

Solution : Suppose the C. P. of 1 lb. of tea is annas x, and that of 1 lb. of coffee is annas y.

By the given conditions

$12x + 14y =$ as. 436 and

$10x + 9y =$ as. 326.

Solve these equations.

$\therefore x =$ as. 20, $\quad y =$ as. 14.

Hence the C. P. of 1 lb. tea = Re. 1-4 as. and that of 1 lb. coffee = as. 14.

[*Verification* : Rs. $12 \times \frac{5}{4}$ + Rs. $14 \times \frac{14}{16}$ = Rs 15 + Rs. 12 4 as. Rs. 27-4-0; also Rs $10 \times \frac{5}{4}$ + Rs. $9 \times \frac{14}{16}$ = Rs. $\frac{25}{2}$ + Rs. $\frac{63}{8}$ = Rs. 20-6].

(4) Five years ago A's age was twice B's and after six years the sum of their ages will be 82. Find their present ages.

Solution : Suppose A's present age is x years and B's, y years

Five years before A was $x-5$ years old while B, $y-5$ years old.

$\therefore$ by the first condition

$$x-5=2(y-5)$$

or, $x-2y=-5$..............(i)

By the second condition,

$$x+6+y+6=82.$$

or, $x+y=70$................(ii)

Solve (i) and (ii)

$\therefore$ $x=45$, $y=25$.

Hence A's present age is 45 years and B's present age is 25 years.

[*Verification* : $45-5=40$, $25-5=20$, $20\times2=40$; also $45+6=51$, $25+6=31$, $51+31=82$].

EXERCISE 36

(1) Of the two numbers if three be added to the greater the sum is equal to two times the other. The difference between the number is 7. Find the numbers.

(2) The sum of two numbers when divided by 5 gives 10 as quotient. If their difference be divided by 2, the quotient is 11. Find the two numbers.

(3) Divide 750 into two parts such that the sum of $\frac{1}{5}$ of the smaller share and $\frac{1}{10}$ of the greater is less than the difference of the numbers by 150

(4) The sum of three numbers is 100. If the second is divided by the first, the quotient is 5 and the remainder is 1. The same is the result when the third is divided by the second. Find the three numbers.

(5) Find two numbers such that the ratio between the greater and the smaller is equal to the ratio of their sum and 42

(6) Find two numbers such that their sum, difference and product are in the ratio 3 : 2 : 5.

(7) If 4 is added to the numerator of a fraction the fraction becomes $\frac{3}{4}$. When 3 is added to the denominator, the fraction becomes $\frac{1}{3}$. Find the fraction

(8) If 3 be subtracted from each of the numerator and denominator of a fraction, the fraction becomes $\frac{2}{3}$ But if 3 be subtracted from the numerator and the same be added to the denominator, the fraction becomes $\frac{1}{3}$. Find the fraction.

(9) Find two fractions such that their denominators are respectively 3 and 4 and their sum is $1\frac{5}{12}$. Also if their numerators be interchanged, the sum of the fractions is $1\frac{1}{3}$.

(10) 13 horses and 5 mules together carry 20 tons and 5 horses and 13 mules together carry 16 tons. How much load can a horse and a mule carry separately ?

(11) I sold 9 horses and 7 cows to a man for Rs. 4,500 and to another man 6 horses and 13 cows for the same price. Find the price of a horse and that of a cow.

(12) Four times the age of A is greater than B's age by 16 years and A's one fifth age equals to B's one sixteenth age. Find their ages.

(13) A's age is twice that of B's. In two years' time B's age will be twice C's, and in 12 years, A's age will be twice C's. Find their present ages.

(14) A's age is three times the age of B. in 11 years A's age will be four times the present age of B two years back. Find their present ages.

(15) A number is made of two digits of which the unit's digit is greater. If the number is divided by the sum of the digits, the quotient is 4. If the digits are reversed and the new number is divided by the sum of the difference of the digits and 2, the quotient is 14. Find the number

Solution : Suppose the unit's digit is x and ten's, y.

Number $= 10y + x$.

By the first condition.

$$\frac{10y+x}{x+y} = 4.$$

or, $2y = x$.. ....(i).

The new number with reversed digits $= 10x + y$

By the second condition.

$$\frac{10x+y}{x-y+2} = 14.$$

or, $-4x + 15y = 28$.............(ii).

Solve (i) and (ii).

$\therefore\ x = 8, y = 4$.

Hence the numcer is 48.

[Verification : No. is 48, sum of the digits $= 12$; $48 \div 12 = 4$; also reversed No. is 84, $84 \div (8 - 4 + 2)$ or $84 \div 6 = 14$].

(16) A number of two digits is greater than the number with digits reversed by 45. When the number is divided by the sum of the digits the quotient is 8. Find the number.

(17) A two digit-number is greater than six times the sum of the digits by I and the ten's digit is greater than the unit's digit by 2. Find the number.

(18) If 54 be added to a number, the digits are reversed. The sum of the number and the reversed number is 132. Find the number.

(19) A man got an increment of £20 in his salary. But on account of the increase in income-tax from 1s. to 1s. 2*d*, per £, his actual increase in salary amounted to £ 10 only. Find his initial salary.

(20) A bicycle dealer takes a profit of 25 per cent. Another dealer buys a cycle for £ 1 less and sells it £ 1 cheaper By so doing he draws a profit of $27\frac{1}{2}$ per cent. For how much does the first dealer buy a cycle?

(21) A cyclist riding with a uniform speed, wishes to cover a distance of 15 miles in $1\frac{1}{2}$ hours. After covering a short distance he is obliged to go 3 miles an hour on account of an accident He reaches his destination 35 minutes late. How far had he gone when he met with the accident?

(22) In an educational exhibition there were 1950 visitors Outsiders were charged Re. 1; teachers, as 8 and students anna 1 as entrance fee per head. The total proceeds were Rs. 350. If the students were 8 times the teachers, find the number of teachers.

(23) To cover every 80 sq. ft., a mason requires 6 stones of one kind and 8 of another. To cover 735 sq. ft., he requires 40 stones of the first and 90 of the second kind. Find the dimensions of each stone.

(24) A sum of Rs. 100 contains 1 anna and 4 anna nickel pieces. The number of 4 anna pieces is greater than 3 times the

number of one anna pieces by 75. Find the number of each kind of coins.

Solution : Suppose there are x one anna pieces and y four anna pieces.

$\therefore$ total amount $=$ as. $(x+4)$.

By the first condition,

$x+4y=100\times16=1600$...........(i)

By the second condition,

$y-3x=75$..............(ii)

Solve (i) and (ii)

$\therefore$ $x=100$. $y=375$

Hence there are 375 four anna pieces and 100 one anna pieces.

(25) A purse contains £8-12s. in half crowns and shillings. If 6 half crowns more are added the number of half crowns is three times the number of shillings. Find the number of each kind of coins,

(26) A purse contains Rs. 120 in rupees and eight anna pieces only. If one fourth of the eight anna pieces and $\frac{1}{5}$ of rupees are spent, the purse contains Rs. 95 only. Find the number of both kinds of coins

(27) A boy paid his fee of Rs. four in two anna and half anna pieces. Three times the number of two anna pieces was greater than the number of half anna pieces by 5. How many pieces were of each kind ?

(28) A man and a boy together do a work in $4\frac{4}{5}$ days. They both work for two days when the man falls ill and boy himself completes the remaining work in 7 days. How many days will each require to finish it ?

(29) If the length and breadth of a tennis court be each increased by 2 yards, the ratio of length and breadth becomes

2 : 1. But if they are decreased by 2 yards, the ratio is 9 : 4, Find the length and the breadth of the court.

(30) A cistern is filled by 3 pipes, two of which are quite similar. If all the three pipes are opened they fill two third of the cistern in 5 hours. But if one of the two equal pipes be closed, the remaining two will fill three-fourth of the cistern in 9 hours. In how many hours does each fill the cistern separately ?

(31) Two trains 92 ft and 84 ft. long respectively running in opposite directions, cross each other in $1\frac{1}{2}$ seconds. Had they been running in the same direction the fast train would have crossed the other in 5 seconds. Find the speed in miles of each train per hour.

(32) In a legislative assembly a resolution was passed by the majority of one-third of voters If 10 of the oppositions had voted in favour the majority would have been only half of the total votes. Find the number of voters.

Solution : Suppose x votes were cast in favour and y, against the proposition.

$\therefore$ Total votes cast$=x+y$.

By the first condition,

$x-y=\frac{1}{3}(x+y)$

or $x-2y=0$.........(i)

By the second condition,

$(x+10)-(y-10)=\frac{1}{2}(x+y)$

or $x-3y=-40$............(ii)

Solve (i) and (ii)

$\therefore\ x=80,\ y=40$

$\therefore$ Total votes$=120$.

(33) In an election 907 votes were cast for A and B. If A got 179 more votes, how many votes did each get ?

(34) In a legislative assembly a resolution was passed by the majority of half the total votes. If 8 more votes were cast in opposition from those in favour, the majority would have been only $\frac{1}{3}$ of the total votes. Find the number of voters on each side.

·(35) A boat goes 20 miles against the current and 55 miles with the current in 10 hours. It goes 30 miles against and 77 miles with the current in $14\frac{1}{3}$ hours. Find the speed of the current and the boat in miles per hour.

(36) A scout party came to know in their camp that if in each tent 3 scouts are lodged, three tents more are required and if 4 scouts live in a tent, two tents are left unoccupied. Find the number of scouts and tents.

(37) The area of a room is 96 sq. ft. and the total length and breadth of its four walls is 40 ft. Find the length and breadth of the room.

(38) A number is made of 3 digits whose sum is 10. The middle digit is equal to the sum of the other two digits. If the digits are reversd the new number is greater than the original number by 99 Find the number.

CHAPTER XIV

IDENTITIES

55. *Consider the following* :

(i) $(a \pm b)(a^2 \mp ab + b^2) = a^3 \pm b^3$.

(ii) $(a+b+c)(a^2+b^2+c^2-ab-bc-ca) = a^3+b^3+c^3-3abc$.

(iii) $x^2(y-z)+y^2(z-x)+z^2(x-y) = -(x-y)(y-z)(z-x)$.

In each of the above statements, the expressions on both sides of the sign of equality are always equal *whatever the value of a, b, c, x, y, z may be.*

Such statements are called 'Identities'.

Identities are different from equations in as much as the sides of the equations are equal for *particular values of the unknown* terms.

56. *Solved Example* :

(1) If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{g}{h}$, prove that each of these ratios is equal to $\frac{a+c+e+g}{b+d+f+h}$.

Solution : Put each of the ratios equal to k.

That is, $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{g}{h}=k$

$\therefore\ a=bk,\ c=dk,\ e=fk,\ g=hk.$

$$\therefore\ \frac{a+c+e+g}{b+d+f+h}=\frac{bk+dk+fk+hk}{b+d+f+h}=\frac{k(b+d+f+h)}{b+d+f+h}=k$$

$$=\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{g}{h}$$

(2) If $(a+bc)^2(1-a^2)=(b+ac)^2(1-b^2)$, prove that
$a^2+b^2+c^2+2abc=1.$

Solution :

$(a+bc)^2(1-a^2)=(a+bc)^2-a^2(a+bc)^2$

and $(b+ac)^2(1-b^2)=(b+ac)^2-b^2(b+ac)^2$

$\therefore\ (a+bc)^2-a^2(a+bc)^2=(b+ac)^2-b^2(b+ac)^2.$

or $(a+bc)^2-(b+ac)^2=a^2(a+bc)^2-b^2(b+ac)^2.$

$=(a^2+abc)^2-(b^2+abc)^2.$

or $(a+bc+b+ac)(a+bc-b-ac).$

$=(a^2+abc+b^2+abc)(a^2+abc-b^2-abc)$

or factorize,

$\therefore\ (a+b)(1+c)(a-b)(1-c)=(a^2+b^2+2abc)(a^2-b^2).$

Divide both sides by a^2-b^2.

$\therefore\ (1+c)(1-c)=a^2+b^2+2abc.$

or $1-c^2 \qquad =a^2+b^2+2abc.$

or $1 \qquad =a^2+b^2+c^2+2abc.$

or $a^2+b^2+c^2+2abc=1.$

(3) If $2s=a+b+c$, prove that

$(s-a)^2+(s-b)^2+(s-c)^2+2(ab+ac+bc)=3s^2.$

Solution :

$$\text{L. H. S.}=s^2-2as+a^2+s^2-2bs+b^2+s^2-2cs+s^2$$
$$+2ab+2ac+2bc$$
$$=3s^2-2s(a+b+c)+a^2+b^2+c^2+2ob+2ac+2bc$$
$$=3s^2-2s.\ 2s+(a+b+c)^2$$
$$=3s^2-4s^2+4s^2$$
$$=3s^2$$
$$=\text{R. H. S.}$$

(4) Prove that $\dfrac{1}{(b-c)^2}+\dfrac{1}{(c-a)^2}+\dfrac{1}{(a-b)^2}$

$$=\left(\frac{1}{b-c}+\frac{1}{c-a}+\frac{1}{a-b}\right)^2$$

Solution : Suppose $b-c=x$, $c-a=y$, $a-b=z$.

$\therefore$ L. H. S. $=\dfrac{1}{x^2}+\dfrac{1}{y^2}+\dfrac{1}{z^2}$ and

$x+y+z=b-c+c-a+a-b=0.$

R. H. S. $=\left(\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}\right)^2$

$$=\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}+\frac{2}{xy}+\frac{2}{yz}+\frac{2}{zx}$$

$$=\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}+\frac{2(x+y+z)}{xz}$$

$$=\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\ [\because x+y+z=0]$$

Restore the value of x, y, z.

$$\therefore \text{R. H. S.} = \frac{1}{(b-c)^2} + \frac{1}{(c-a)^2} + \frac{1}{(a-b)^2}$$
$$= \text{L. H. S.}$$

EXERCISE 37

(1) If $x - \frac{1}{x} = 1$, prove that $x^3 - \frac{1}{x^3} = 4$.

(2) If $a(x^2 - yz) + b(y^2 - zx) + c(z^2 - xy) = 0$
and $a + b + c = 0$. prove that $ax + by + cz = 0$.

(3) If $a + b + c = 2x$ and
$a^2 + ab + b^2 + x^2 = 2x(a + b)$, prove that
$(a - x)^2 + (b - x)^2 + (c - x)^2 = x^2$.

(4) If $\frac{a}{b} = \frac{c}{d}$, prove that $abcd = \dfrac{a^2 + b^2 + c^2 + d^2}{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + \frac{1}{d^2}}$

(5) Prove that
$(x-a)(x-b)(a-b) + (x-b)(x-c)(b-c) + (x-c)(x-a)$
$(c-a) \equiv -(b-c)(c-a)(a-b)$

(6) If $a + b + c = 0$, prove that $a^3 + b^3 + c^3 = 3abc$.

(7) Prove that
$(y - z)^3 + (z - x)^3 + (x - y)^3 = 3(y - z)(z - x)(x - y)$.

If $s = a + b + c$, prove that :

(8) $(s-a)(s-b)(s-c) = (a+b+c)(bc+ca+ab) - abc$.

(9) $(b+c)s(s-a) + a(s-b)(s-c) - 2bcs$
$\equiv (c+a)s(s-b) + b(s-c)(s-a) - 2cas$.

(10) If $2s = a + b + c$ prove that :

$$a^2 - \left(\frac{a^2 + b^2 - c^2}{2b}\right)^2 = \frac{4s(s-a)(s-b)(s-c)}{b^2}.$$

(11) $(s-a)^3+(s-b)^3+(s-c)^3+3abc=s^3$.

(12) $a(s-b)(s-c)+b(s-c)(s-a)+c(s-a)(s-b)$
$+2(s-a)(s-b)(s-c)=abc$.

(13) If $x+y+z=0$, prove that :

$$\frac{x^2}{2x^2+yz}+\frac{y^2}{2y^2+xz}+\frac{z^2}{2z^2+xy}=1.$$

(14) If $\frac{x}{y+z}=a, \frac{y}{z+x}=b, \frac{z}{x+y}=c$, prove that :

$$\frac{x^2}{a(1-bc)}=\frac{y^2}{b(1-ca)}=\frac{z^2}{c(1-ab)}$$

(15) If $x+y+z=xyz$, prove that :

$$\frac{2x}{1-x^2}+\frac{2y}{1-y^2}+\frac{2z}{1-z^2}=\frac{2x}{1-x^2}\times\frac{2y}{1-y^2}\times\frac{2z}{1-z^2}.$$

(16) If $x+y+z=0$, prove that :

$$\left(\frac{x+y}{z}+\frac{z+x}{y}+\frac{y+z}{x}\right)\times\left(\frac{x}{y+z}+\frac{y}{z+x}+\frac{z}{x+y}\right)=9.$$

Solution : $\because x+y+z=0$,

$\therefore y+z=-x, z+x=-y, x+y=-z$.

L. H. S. $=\left(-\frac{z}{z}-\frac{y}{y}-\frac{x}{x}\right)\left(-\frac{x}{x}-\frac{y}{y}+\frac{z}{z}\right)$

$=-3\times-3$.

$=9$.

(17) If $y=\frac{1+x}{1-x}$ prove that :

$$\left(x-\frac{1}{x}\right)\left(y-\frac{1}{y}\right)=4.\frac{xy+1}{x-y}.$$

(18) Prove that $\frac{bc}{(x-b)(x-c)}+\frac{ca}{(x-c)(x-a)}+\frac{ab}{(x-a)(x-b)}=0$,

If $x=\frac{3abc}{ab+bc+ca}$

(19) If $x=\dfrac{b^2+c^2-a^2}{2bc}$, $y=\dfrac{c^2+a^2-b^2}{2ca}$, $z=\dfrac{a^2+b^2-c^2}{2ab}$.

prove that $(b+c)x+(c+a)y+(a+b)z=a+b+c$.

(20) If $x^2+y^2+z^2=xy+yz+zx$, prove that :

$(y-z)^2+(z-x)^2+(x-y)^2=0$

(21) If $x=b+c$, $y=c+a$, $z=a+b$, prove that :

$\dfrac{x^3+y^3+z^3-3xyz}{a^3+b^3+c^3-3abc}=2.$

(22) Prove that $x(y+z)^2+y(z+x)^2+z(x+y)^2$

$=(x+y)(y+z)(z+x)+4xyz$

(23) If $a+b+c=0$, prove that

$a^2+ab+b^2=b^2+bc+c^2=c^2+ca+a^2$.

(24) If $\dfrac{b^2+c^2-a^2}{2bc}+\dfrac{c^2+a^2-b^2}{2ac}+\dfrac{a^2+b^2-c^2}{2ab}=0,$

prove that $(b+c-a)(c+a-b)(a+b-c)=0.$

(25) If $x=\dfrac{ab}{a+b}$, prove that:

$\dfrac{x+2a}{2b-x}+\dfrac{x-2a}{2b+x}+\dfrac{4ab}{x^2-4b^2}=0.$

(26) If $x+\dfrac{1}{x}=a$, $y+\dfrac{1}{y}=b$, $z+\dfrac{1}{z}=c$, prove that :

$a^2+b^2+c^2=4+(x^2+\dfrac{1}{x^2}+y^2+\dfrac{1}{y^2}+z^2+\dfrac{1}{z^2}+2)$

(27) If $x^2-7x+10\equiv A(x^2-1)+B(x^2-x-2)$ $+C(x-1)(x-2)$, prove that $A=0$, $B=-2$, $C=3$.

(28) Prove that $(a+b+c)^3-a^3-b^3-c^3$

$=3(a+b)(b+c)(c+a).$

(29) If $3x^2+x-2\equiv a(x-2)^2+b(1-2x)(x-2)+c(1-2x)$, prove that $a=-\frac{1}{8}$, $b=-\frac{5}{8}$. $c=-4$

(30) If $\left(x+\frac{1}{x}\right)^2=3$, prove that $x^3+\frac{1}{x^3}=0$.

(31) Prove that $a^2(b+c)+b^2(c+a)+c^2(a+b)+3abc$
$=(a+b+c)\,(ab+bc+ca)$.

(32) If $x+y+z=p$ and $x^2+y^2+z^2=q$. prove that

$yz\,(y+z)+zx(z+x)+xy(x+y)+3xyz$

$=\frac{1}{2}(p^3-pq)$.

CHAPTER XV

RATIO AND PROPORTION

57. *Ratio* : The age of a man is 45 years, and that of his son, 15 years. The man is three times older than his son or the ratio between their ages is 3. In the same way the length of a fort is 40′ and breadth, 25′, the ratio of the length and breadth is $\frac{40}{25}$ or $\frac{8}{5}$.

Definition : The ratio between two quantities of the same kind is the arithmetical abstract number which expresses what fraction or multiple the first quantity is of the other. The ratio between two quantities is expressed by the sign (:) ; as for instance, the ratio between a and b is expressed as $a : b$ In this ratio a and b are the *quantities* or *terms*, a is called the *antecedent* and b, the *consequent*.

When both the terms of a ratio are equal. the ratio is called the Ratio of Equality and is equal to 1, as 8 : 8

When the antecedent of a ratio is greater than its consequent the ratio is called the Ratio of Greater Inequality and it is greater than 1 is 4 : 3 when the antecedent is less than the consequent, the ratio as called the Ratio of Lesser Inequality and it is less than 1, as 5 : 9.

If the terms of a ratio are multiplied or divided by another quantity, the value of the ratio does not change, as

$3 : 4 = 3 \times 5 : 4 \times 5$ and $3 \div 6 : 4 \div 6$.

When the product of two antecedents of two ratios is the antecedent of a new ratio and the product of two consequents of two ratios is the consequent of a new ratio, the new ratio is called the Compound Ratio, as $\frac{3}{4}$ and $\frac{5}{24}$ form $\frac{15}{96}$ as the Compound Ratio.

Similarly $2 : 3$, $3 : 4$, $4 : 5$ form $24 : 60 = 2 : 5$ as the compound ratio, or $a : b$, $c : d$ $e : f$ form $ace : bdf$ as the compound ratio.

When $\frac{a}{b}$ is multiplied by $\frac{a}{b}$, the result $\frac{a^2}{b^2}$ is called the Duplicate Ratio. Similarly $\frac{a^3}{b^3}$ is called the Triplicate Ratio.

The square root of $9x^2 : 16y^2$ is $3x : 4y$. $3x : 4y$ is called the sub-duplicate Ratio of $9x^2 : 16y^2$. Similarly $5a : 4b$ is the Sub-triplicate Ratio of $125a^3 : 64b^3$.

58. IMPORTANT THEOREMS

(1) If a, b, x are positive integers, $a : b \gtrless (a+x) : (b+x)$ as $a \gtrless b$.

Proof : $\frac{a+x}{b+x} - \frac{a}{b} = \frac{b(a+x) - a(b+x)}{b(b+x)} = \frac{x(b-a)}{b(b+x)}$.

Since a and b are positive, therefore, $\frac{a+x}{b+x} \gtrless \frac{a}{b}$ as $a \gtrless b$

i. e. $\frac{a}{b} \gtrless \frac{a+x}{b+x}$ as $a \gtrless b$

Hence, (i) when a positive quantity is added to each of the antecedent and consequent of a ratio (of greater inequality) the value of the ratio diminishes.

(ii) When a positive quantity is added to each of the

antecedent and consequent of a given ratio of lesser inequality, the value of the ratio increases.

(iii) When a positive quantity is subtracted from each of the antecedent and consequent of a ratio of greater inequality, the value of the ratio increases, and

(iv) When a positive quantity is subtracted from each of the antecedent and consequent of a ratio of lesser inequality, the value of the ratio diminishes.

59. *Solved Examples* : (1) The ratio between two quantities is 4 : 5. If 6 be added to each of the quantities the ratio becomes 5 : 6. Find the quantities.

Solution : Since the ratio is 4 : 5, the quantities may be denoted as $4x : 5x$.

$\therefore$ By the given condition,

$$\frac{4x+6}{5x+6}=\frac{5}{6}.$$

Solve the equation

$\therefore x=6$.

$\therefore$ the quantities are 24 and 30.

(2) For what value of x is $\frac{3x+17}{5x+13}$ equal to $\frac{7}{4}$.

Solution : $\frac{3x+17}{5x+13}=\frac{7}{4}$

Multiply across,

$$35x+91=12x+68$$

or $x=-1$

(3) Find the ratio between x and y from $\frac{x-3y}{2y}=\frac{6x-5y}{5x}$.

Solution :

$$\frac{x-3y}{2y}=\frac{6x-5y}{5x}.$$

Multiply across,

or $5x^2-15xy=12xy-10y^2$

or $5x^2-27xy+10y^2=0$

Divide both sides by y^2

$$\therefore \frac{5x^2}{y^2}-\frac{27xy}{y^2}+\frac{10y^2}{y^2}=0$$

or $$5\left(\frac{x}{y}\right)^2-27\left(\frac{x}{y}\right)+10=0$$

Factorize,

$$\left(\frac{5x}{y}-2\right)\left(\frac{x}{y}-5\right)=0$$

$$\therefore \quad \frac{x}{y}=\frac{2}{5} \text{ or } \frac{x}{y}=5.$$

(4) If b is positive and $a > b$, prove that :

$$\frac{a-b}{a+b}<\frac{a^2-b^2}{a^2+b^2}.$$

Solution : Multiply the quantities on L. H. S. by $a+b$.

$$\text{Product}=\frac{(a-b)(a+b)}{(a+b)^2}=\frac{a^2-b^2}{(a+b)^2}$$

$$=\frac{a^2-b^2}{a^2+2ab+b^2}$$

Now since b is positive and $a>b$,

$$\therefore \quad a^2+2ab+b^2>a^2+b^2$$

or $$\frac{a^2-b^2}{a^2+2ab+b^2}<\frac{a^2-b^2}{a^2+b^2}$$

$$\therefore \frac{a-b}{a+b}<\frac{a^2-b^2}{a^2+b^2}.$$

EXERCISE 38

(1) If $5+x : 7+x=4 : 5$, find the value of x.

(2) Two numbers are in the ratio of $2 : 3$. If 6 is added, to each of them the ratio becomes $7 : 9$. Find the numbers.

(3) If $x : y = 1 : 2$ and $x - 5 : y - 5 = 2 : 5$, find the values of x and y.

(4) If $x - 4y : x + 4y = 1 : 2$, find the value of

$$3x + 5y : 3x - 5y.$$

(5) Find the compound ratio which is formed with $a + x : a - x$, $a^2 + x^2 : (a + x)^2$ and $(a^2 - x^2)^2 : a^4 - x^4$.

(6) Two numbers are in the ratio of $7 : 8$ and their sum is 135. Find them.

(7) Find the ratio which is formed with the duplicate ratio of $10 : 3$ and triplicate ratio of $2 : 5$.

(8) If $9x^2 - 6xy + y^2 = 0$, find the value of $x : y$

(9) If $3x^2 - 7xy + 2y^2 = 0$, find the value of $x : y$.

(10) Divide a straight line a'' long into the ratio of $p : q$ and find the length of each part.

(11) If $x : a = y : b = z : c$, prove that

$$\frac{a^3 + b^3 + c^3}{x^3 + y^3 + z^3} = \frac{abc}{xyz}$$

Solution: Suppose $\dfrac{x}{a} = \dfrac{y}{b} = \dfrac{z}{c} = k$.

Then $x = ak$, $y = bk$, $z = ck$.

$$\text{R. H. S.} = \frac{abc}{ak.\,bk.\,ck.} = \frac{1}{k^3}$$

$$\text{L. H. S.} = \frac{a^3 + b^3 + c^3}{a^3k^3 + b^3k^3 + c^3k^3} = \frac{a^3 + b^3 + c^3}{k^3(a^3 + b^3 + c^3)}$$

$$= \frac{1}{k^3}.$$

$\therefore$ R. H. S. = L. H. S.

(12) If $(a^2 + b^2)(x^2 + y^2) = (ax + by)^2$. prove that $x : a = y : b$.

(13) $x : a = y : b = z : c$, prove that each of the ratios $= \sqrt{\dfrac{yz+zx+xy}{ab+bc+ca}}$

(14) If $a : x+y = b : y+z = c : z+x$, prove that each ratio $= \dfrac{a+b+c}{2(x+y+z)}$.

(15) If $\dfrac{x}{b+c} = \dfrac{y}{c+a} = \dfrac{z}{a+b}$, prove that :

$$\frac{a}{y+z-x} = \frac{b}{z+x-y} = \frac{c}{x+y-z}.$$

(16) The ages of two persons are in the ratio of 2 : 3. After 7 years the ratio will become 3 : 4. Find their present ages.

If $\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{e}{f}$. prove that each of the ratios is equal to :

(17) $\sqrt[3]{\dfrac{a^3+c^3+e^3}{b^3+d^3+f^3}}$.

(18) $\sqrt[3]{\dfrac{4ac^2-3ce^2+2ace}{4bd^2-3df^2+2bdf}}$.

(19) Two troops have respectively 11000 and 7000 soldiers. Each troop is reinforced by 1000 men more. Which troop is the gainer in ratio ?

(20) In a purse the ratio between rupees, eight-anna and two-anna pieces is 2 : 8 : 9. If the total amount is Rs. 92-10as, find the nuumber of each kind of coins.

PROPORTION

60. *Definitions* : The equality of two ratios is called *Proportion* As for example, the ratio between 4 and 12 is the

same as between 6 and 18. Therefore 4, 12, 6, 18 are in proportion, and are written as $4 : 12 = 6 : 18$ or $4 : 12 :: 6 : 18$. Similarly if a, b, c, d are in proportion they are written as $a : b = c : d$ or $a : b :: c : d$. Here a, d are called *extremes* and b, c, *means*. a is called the *first proportional*, b, the *second proportional*; c, the *third proportional* and d, the *fourth proportional*.

61. If $\frac{a}{b} = \frac{c}{d}$, then by multiplying across $ad = bc$.

That is, the product of the extremes is equal to the product of the means.

If $\frac{a}{b} = \frac{b}{c}$, then by multiplying across, $ac = b^2$.

That is, the product of the first and third proportionals is equal to the square of the second proportional. In this case a, b, c, are said to be in *continued proportion*, and the middle term b is called the *mean proportional* and a and c are called the *extremes*.

62. Some Important Theorems.

(1) If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$.

Proof: Divide 1 by each ratio,

$\therefore 1 \div \frac{a}{b} = 1 \div \frac{c}{d}$, or $\frac{b}{a} = \frac{d}{c}$.

This ratio is called '*Invertendo*'.

(2) If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$.

Proof: Multiply each ratio by $\frac{b}{c}$.

$\therefore \frac{a}{b} \times \frac{b}{c} = \frac{c}{d} \times \frac{b}{c}$.

or $\frac{a}{c} = \frac{b}{d}$.

This ratio is called '*Alternando*'.

(3) If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a+b}{b}=\frac{c+d}{d}$.

Proof: Add 1 to each ratio.

Then $1+\frac{a}{b}=1+\frac{c}{d}$.

or $\frac{a+b}{b}=\frac{c+d}{d}$.

This result is called "*Componendo.*"

(4) If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a-b}{b}=\frac{c-d}{d}$.

Proof: Subtract 1 from each ratio.

Then $\frac{a}{b}-1=\frac{c}{d}-1$.

or $\frac{a-b}{b}=\frac{c-d}{d}$.

This result is called "*Dividendo.*"

(5) If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a+b}{a-b}=\frac{c+d}{c-d}$.

Proof: Because $\frac{a}{b}=\frac{c}{d}$

$\therefore \frac{a+b}{b}=\frac{c+d}{d}$ ....(i) (*Componendo*)

and $\frac{a-b}{b}=\frac{c-d}{d}$ ....(ii) (*Dividendo*)

$\therefore \frac{a+b}{b}\div\frac{a-b}{b}=\frac{c+d}{d}\div\frac{c-d}{d}$

or $\frac{a+b}{a-b}=\frac{c+d}{c-a}$.

This result is called '*Componendo and Dividendo.*'

63. *Solved Examples :*

(1) Find the fourth proportional of 3, 4, 12.

Solution : Suppose the fourth proportional is x.

Then $\dfrac{3}{4}=\dfrac{12}{x}$.

or $3x=48$. $\therefore x=16$,

(2) Find the third proportional of 4, 7.

Solution : Suppose the third proportional is x.

Then $4:7=7:x$.

or $4x=49$.

$\therefore$ $x=12\frac{1}{4}$.

(3) Find the mean proportional 3, 27.

Solution : Suppose the mean proportional is x.

Then $\dfrac{3}{x}=\dfrac{x}{27}$.

or $x^2=81$. $\therefore x=9$.

(4) Find the value of $\dfrac{x+2a}{x-2a}+\dfrac{x+2b}{x-2b}$ when $x=\dfrac{4ab}{a+b}$

Solution : Because $x=\dfrac{4ab}{a+b}$

$$\therefore \frac{x}{2a}=\frac{2b}{a+b} \text{ and } \frac{x}{2b}=\frac{2a}{a+b}$$

Therefore by Componendo and Dividendo

$$\frac{x+2a}{x-2a}=\frac{2b+a+b}{2b-a-b}=\frac{a+3b}{b-a}$$

and $\dfrac{x+2b}{x-2b}=\dfrac{2a+a+b}{2a-a-b}=\dfrac{3a+b}{a-b}$.

$\therefore$ the exp. $=\dfrac{a+3b}{b-a}+\dfrac{3a+b}{a-b}$

$$=\frac{3a+b}{a-b}-\frac{a+3b}{a-b}$$

$$=\frac{2(a-b)}{a-b}=2.$$

Second Method :

$$\frac{x+2a}{x-2a}+\frac{x+2b}{x-2b}$$

$$=1+\frac{4a}{x-2a}+1+\frac{4b}{x-2b}$$

$$=2+4\left\{\frac{a}{x-2a}+\frac{b}{x-2b}\right\}$$

$$=2+4\left\{\frac{a(x-2b)+b(x-2a)}{(x-2a)(x-2b)}\right\}$$

$$=2+4\left\{\frac{ax-2ab+bx-2ab}{(x-2a)(x-2b)}\right\}$$

$$=2+4\left\{\frac{x(a+b)-4ab}{(x-2a)(x-2b)}\right\},\left[\because x=\frac{4ab}{a+b}\ \therefore\ x(a+b)=4ab\right]$$

$$=2+4\times\frac{0}{(x-2a)(x-2b)}=2+0=2.$$

(5) If $(a+b+c+d)(a-b-c+d)=(a+b-c-d)(a-b+c-d)$ prove that $a:b=c:d$.

Solution : Because $(a+b+c+d)(a-b-c+d)$

$$=(a+b-c-d)(a-b+c-d)$$

$$\therefore\frac{a+b+c+d}{a+b-c-d}=\frac{a-b+c-d}{a-b-c+d}$$

By Componendo and Dividendo,

$$\frac{a+b+c+d+a+b-c-d}{a+b+c+d-a-b+c+d}=\frac{a-b+c-d+a-b-c+d}{a-b+c-d-a+b+c-d}$$

or, $\dfrac{a+b}{c+d}=\dfrac{a-b}{c-d}.$

By Alternando,

$$\frac{a+b}{a-b}=\frac{c+d}{c-d}.$$

Again by Componendo and Dividendo.

$$\frac{a+b+a-b}{a+b-a+b}=\frac{c+d+c-d}{c+d-c+d}\text{ or, }\frac{a}{b}=\frac{c}{d}.$$

EXERCISE 39

Find the fourth proportional of the following:—

(1) 2, 3, 6. (2) a, $3b$, $2a$.

(3) a, ab, c. (4) $15x$, $12y$, $15z$.

(5) $7p$, $11q$, $14r$.

Find the third proportional of the following:—

(6) 3, 8. (7) 9, 15. (8) x^2, $xy+y^2$

If $\frac{a}{b}=\frac{c}{d}$ prove that:— (9) $\frac{la^2+mc^2}{lb^2+md^2}=\frac{ac}{bd}$

(10) $\frac{a^2+ab+b^2}{a^2-ab+b^2}=\frac{c^2+cd+d^2}{c^2-cd+d^2}$. (11) $\frac{ma+nb}{mc+nd}=\frac{b^2c}{d^2a}$

(12) $\frac{a}{a+c}=\frac{a+b}{a+b+c+d}$.

(13) $\frac{a^2b-3ac}{b^3-3bd^2}=\frac{a^2+5c^2}{b^2+5d^2}$.

(14) $\frac{c}{d}=\frac{\sqrt{a^2+c^2}}{\sqrt{b^2+d^2}}$.

(15) If $a-b:b-c=b:c$, prove that a, b, c are in continued proportion.

(16) If $a^2+c^2:ab+cd=ab+cd:b^2+c^2$, prove that $a:b=c:d$.

(17) If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$, prove that each ratio is equal to

$$\left[\frac{pa^n+qc^n+re^n}{pb^n+qd^n+rf^n}\right]^{\frac{1}{n}}$$

when p, q, r n are any integers.

Solution : Suppose each ratio $= k$.

$\therefore\ a = bk,\ c = dk,\ e = fk.$

or $pa^n = pb^n k^n,\ qc^n = qd^n k^n$ and $re^n = rf^n k^n$

or $pa^n + qc^n + re^n = k^n (pb^n + qd^n + rf^n)$.

or $k^n = \left[\dfrac{pa^n + qc^n + re^n}{pb^n + qd^n + rf^n}\right]$

$$\therefore\ \left[\frac{pa^n + qc^n + re^n}{pb^n + qd^n + rf^n}\right]^{\frac{1}{n}} = k^{n \cdot \frac{1}{n}}$$

$$= k = \frac{a}{b} = \frac{c}{d} = \frac{e}{f} .$$

If $\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{e}{f}$, prove that each ratio is equal to : —

(18) $\sqrt[5]{\dfrac{a^5 + c^5 + e^5}{b^5 + d^5 + f^5}}$. (19) $\sqrt{\dfrac{ac - e^2}{a^2 + c^2}} = \sqrt{\dfrac{bd - f^2}{b^2 + d^2}}$.

(20) $\sqrt[5]{\dfrac{6a^2c^2e - c^4ef + 7bc^5}{6b^2d^2f - d^4f^2 + 7bd^5}}$.

(21) If $\dfrac{x}{(b-c)(b+c-2a)} = \dfrac{y}{(c-a)(c+a-2b)}$.

$= \dfrac{z}{(a-b)(a+b-2c)}$, find the value of $x + y + z$.

(22) If $\dfrac{x}{lm - n^2} = \dfrac{y}{mn - l^2} = \dfrac{z}{nl - m^2}$

prove that $mx + ny + lz = 0$ and

$lx + my + nz = 0$

(23) If $x : a = y : b$, prove that

$$\frac{x^2 + a^2}{x + a} + \frac{y^2 + b^2}{y + b} = \frac{(x+y)^2 + (a+b)^2}{x + y + a + b} .$$

(24) If $x=\dfrac{ab}{a+b}$ find the value of :

$$\frac{x+2a}{2b-x}+\frac{x-2a}{2b+x}+\frac{4ab}{x^2-4b^2}.$$

Find the value of $x:y:z$ from the following equations :

(25) $4x=7y+5z,\ 2x+y=z.$

Solution :

$$4x-7y-5z=0$$
$$2x+y-z=0$$

Cross multiply,

$$\frac{x}{(-7).(-1)-(-5).1}=\frac{y}{(-5).2-(-1).4}=\frac{z}{4.1-(-7)2}$$

or $\dfrac{x}{12}=\dfrac{y}{-6}=\dfrac{z}{18}$

or $\dfrac{x}{2}=-y=\dfrac{z}{3}$

$\therefore\ x:y:z=2:-1:3$

(26) $4x-2y-7z=0.$

$x+y-4x=0.$

(27) $3x-2y=3z.$

$10y-6z=x.$

Solve the following equation :

(28) $3x+4y=13,\ \dfrac{2x+3y}{2x-3y}=3.$

Solution :

By componendo and dividendo, the (ii) equation

$$=\frac{2x+3y+2x-3y}{2x+3y-2x+3y}=\frac{3+1}{3-1}.$$

or, $\dfrac{4x}{6y}=\frac{4}{2}.$

or, $x=3y.$

Substitute $x=3y$ in the (i) equation

Then $9y+4y=13$,

or $y=1$.

$\therefore\ x=3, \qquad y=1.$

(29) $3x-4y=7,\ \dfrac{3x+4y}{3x-4y}=2\frac{3}{7}$.

(30) $\dfrac{(x-2)(x-8)}{(x-3)(x-7)}=\dfrac{(x-5)(x-11)}{(x-6)(x-10)}$.

CHAPTER XVI

GRAPHS

64. On a piece of graph paper draw two straight lines XOX' and YOY' cutting each other at right angles at O (Fig. 1). Distances measured from O along OX or parallel to OX in the right direction are *positive*; those measured along OX' in the left direction are *negative*. Similarly, distances measured from O along OY or parallel to OY upwards are *positive*; those measured along OY' downwards are *negative*.

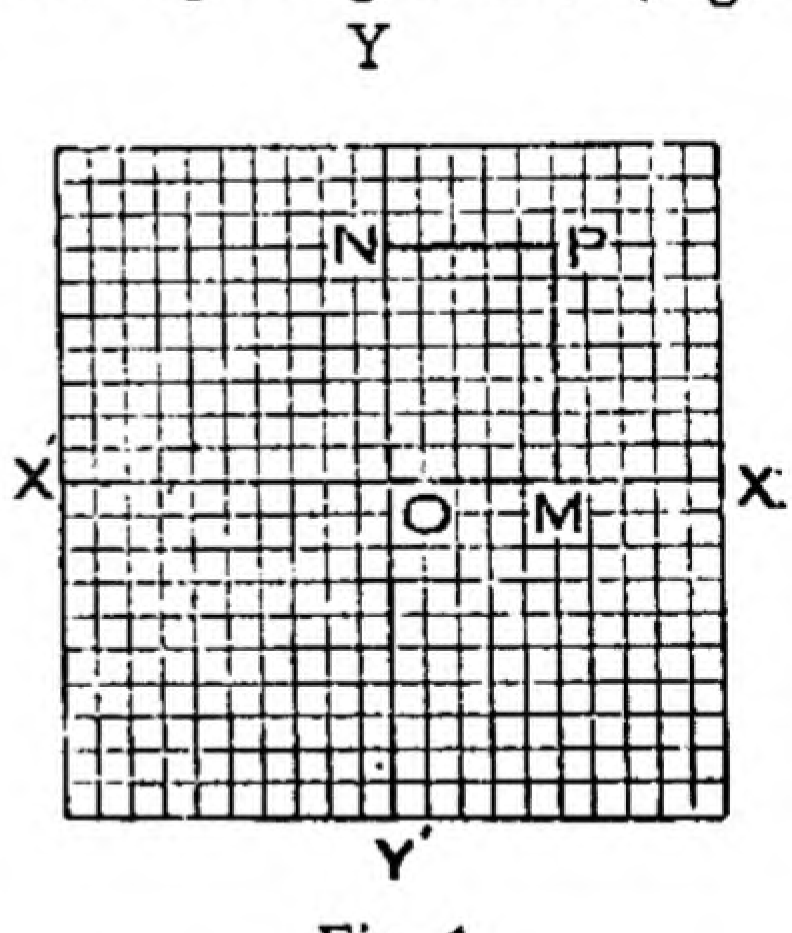


Fig. 1.

Take a point P and from it draw PM and PN at right angles to OX and OY respectively. If we know the lengths of PN and PM or of OM and ON we can determine the position of P on the graph paper. In fig. (1) OM=5 smallest divisions and ON=7 smallest divisions.

65. DEFS. *Origin.* The point O at which XOX′ and YOY′ intersect each other at right angles is called the *Origin.* The origin is therefore the point of intersection from which all distances are measured.

Coordinates. Distances OM and MP are the *Coordinates* of P. OM, the distance along OX is the *Abscissa* and ON, the distance along OY, the *Ordinate* of P.

Axes of Reference. XOX′ and YOY′ are the *Axes of Reference,* of which XOX′ is the *x-axis* and′, YOY′ the *y-axis.*

Plotting of a point is to determine its position on the graph paper. The points whose Co-ordinates are x and y is written as (x, y) In fig 1. P is written as (5, 7) The abscissa always preceeds the ordinate. Therefore (5, 7) and (7, 5) are two different points.

66. The graph paper is divided into four parts by the axes XOX′ and YOY′. Each part is called a *Quadrant.* In fig. 1 XOY is the first, YOX′, the second, X′OY′, the third and Y′OX, the fourth quadrant. In the first quadrant XOY, the abscissa and ordinates, that is, the co-ordinates of all points are *positive.* As for example, in fig. 2 the co-ordinates of P are positive, the abscissa of Q is *negative* and its ordinate is *positive*, the co- ordinates of R are *negative* while the abscissa of S is *positive* and its ordinate is *negative.*

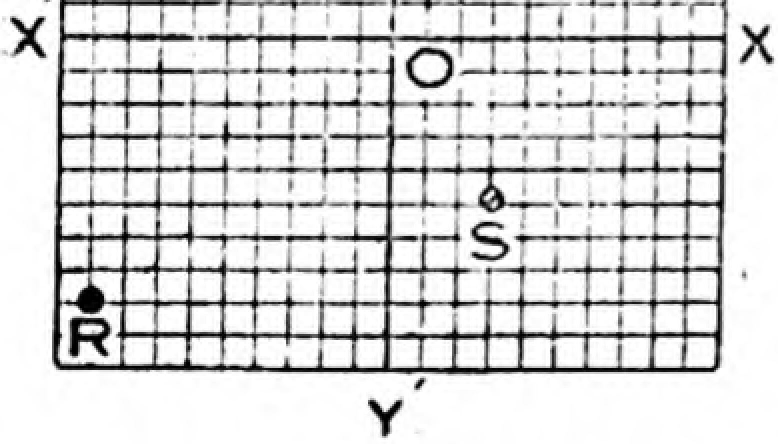


Fig. 2.

The co-ordinates of P, Q; R and S are (5,5); (−8, 6), (−9, −8), and (3, −5) respectively.

67. *Solved Examples* :

(1) Plot the following points :

(4, 4), (-6, 6), (-8, -5), and (9, -8).

Solution : Draw XOX′ and YOY′ on the graph paper (fig. 3.)

In order to plot (4,4 count 4 divisions on OX to the right of O and also 4 divisions on OY upwards. Let these distances be OM and OM′ respectively. From M and M′ draw perpendiculars to the x-axis and y-axis respectively meeting at P. P is therefore the required point.

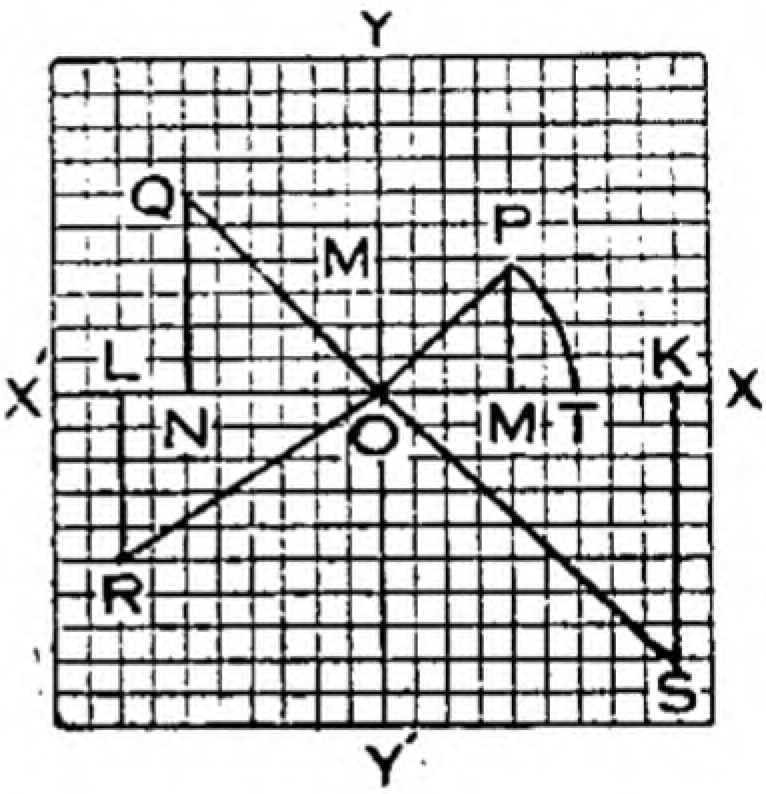


Fig. 3.

In the case of (-6, 6), the abscissa is negative and the ordinate, positive. Therefore the point lies in the second quadrant. Plot it as above. Q is the required point.

The co-ordinates (-8, -5) are negative. Therefore the point lies in the third quadrant. R is the point. Similarly. (9. -8) is in the fourth quadrant and S represents it.

(2) Find the distances from O of the points P. Q, R, S, in fig. 3.

Solution : OMP is a right angled triangle,

$\therefore OP^2 = OM^2 + MP^2$

$= 4^2 + 4^2 = 16 + 16 = 32.$

$\therefore$ OP $= \sqrt{32}$. units of length ($\frac{1}{10}$th of an inch.)

Similarly the distances of Q. R and S are respectively $\sqrt{72}$ units of length, $\sqrt{89}$ units of length and $\sqrt{145}$ units of length.

Second Method (by measurement) :

With O as centre and OP, radius draw an arc cutting OX at T Then OP=OT. Measure OT directly. Do this in the case of Q. R and S also.

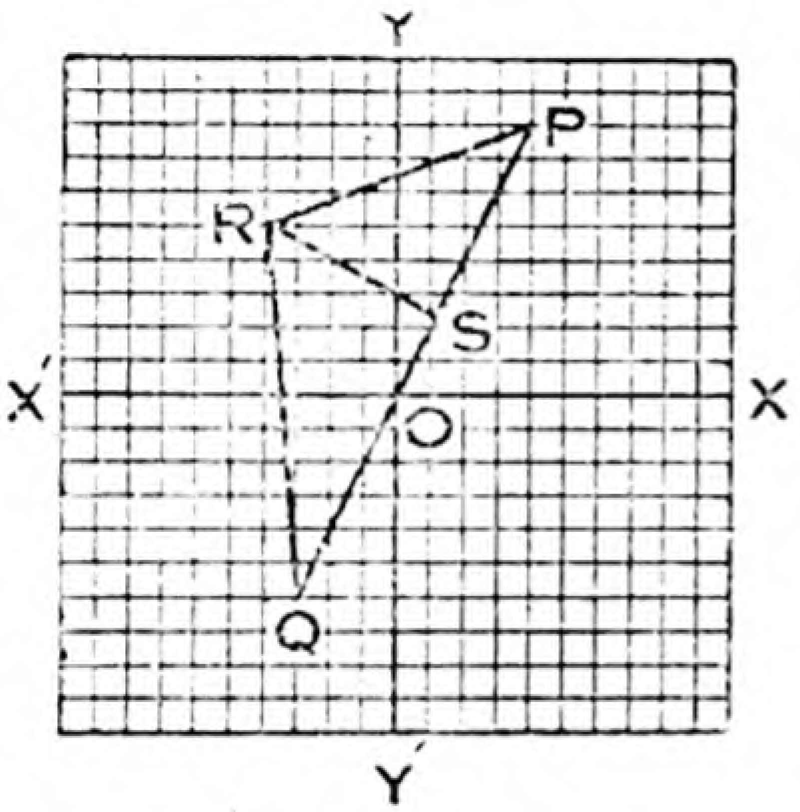


Fig. 4.

Both the above processes are employed to find the distance between two given points on the graph paper.

(3) Plot the points $(-3, -6)$, $(4, 8)$ and $(-4, 5)$ and find he aıea of the triangle formed by joining them.

Solution : In Fig. 4, Q. P. R. represent $(-3, -6)$, $(4, 8)$ and $(-4, 5)$ repectively and PQR is the triangle formed From R draw RS perpendicular to PQ.

Area of PQR$=\frac{1}{2}$ RS. PQ square units of length.

$=\frac{1}{2}.6.16$,, ,, ,,

$=48$,, ,, ,,

$=48$ square inch.

Second Method (by counting small squares):

Count the small squares in the $\triangle$ PQR to get its area. In counting small squares consider half or greater than half the square as one and leave off squares less than half.

EXERCISE 40

(1) Plot the following points :

$(-1, 1)$, $(3, 4)$, $(-3, 2)$, $(0, 2)$, $(5, -8)$, $(-2, 5)$, $(1, -2)$, $(-1, -2)$, $(-7, 13)$, $(-8, -9)$ $(3, 0)$, $(-4, 0)$, $(0, -5)$.

(2) Plot $(-3, 2)$, $(-5, 3)$ and find the co-ordinates of the middle point of the line joining them.

(3) Plot (6, 8) and find its distance from the origin.

(4) Find the distance between each pair of the following :

(i) (−2, 2), and (1, −2).

(ii) (0, 0) and (1, 2).

(iii) (1, −1) and (2, 2)

(5) Plot the points A=(−1, 1), B=(0, 2), C=(2, 1), D=(1, −2) and measure the lengths of the sides of the figure ABCD.

(6) Find the distances of A, B, C and D in Q 5 from the origin.

(7) Plot (−2, 3), (0, 3), (3, 3): What is the peculiarity about these points ?

(8) Plot (4, 2), (4, 1), (4, −2) and (4, −3). What do you observe ?

(9) Plot A=(3, 2), B=(2, 2) C=(11, 8) and D=(2, 8) and find the area of the figure A B C D by counting Squares. Find also the coordinates of the middle points of A C and B D respectively.

(10) Plot the points A=(3, 2) B=(3, 7) and C=(8,5). Find the area of the triangle ABC.

(11) On the graph paper construct a triangle with base=6 cm. and other two sides 5 cm. and 3 cm. respectively. Find the height of the triangle.

(12) Plot A=(5, 2), B=(6, 8) and C=(7, 12) and find the area of the triangle ABC.

(13) Plot two points P=(1, 4), R=(3, 2.). Join PR and find the co-ordinates of the mid-point of the line PR.

(14) Plot the following sets of points and observe that each set lies in one straight line :

(1) (2, 3) (6, 9), (0, 0), (4, 6) (−6, −9).

(2) (3, 1), (9, 10), (−3, −8), (−1, −5).

(3) (5, 4), (5, 7), (5, 8), (5, −11).

(4) (4, 8) (1, 1), (−2, −4), (−3, −6).

(5) (0, 0), (9, −8), (4 −4)

(6) (−4, 5), (−3, −6), (−3 −2).

(7) (5, 5), (0, 0), (−5, −5).

(8) (−8, 4), (0, 5), (,3 −8).

(9) (0, 7), (10, 5), (7, −10),

(10) (4, 6), (−3, 6), (−7, 6) and (0, 6).

(15) Find the area of the figures formed by joining the following points :

(1) (−2, 4), (−5, −4), (6, −3).

(2) (6, 4), (−5, −4), (6, −3).

(3) (−2, 4), (−5, 4), (−3, −4).

(4) (3, 6), (5, −5), (−9, 0).

(5) (7, 10), (−5, −5), (14, −5).

(16) Show that the following set of points are the vertices of isosceles triangle. Calculate and measure the lengths of equal sides.

(i) (0, 5), (5, −5), (−5, −5)

(ii) (9, −2). (2 −10), (−5, −2).

(iii) (−8, 0), (14, 0), (3, 14)

(17) Plot the points (5, 0), (5, 5), (0, 10), (—5, 0), and find the area of the resulting figure.

(18) Plot the points (5, 5), (5, −5) (−5, −5), (−5, 5) and show that the resulting figure is a square. Find its area by counting squares and verify your result by calculation.

(19) Plot the points (6, 1), (−7, 1), (−7, −2), (6, −2) and find its area by counting squares and by calculation. What kind of figure is it ?

(20) Plot the points $(-5, 5)$, $(5, 4)$, $(4, -4)$ and $(-5, -6)$ and find the area of the quadrilateral so formed.

(21) Plot the points $(-3, 2)$ $(-3, -5)$, $(8, -5)$ and $(8, 2)$. Find the area of the resulting figure and show that the figure is a rectangle

(22) Plot the points $(8, 0)$, $(-7, 0)$, and $(0, -10)$ and find the area of the resulting triangle

(23) Plot the points $A=(6, 0)$, $B=(4, 5)$, $C=(5, 7)$, $D=(-8, 0)$, $E=(0, -6)$ and find, by counting squares, the area of the figure ABCDE.

(24) Plot the points $A=(5, -12)$, $B=(12, -8)$, $C=(15, 0)$, $D=(12, 5)$, $E=(5, 0)$ and find, by counting squares the area of the rectilineal figure ABCDE.

(25) Plot the points $A=(-9, -3)$, $B=(7, -3)$ $C=(-1, 7)$, $D=(-1, -5)$. Join AB and CD and show that AB is perpendicular to CD.

68. LINEAR GRAPHS.

Units. The position of a point on graph paper is determined when its co-ordinates with reference to the axes of co-ordinates are known. This is done by measuring its distances from the origin.

These distances may be measured in suitable units of length. The units may be ;

(1) the side of a small square on x-axis and y-axis,

(2) the sides of two or more small squares on both axes.

(3) any fraction of the side of a small square on both axes.

or,

(4) different for different axes, that is the abscissa may be measured in one unit and the ordinate in a different unit

The choice of the units is important in the graphical solution of simultaneous equations and problems.

69. *Solved Examples :*

(1) Plot the following sets of points with units mentioned against each set :

(*a*) (11, 10), (−15, 6), (−10, −7) and (−10, −10); unit =1 side of a small square.

(*b*) (5, 3), (−2, 4), (−8, −5) and (3, −4); unit=twice the side of a small square.

(*c*) (3, 1), (−2, 2), (−1, −4) and (5, 0) ; unit=three times the side of a small square.

(*d*) (8, 4), (−4, 8), (−12, − 12) and (16,−8); unit=half the side of a small square

Solution : The plotted points, as shown in the graph, are :

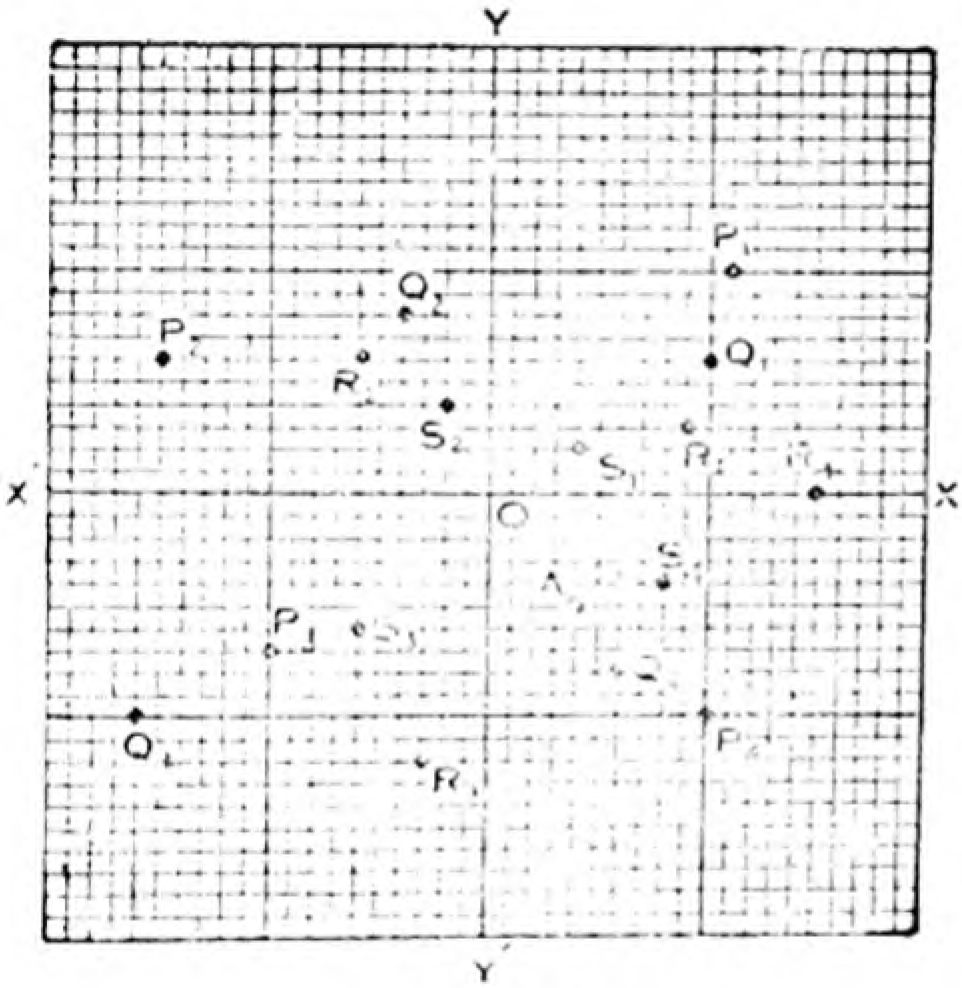


Fig. 5.

(*a*) P_1, P_2, P_3 and P_4.

(*b*) Q_1, Q_2, Q_3 and Q_4.

(*c*) R_1, R_2, R_3 and R_4.

(*d*) S_1, S_2, S_3 and S_4.

(2) Plot the following points respectively with the units mentioned against each :

(*a*) (−3, 2); unit for the abscissa=twice the side of a small square and unit for the ordinate=3 times the side of a small square.

(*b*) (20, −15) ; unit for abscissa=$\frac{1}{5}$th of the side of a small square and unit for ordinate=$\frac{1}{3}$rd of the side of a small square.

Solution: The plotted points as shown in the above figure are:

(*a*) R_2

(*b*) A.

70. *Graphical Representation of Functions :*

Definitions : *Variable Quantity* is the quantity of which the value is not always the same, as for example, the temperature during the day, the recurring expenses of a hostel, the rainfall of a town etc.

Function of x is that expression containing a variable quantity x whose value depends on the value of x.

As for example, $2x+5$ is a function of x since its value depends on that of x.

If	$x=-3$	-2	-1	0	1	2
then $2x+5$	$=-1$	1	3	5	7	9

If we plot the points $(-3, -1)$, $(-2, 1)$, $(-1, 3)$ $(0, 5)$, $(1, 7)$, $(2, 9)$ and join them we get the graph of the function of x, namely, $2x+5$.

These points are respectively plotted as K, L, M, N, P, Q in the graph,

Therefore the line KLMNPQ represents the graph $2x+5$.

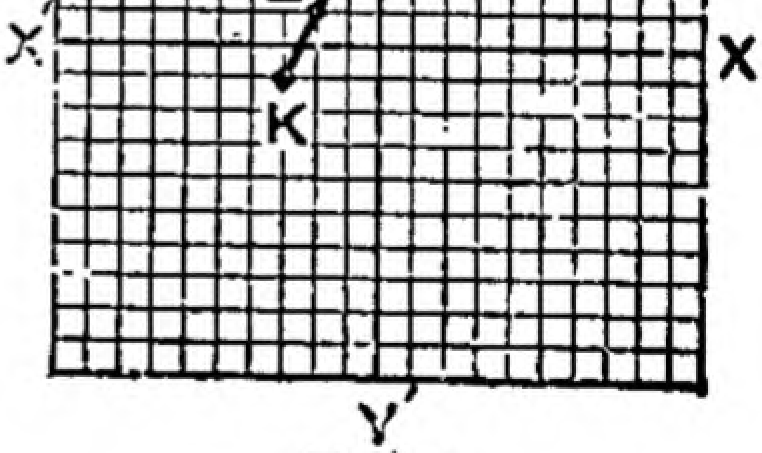


Fig 6.

71. When $2x+5$ is put equal to y, then $y=2x+5$ is a linear equation and y changes as x changes.

As for example

When	$x=-3$	-2	-1	0	1	2
	$y=-1$	1	3	5	7	9

Therefore the line KLMNPQ is also the graph of the equation $y=2x+5$.

72. *Solved Examples* :

(1) *Draw the graph of* $y=2x$.

Solution : Find the coresponding values of y for different values of x and tabulate these values.

As for example,

When	$x=-4$	-3	-2	-1	0	1	2	3	4
	$y=-8$	-6	-4	-2	0	2	4	6	8

Plot the points $(-4, -8)$, $(-3, -6)$, $(-2, -4)$, $(-1, -2)$, $(0, 0)$, $(1, 2)$, $(2, 4)$, $(3, 6)$ and $(4, 8)$ and notice that all the points lie on one straight line R S (which can be produced). The co-ordinates of all the above points satisfy the equation $y=2x$.

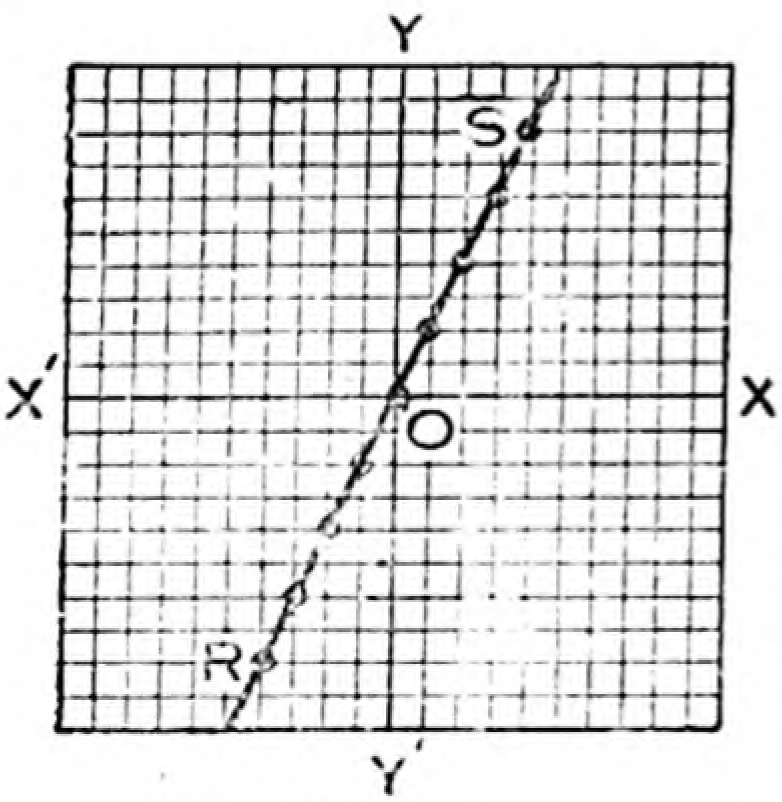


Fig. 7.

(2) Draw the graph of $x=5$.

Solution : Here the abscissae of a series of points is given as 5 units of length and nothing is said about the ordinates of those points (or of a moving point with abscissa $=5$ units of length).

Hence we can take any ordinate such as $2, -3, 4, 0$

etc. and plot the points (5, 2), (5, −3), (5, 4) and (5, 0) to get the required graph.

Unit=twice the side of a small square.

The points P, Q, R and S represent (5, 2), (5, −3), (5, 4) and (5, 0) respectively.

Therefore RQ (produced) is the graph of $x=5$.

(3) Draw the graph of $x=-3$.

Solution: As above let the ordinate be 0, 1, 4, −3.

The points to be plotted are therefore (−3, 0), (−3, 1), (−3, 4), (−3, −3).

Unit = Three times the side of a small square.

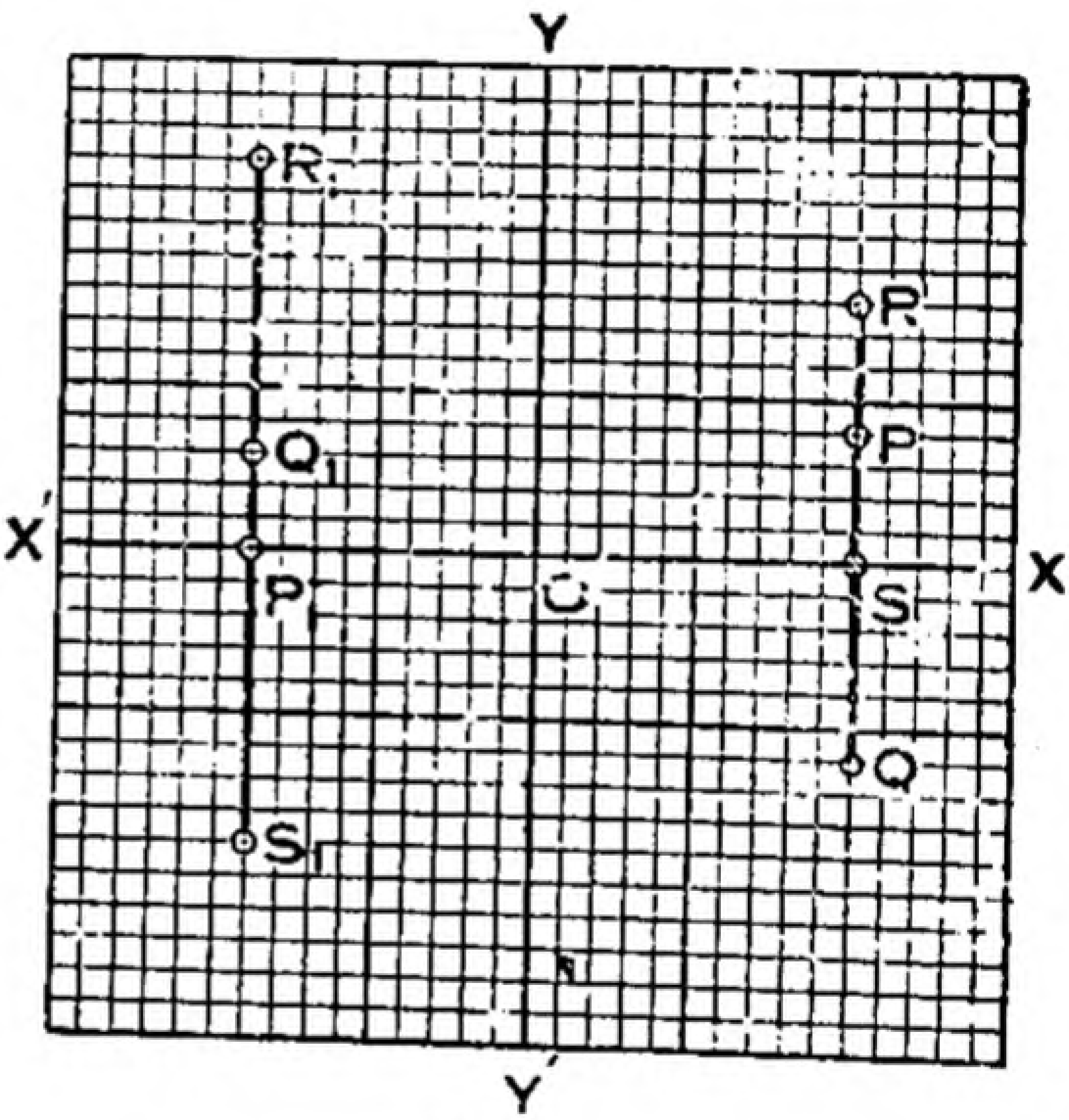


Fig. 8

The points P_1, Q_1, R_1 and S_1 represent (−3, 0), (−3, 1), (−3, 4) and (−3, −3) respectively on the above graph.

Therefore R_1 S_1 (which can be produced both ways) is the required graph.

(4) Draw the graph of $y=4$.

Solution: Here the ordinate of the moving point is always 4 units of length and we can have, as above, any number of abscissae, namely, 0, 1, −1, 2, −2 etc. The points to be plotted are therefore (0, 4), (1, 4), (−1, 4), (2, 4) and (−2, 4).

Units = four times the side of a small square on x-axis and three times the side of a small square on y-axis.

In the above figure A, B, C, D and E represent (0, 4), (1, 4), (-1, 4), (2, 4) and (-2, 4) respectively with the given units. Therefore DE (produced both sides) represents the graph of $y=4$.

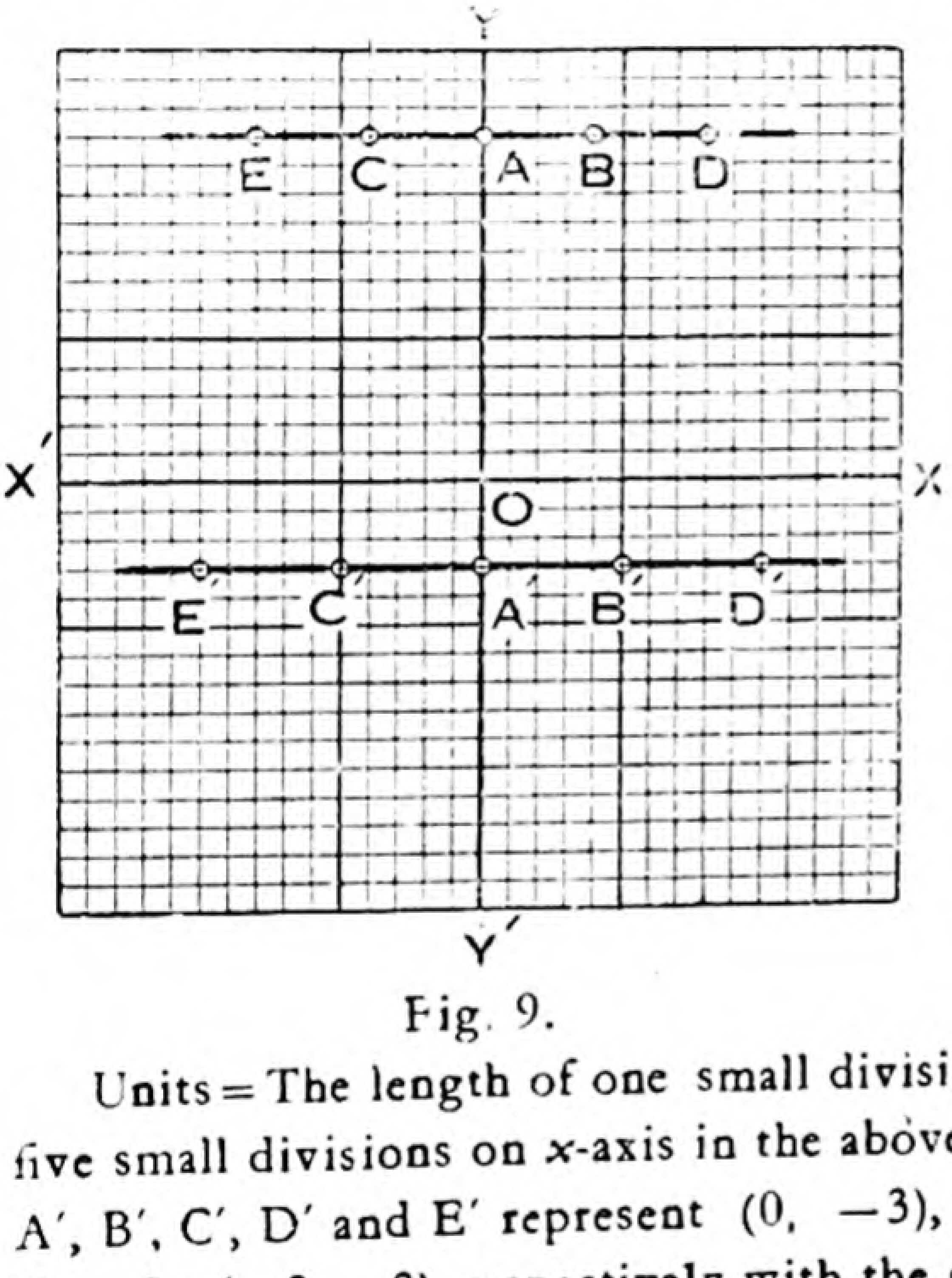


Fig. 9.

(5) Draw the graph of $y=-3$.

Solution : As above, let the abscissae be 0, 1, -1, 2, -2.

The points to be plotted are therefore (0, -3), (1, -3), (-1, -3), (2, -3) and (-2, -3).

Units = The length of one small division on y-axis and the five small divisions on x-axis in the above figure. The points A′, B′, C′, D′ and E′ represent (0, -3), (1, -3), (-1, -3), (2, -3), (-2 -3) respectively with the given units.

Then D′ E′ (produced) represents the graph of $y=-3$.

(6) Draw the graph of $x=a$.

Solution : Here the abscissa is constant, that is, a units of length while the ordinates vary.

Suppose $x=a=7$ and $y=3, 4, -5, 5, 2$. Then (7, 3). (7, 4), (7, $-$ 5), (7, 5), (7, 2) are the points to be plotted. They all lie on one straight line AB parallel to y-axis,

(Unit = the side of a small square)

Therefore AB represents the graph of $x = a$

Similarly, $y = b$ represents the straight line A′ B′ in the above figure parallel to x-axis.

Note : (i) If a and b are negative quantities, *e. g.*, -7 and -9 respectively the graph of $x = a$ and $y = b$ will be on the left of YOY′ and below XOX′ respectively as indicated by CD and C′ D′ respectively in the above figure.

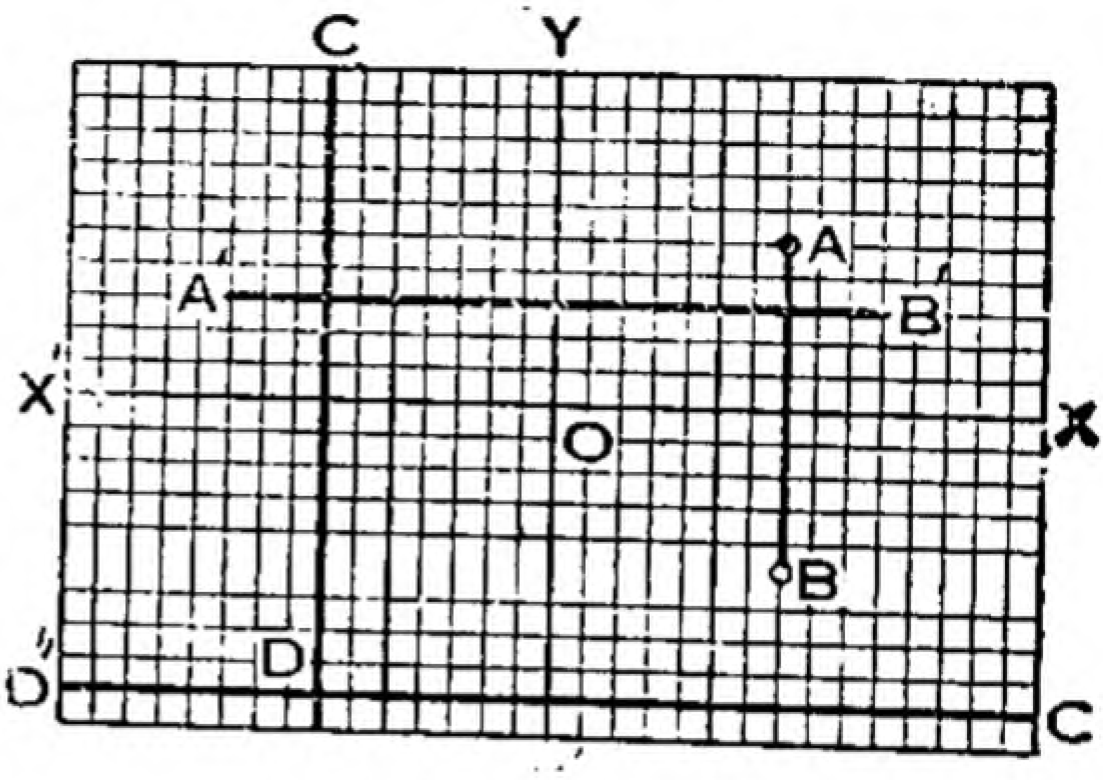


Fig. 10.

(ii) The graph of simple equations in x or y is a straight line.

(iii) The graphs of the equations of the form $y = ax$ always pass through the origin.

(7) Draw the graph of $3y = -4x$.

Solution : Here $y = \dfrac{-4x}{3}$

When					
$x = 0$	3	-3	6	-6
$y = 0$	-4	4	-8	8

Unit = twice the side of a small square.

Plot the points (0, 0), (3 -4), (-3, 4) (6, -8), (-6, 8). They are represented by O, A, B, C, D respectively.

Join A, B, C, D.

The straight line CD passing through the origin is the required graph. (Fig. 11)

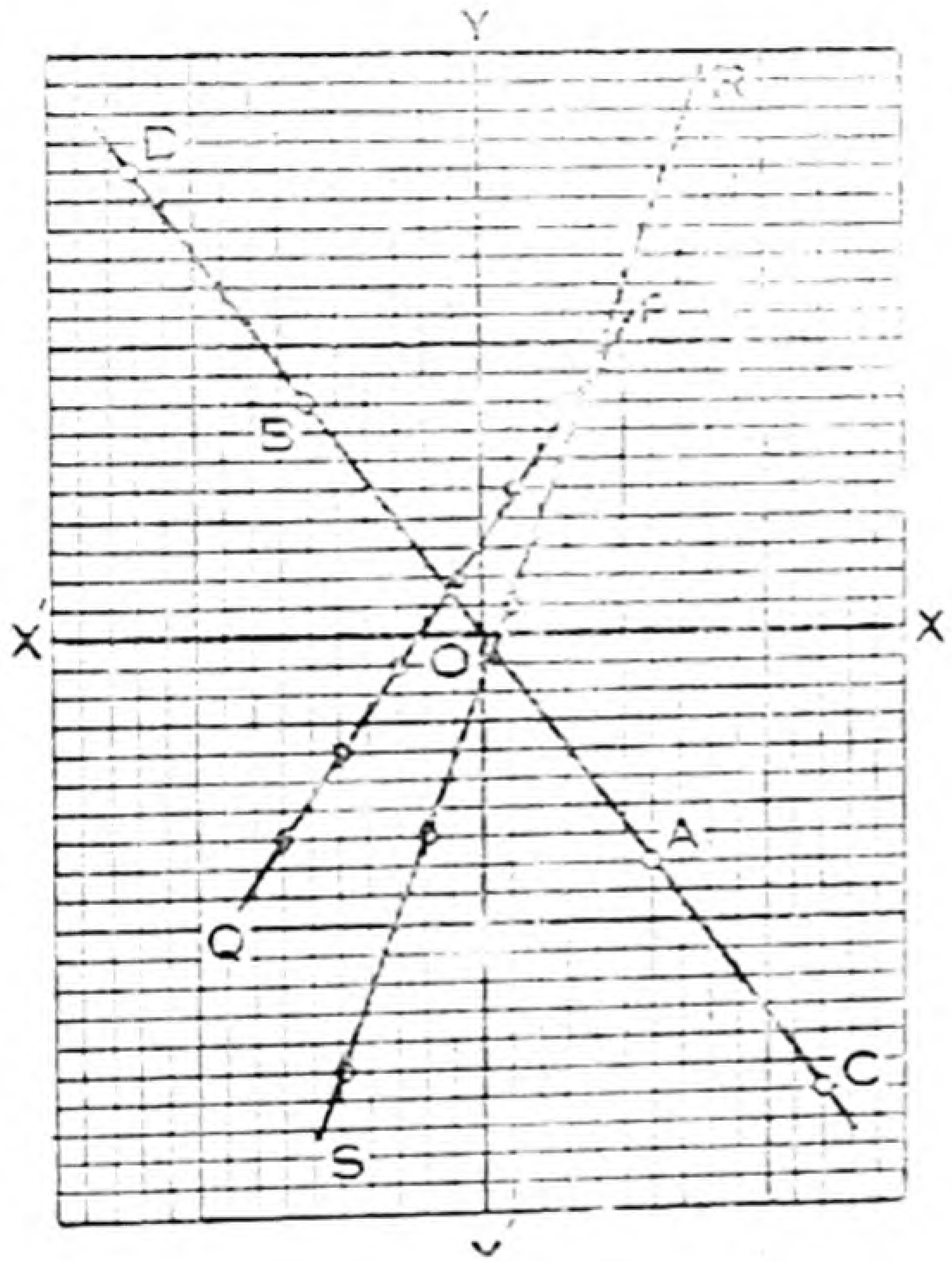


Fig. 11.

(8) On the same diagram draw the graphs of the following equations.

(i) $2y = 3x + 7$.

(ii) $3y = 8x - 5$.

Solution : In each case we find the values of y corresponding to those of x.

From (i),

$$y = \frac{3x+7}{2}.$$

From (ii),

$$y = \frac{8x-5}{3}.$$

Thus when in (i)	$x=1$	-1	3	-3	-7
	$y=5$	2	8	-1	-7
and when in (ii)	$x=1$	-2	4	-5	7
	$y=1$	-7	9	-15	17

Plot the points (1, 5), (−1, 2), (3, 8), (−3, −1), (−7, −7) and (1, 1), (−2, −7), (4. 9), (−5, −15) (7, 17) respectively.

Join the two series of points by straight lines PQ and RS (Fig. 11), which represent the graphs of $2y=3x+7$ and $3y=8x-5$ respectively.

(9) Draw the graph of $2x+3y=5$.

Solution : Here $y=\dfrac{5-2x}{3}$

When	$x=0$	1	2	-1	-2
	$y=\frac{5}{3}$	1	$\frac{1}{3}$	$\frac{7}{3}$	3

Taking 3 times the side of a small square as the unit, plot the points $(0, \frac{5}{3})$, $(1, 1)$, $(2, \frac{1}{3})$, $(-1, \frac{7}{3})$, $(-2, 3)$.

Join them and notice that they lie on the straight line AB which is therefore the graph of $2x+3y=5$.

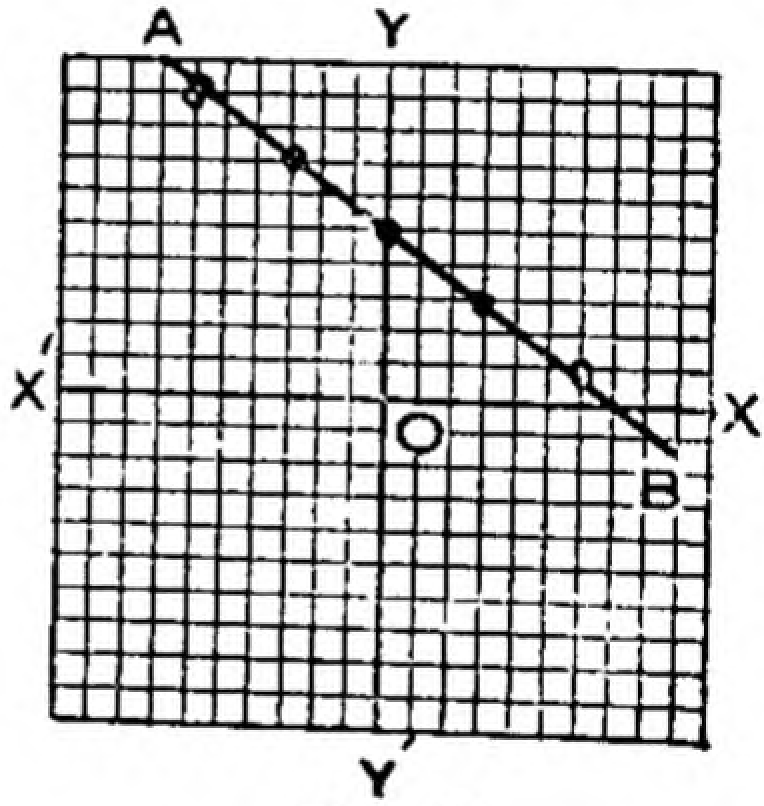


Fig. 12

73. Find the equation of the straight line which passes through the points (0, 1) and (2, -3).

Solution : Let $y=mx+c$ be the required equation.

Since (0, 1), (2, -3) satisfy $y=mx+c$, we have

$1=c$............(i). ($\because x=0, y=1$) and

$-3=2m+1$.......... (ii), ($\because x=2, y=-3$).

or, $-4=2m$ $\therefore$ $m=-2$.

Thus the required equation is,

$y=-2x+1$ or $2x+y=1$

EXERCISE 41

Draw the graphs of the following equations :

(1) $x=3$. (2) $y=4$.

(3) $x=-4$. (4) $y=-5$.

(5) $x=y$. (6) $y=-x$.

(7) $x+10=0$. (8) $y-9=0$.

(9) $3y=5x$. (10) $3y=-7x$.

(11) $3x+4y=0$. (12) $6y+13=0$.

(13) $\frac{x}{2}-\frac{y}{3}=1$. (14) $\frac{x}{4}+\frac{y}{5}=1$.

(15) $x+y=8$. (16) $3x+y-5=0$.

(17) $y=\frac{1}{2}x-2$. (18) $y=\frac{9x-13}{4}$.

(19) $x=\frac{3y-5}{2}$. (20) $\frac{x}{8}-\frac{y}{13}=-1$.

(21) $\frac{4x}{3}-\frac{3x}{4}=1$. (22) $2y=\frac{x}{3}+\frac{1}{3}$.

(23) $3y=\frac{4x-7}{2}$. (24) $\frac{x}{4}-\frac{y}{3}=5$.

(25) $\frac{4x+3y}{7}=1$. (26) $y=\frac{2x-1}{3}$.

(27) $y=\frac{3x-1}{5}$. (28) $7y=5x-35$.

(29) $4x+5y+6=0$.

Draw the graphs of the following expressions :—

(30) $x+2$. (31) $x-3$.

(32) $3x+5$. (33) $\frac{8x+11}{2}$.

(34) $-3x+4$. (35) $3x+13$.

(36) $\frac{5x-9}{3}$. (37) $\frac{x}{2}+\frac{3}{5}$.

(38) $\frac{2x+3}{5}$. (39) $\frac{5}{2}-\frac{3x}{5}$.

Find the equations of the lines passing through the following pairs of points :—

(40) (0, 0), (2, 3). (41) (−1, 4), (4, −5).

(42) (5, 6), (−2, −3). (43) $(-\frac{1}{2}, \frac{1}{3})$, (0, 1).

(44) $(\frac{1}{3}, \frac{1}{4})$, $(\frac{1}{4}, \frac{1}{3})$. (45) (2, −8), (6, −2).

(46) (2, −3), (−3, 2). (47) (0, 2), (20, 8).

(48) (3,—5), (−5, 3). (49) $(4\frac{1}{2}, 3\frac{1}{2})$, $(-2\frac{1}{2}, -1\frac{1}{2})$.

Show that the point (1, 1) is also on the straight line.

74. GRAPHICAL SOLUTION OF SIMULTANEOUS EQUATIONS.

Solved Examples :

Solve the following equations graphically :

(1) $x+y=15$

$2x-3y=5$.

Solution : From the first equation $y=15-x$.

Therefore

when $x=7$	8	10	5
$y=8$	7	5	10

Plot the points (7, 8), (8, 7), (10, 5) (5, 10). Join them and notice that they lie on the straight line AB,

From the second equation $y=\frac{2x-5}{3}$

Therefore

when $x=4$	-2	-5
$y=1$	-3	-5

Plot the points (4, 1), (−2, −3), (−5, −5).

Join them and notice that they lie on the straight line A′ B′ which when produced cuts AB at B.

Find the co-ordinate of B, the point of intersection.

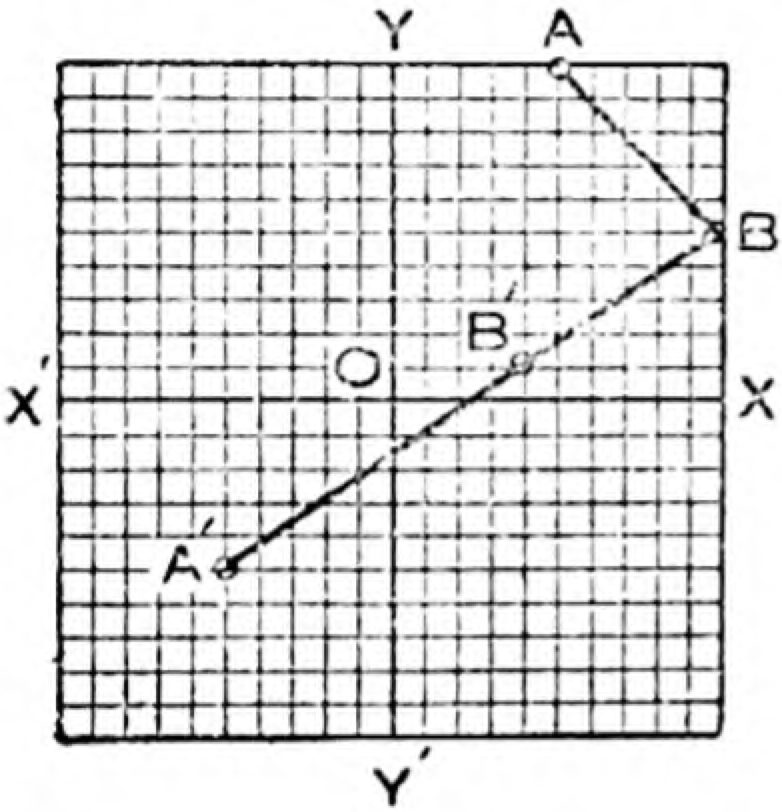


Fig. 13.

From the figure the co-ordinates are (10, 5).

Therefore $x=10, y=5$.

Note : Verify the result in the ordinary way.

(2) $$\frac{x}{3}+\frac{y}{2}=12$$

$$\frac{x}{2}+\frac{y}{3}=\frac{34}{3}$$

Solution : From the first equation, $y=\dfrac{72-2x}{3}$

Therefore

when $x=0$	3	−3	12
$y=24$	22	26	16

Plot the points (0, 24), (03, 22), (−3, 26) and (12, 16) and join them.

Notice that they lie on the straight line AB.

From the second equation, $y=\dfrac{68-3x}{2}$.

Therefore,

when $x=18$	12	8
$y=7$	16	22

Plot the points (18, 7), (12, 16), (8, 22) and join them. Notice that they lie on the same straight line CD which cuts AB at P. Find the co-ordinates of P

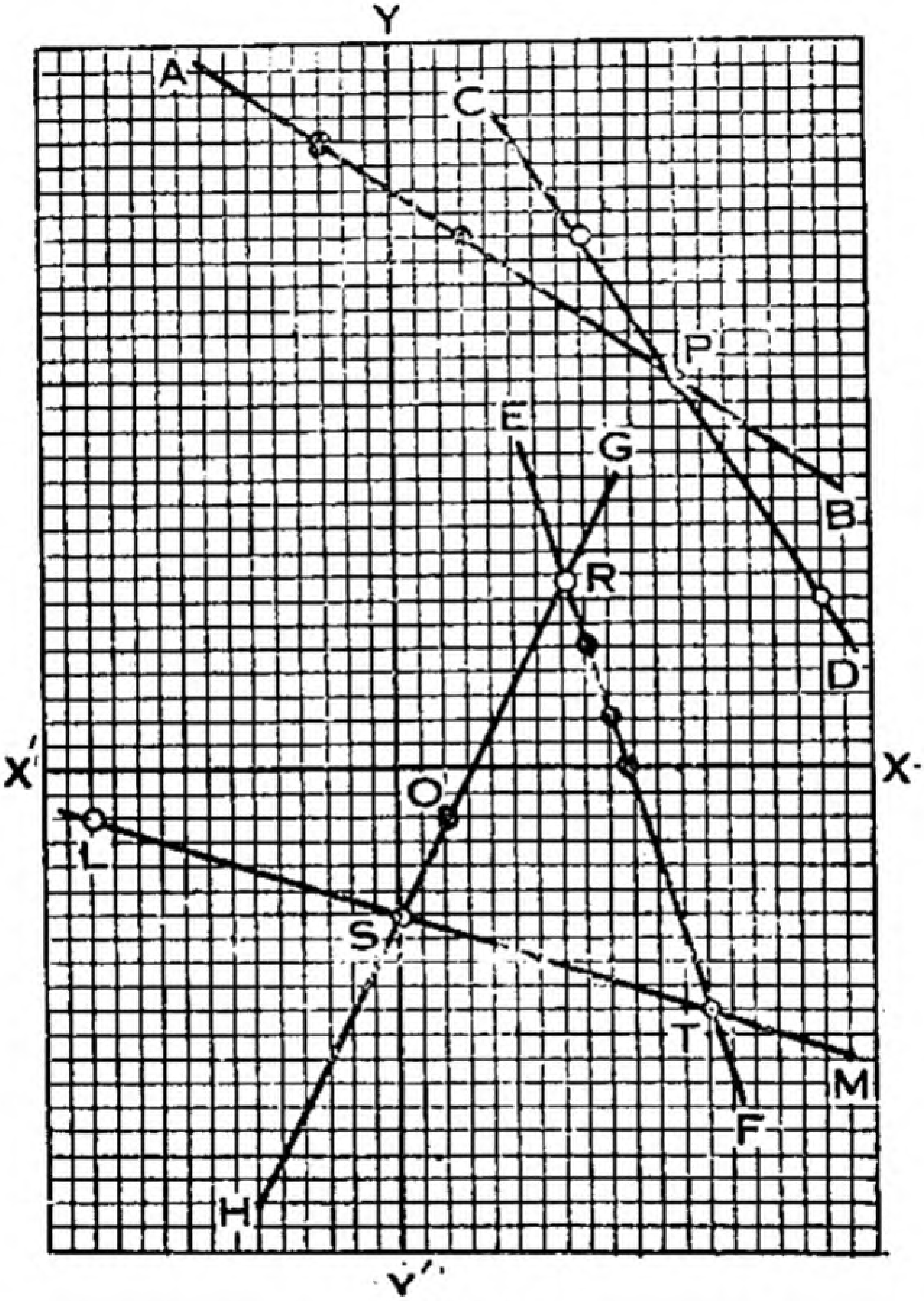


Fig. 14

From figure 14 the co-ordinates are (12, 16)

Therefore $x=12, y=16$

Note : Verify the result in the ordinary way.

(3) Find the co-ordinates of the vertices of the triangle

whose sides are given by the equations $3x+y=29$, $y-2x+6=0$ and $4x+13y+78=0$.

Solution : From the first equation $y=29-3x$.

Therefore,

when $x=9$	10	8
$y=2$	-1	5

Plot the points (9, 2), (10, -1) and (8, 5). Join them and notice that they lie on the straight line EF in the above figure.

From the second equation $y=2x-6$.

Therefore,

when $x=0$	2	4
$y=-6$	-2	2

Plot the points (0, -6), (2, -2) and (4, 2). Join them and notice that they lie on the straight line GH in the above figure.

Also, from the third equation $y=\dfrac{-4x-78}{13}$.

Therefore,

when $x=0$	13	-13
$y=-6$	-10	-2

Plot the points (0, -6), (13, -10) and (-13, -2) and join them. Notice that they lie on the straight line LM, in the above figure.

The co-ordinates of the verities R, S, T of the triangle RST are (7, 8), (0, −6) and (13, −10) respectively.

(4) Find graphically the co-ordinates of the vertices of the quadrilateral whose sides are $x+y=15$, $x-y=15$, $x+y+15=0$ and $x-y+15=0$. Prove that the quadrilateral is a square and find its area.

Solution : From the first equation $y=15-x$.

Therefore,

when $x=0$	12	15
$y=15$	3	0

Plot the points (0, 15), (12, 3) and (15, 0) and join them. Notice that they lie on the straight line AB.

.From the second equation $y=x-15$.

Therefore,

when $x=0$	12	15
$y=-15$	−3	0

Plot the points (0, −15), (12, −3) and (15. 0) and join them. Notice that they lie on the straight line BC.

From the third equation $y=-x-15$.

Therefore,

when $x=0$	−12	−15
$y=-15$	−3	0

Plot the points $(0, -15)$, $(-12, -3)$ and $(-15, 0)$ and join them. They lie on the straight line CD.

Similarly, from the fourth equation $y = x + 15$.

Therefore,

when $x = 0$	-10	-15
$y = 15$	5	0

Plot the points $(0, 15)$, $(-10, 5)$ and $(-15, 0)$ and join them. They lie on the straight line DA.

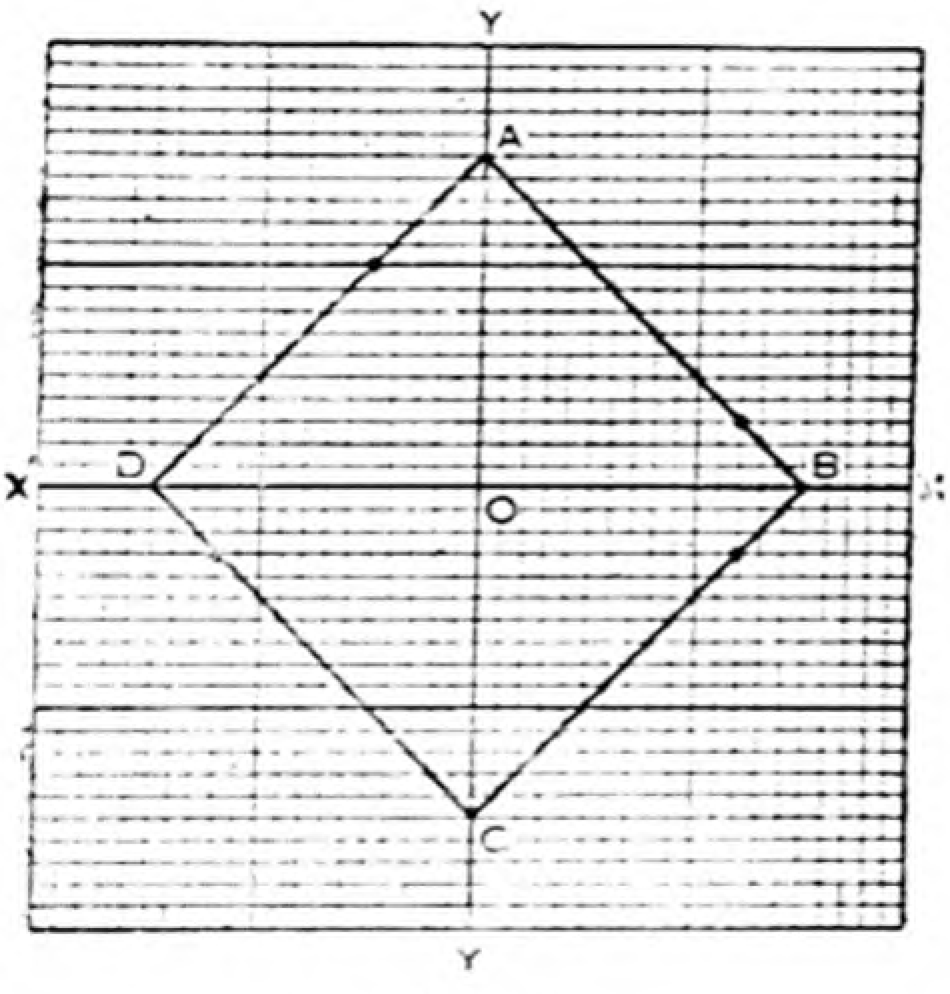


Fig. 15.

Then ABCD is the required figure, the co-ordinates of whose vertices A, B, C and D are $(0, 15)$, $(15, 0)$, $(0, -15)$ and $(-15, 0)$ respectively.

It is clear from the figure that OA = OB = OC = OD and the diagonals AC and BD bisect each other at right angles.

Therefore figure ABCD is a square. Area of the figure ABCD

$= \text{area of the } \triangle \text{ ABD} + \text{area of the } \triangle \text{ CBD}$

$= \frac{1}{2} \text{ BD} \times \text{AO} + \frac{1}{2} \text{ BD} \times \text{OC}$

$= \frac{1}{2} \text{ BD (AO} + \text{OC)} = \frac{1}{2} \text{ BD. AC}$

$= \frac{1}{2} \times 30 \times 30 = 450$ units of area.

EXERCISE 42

Solve graphically :

(1) $x-y+1=0$,
$3x-2y=0$.

(2) $3x+y=5$.
$x+4y=9$

(3) $3x-y=2$.
$x+y=6$.

(4) $2x=y+\frac{5}{4}$.
$x+y=8(y-x)$.

(5) $5x+y=23$.
$x+5y=19$.

(6) $5x+4y=23$.
$2x+9y=24$.

(7) $3x-4y=-7$.
$3y-4x=-7$.

(8) $21x+8y+66=0$.
$23y-28x+13=0$.

(9) $\frac{x}{6}+\frac{y}{9}=4$.
$\frac{x}{4}-\frac{y}{3}=-3$.

(10) $\frac{x+y}{5}=8$.
$\frac{x-y}{3}=2$.

(11) $5x=11y$.
$17x-72=23y$.

(12) $3(x-2)-2(y+3)=1$.
$2(x-3)+(y+2)=0$.

(13) $\frac{x}{2}+\frac{y}{5}=11$.
$\frac{x}{7}+\frac{y}{4}=7$.

(14) $\frac{6x}{7}-\frac{3y}{10}=6$.
$\frac{5x}{2}-\frac{7y}{5}=7$.

(15) $2(x-2)=\frac{y-3}{5}$
$3(y-3)=\frac{2-x}{3}$

(16) Find four points on each of the graphs $5x+4y=10$ and $3x+2y=6$ and thence solve the equations. (A. U. 1911.)

(17) Show graphically that the following equations have a common solution and find it.

(i) $2x-3y+12=0$.
(ii) $2x+3y=0$.
(iii) $4x+3y+6=0$.

(B. U. 1909.)

(18) Draw the graphs represented by $x-3y=2$, $2x-5y=5$ and $x+1=6y$. Show that the straight lines represented by them meet at a common point and find its co-ordinates. (B. U. 1911).

(19) Show by means of graph that the values of x and y which satisfy the equations $x+2y=1$ and $3x-y=2$, also satisfy the equation $2x-3y=1$.

(20) Draw the graphs of $y=x+\frac{5}{4}$ and $y=\frac{x}{2}+\frac{3}{2}$ by taking the values of x between -5 and $+5$. Find the co-ordinates of their point of intersection and verify by the ordinary method.

(21) Find graphically the value of $13x+6$ when $x=1{\cdot}4$ and find for what value of x this expression is equal to 14.

(22) Draw with the same axes the graphs of (i) $y+x=5$, (ii) $x=2y-3$ and (iii) $x=7$ and find the co-ordinates of the vertices of the triangle formed by them; also show that $2x-y-2=0$ passes through the intersection of (i) and (ii) (A. B. 1930)

(23) With the same origin and axes draw the graphs of $3y-x=7$, $y=x-1$ and $3x+2y=12$. Hence, write down the co-ordinates of the points of intersection of each pair. (Unit $\frac{1}{2}$ inch) (B. U. 1916)

(24) Draw the graphs of the following straight lines :—

(i) $4x-y=10$ (ii) $2x-y=4$

(iii) $x=3$ (iv) $y=2$.

Solve graphically the first two simultaneous equations.

Draw a straight line which cuts off intercepts of 3 and 4 on the axes of x and y respectively and find its equation. (The same units are taken in all cases). (All. 1934).

(25) Draw the graph of the equation $14x+10y=35$ and find the co-ordinates of the points of intersection of this line with the lines $x=0$ and $y=0$. (Dacca 1941).

75. STATISTICAL GRAPHS

Solved Examples :

(1) Plot a graph to show the variations of population of certain country from the following statistics : where P is the number in millions at the beginning of each of the years specified :

Year	1830	1835	1840	1850	1860	1865	1870	1880
P	20	22·1	23·5	29	34·2	38·2	41	49·4

Find what the populations were at the beginning of the year 1848 and 1875.

Solution : Mark off years along OX representing 1 small division=1 year beginning with the year 1830. Also measure population along OY beginning with 20 millions from O and representing 1 million=1 small division

Then plot the points for the given data and join them by means of a smooth curve.

The graph shows that the population in the years 1884 and 1875 was 27·8 million and 45·2 million respectively.

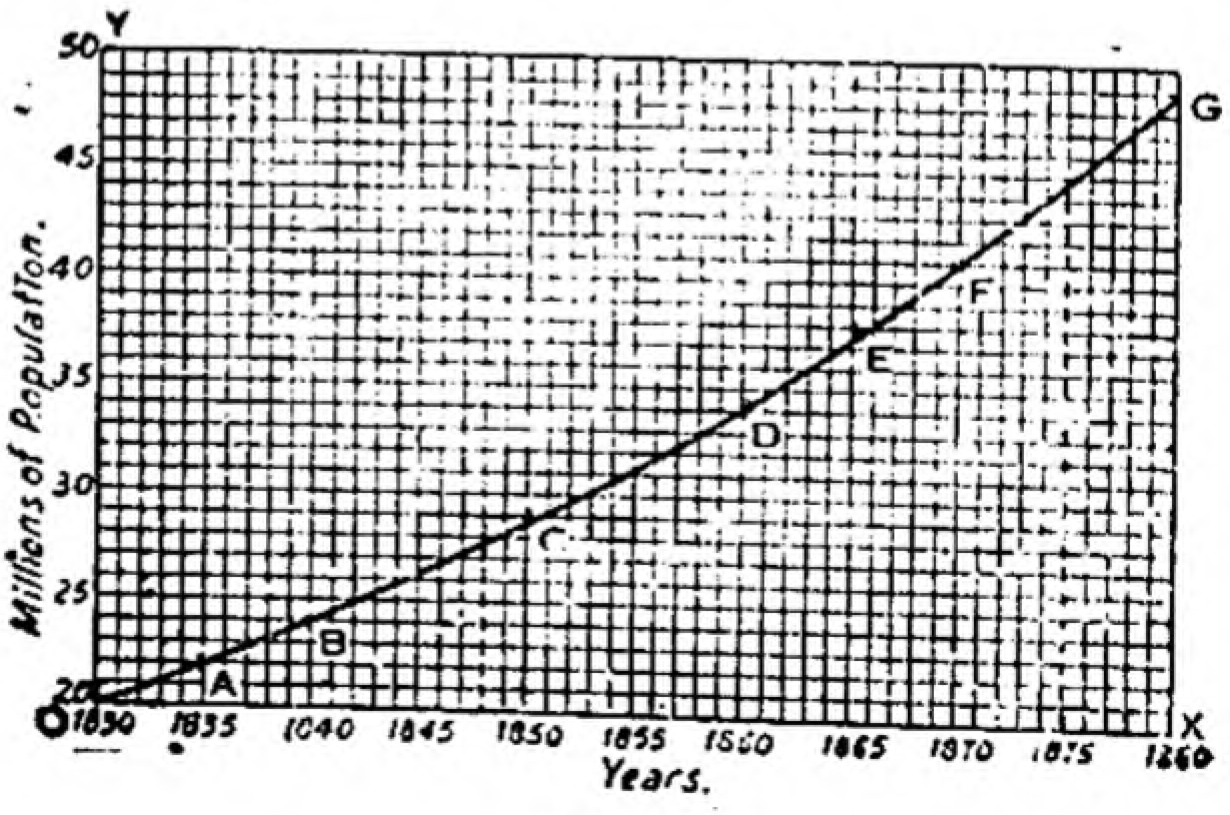


Fig. 16.

(2) The temperatures in degrees Fahrenheit taken at the different times of a day were as follows :

Time	8 A.M.	10 A.M.	12 Noon	2 P.M.	4 P.M	6 P.M.	8 P.M.
Temperature	42°	45 5°	58°	58·5°	53°	49°	43°

Exhibit graphically the variation of temperature throughout the day. Calculate the average temperature of the day and find from the graph the time at which the day experiences the average temperature.

Solution : Take the hours along OX representing 5 divisions =1 hour beginning from 8 A. M. Mark off degrees of temperature along OY representing 2 divisions=1 degree.

Plot the points according to the given data and join them by means of a smooth curve as shown in the figure.

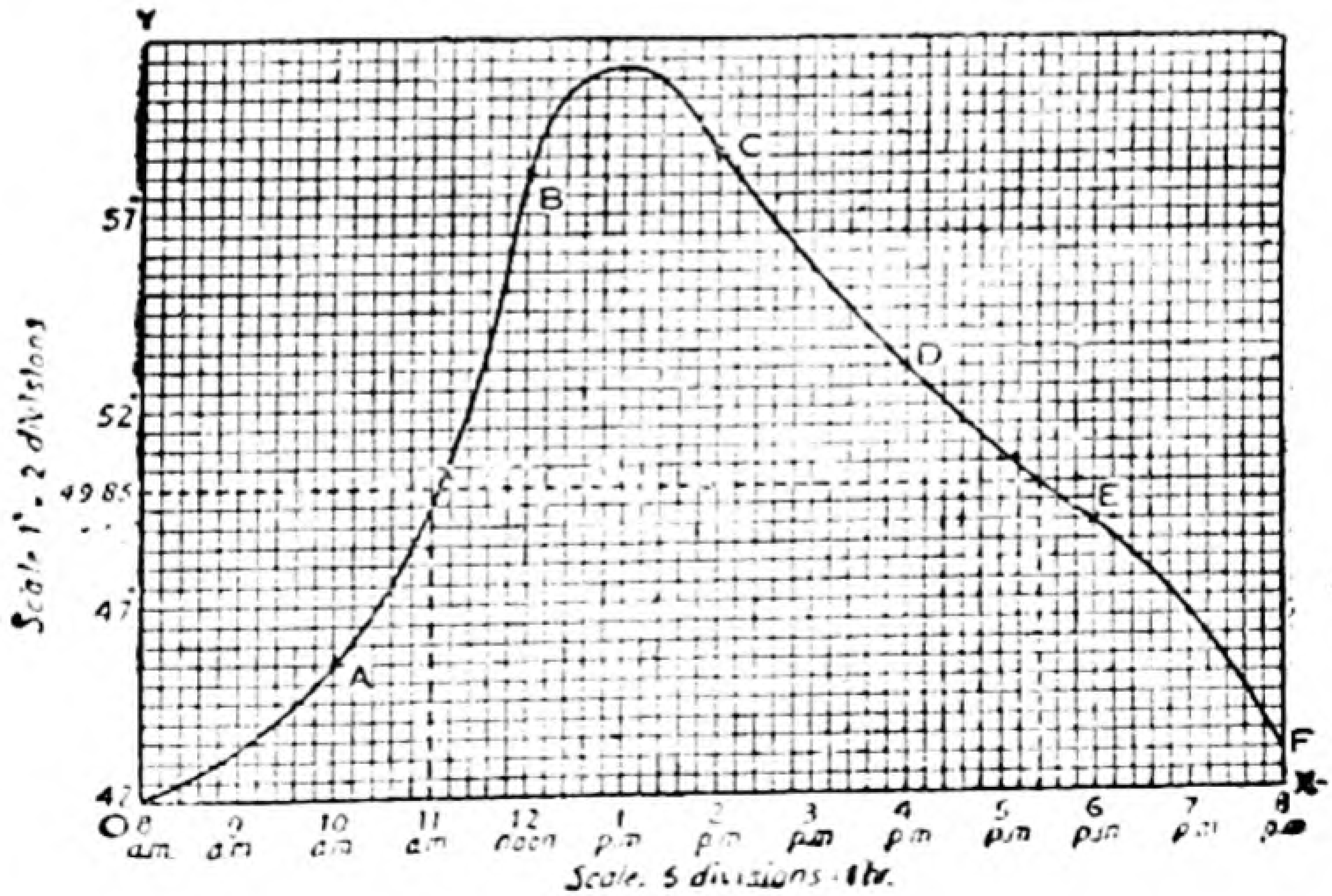


Fig. 17.

The average temperature of the day comes to about $49{\cdot}85^{c}$. From the figure it can be seen that this temperature was reached at about 11 A. M. and 5. 24 P. M.

(3) The following table gives the average weight and chest measurements corresponding to different heights. H is the height, C, the chest measurement and W, the weight, in ft. and inches, inches and pounds (lb) respectively.

(U. P. 1930)

H	W	C
5 ft. 0 inches	122	34
5 ft. 3 inches	131	35·5
5 ft. 6 inches	137·5	37
5 ft. 9 inches	156·5	39
6 ft. 0 inches	173	40·5

Represent these graphically and deduce the weights and chest measurements of the average men of 5 ft. 4 inches and 5 ft 8 inches.

Solution : Measure height in inches along OX representing 5 divisions = 1." Represent weight and chest measurements along OY taking 1 lb = 1 division and 2 divisions = 1" of the chest measurement.

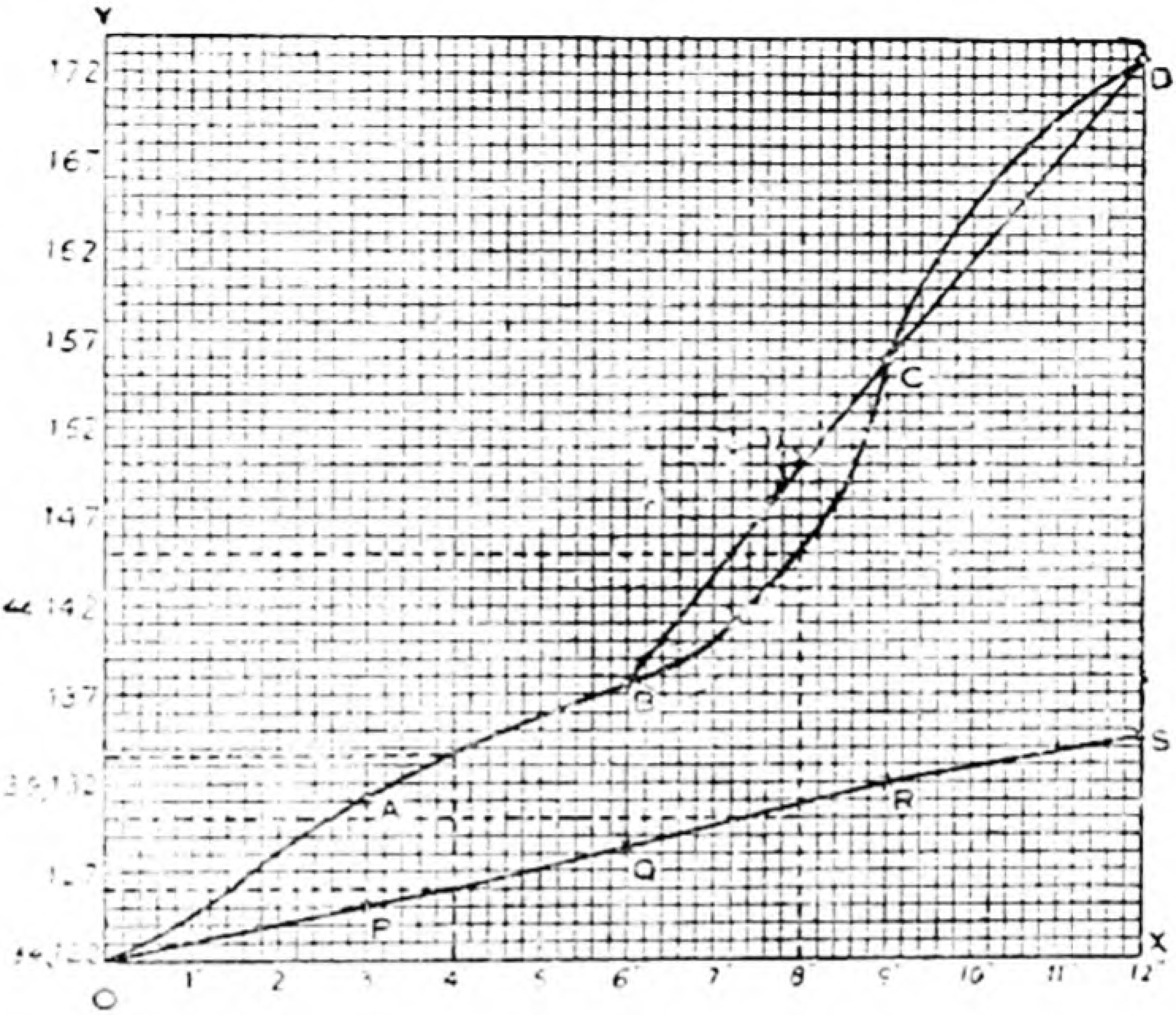


Fig 18

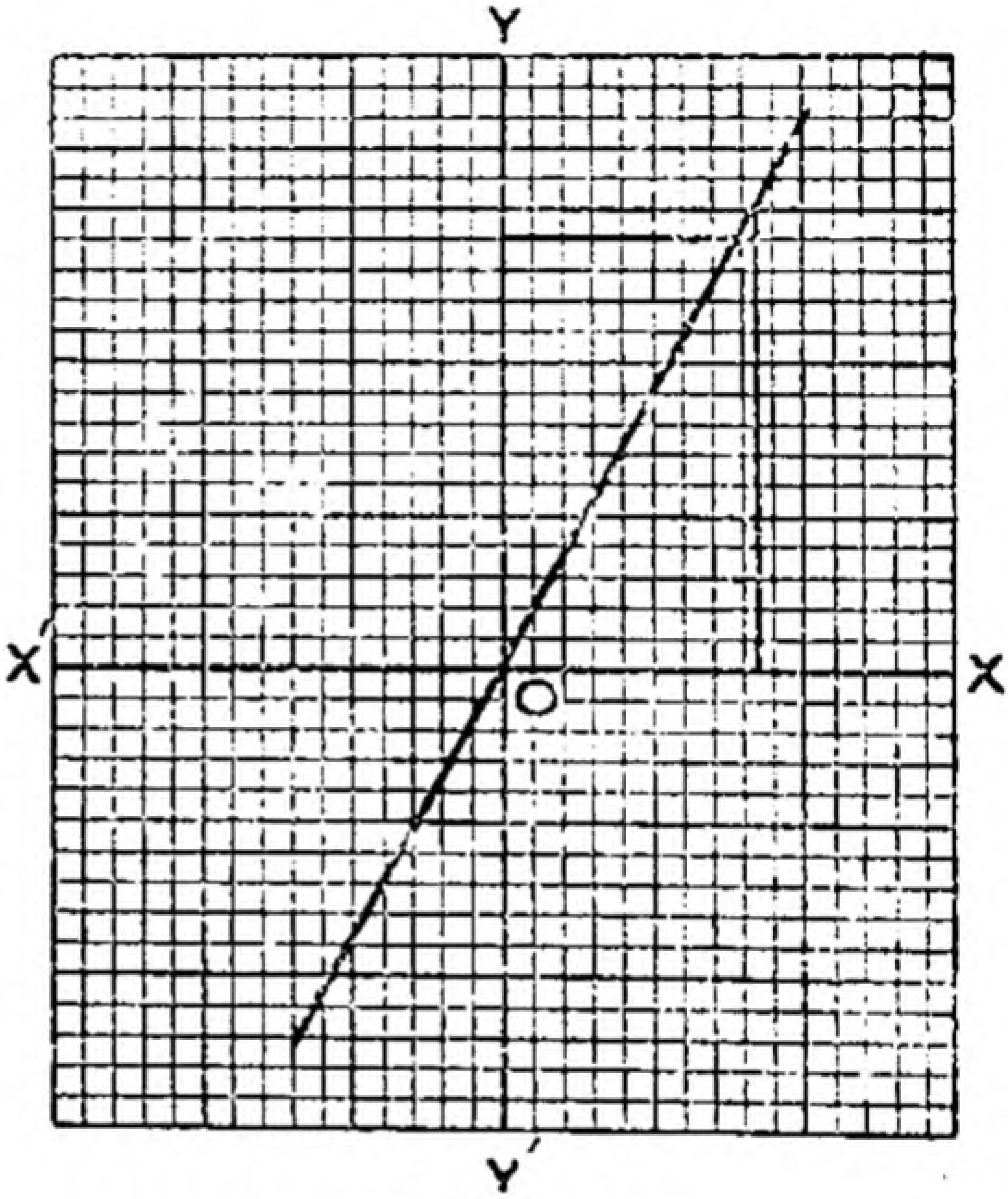
The graph OABCD represents the weight graph and OPQRS, the chest graph

From these figures the weight and chest measurements at the heights of 5′. 4″ and 5′. 8″ are 133·5 lb., 145 lb. and 36″ and 38″ respectively.

(4) The following table gives the temperatures in Centigrade degrees corresponding to those in degrees Fahrenheit. Exhibit the temperatures on a graph and find from it the degree F. corresponding to 55° C and the degree C corresponding to 134°F.

Readings on Centigrade Thermometer	60°	50°	40°
Corresponding Readings on Fahrenheit Thermometer	140°	122°	104°

Solution : Units : 1 small division on x-axis represents 2°C and 1 small division on y-axis represents 2°F. Let O represent 40°C, 104°F.



Plot the points (60, 140), (50, 122), and (40, 104). Join them by AB which therefore represents the required graph.

From the graph it is clear that 55°C =131°F and 134°F =57° C nearly.

Fig. 19.

(5) The daily expenses per head of a hotel corresponding to the number of inmates are as follows : (Ajmer 1934)

No. of inmates	150	200	250	300	350	400
Expenses per head	Rs. 6	Rs. 4-14	Rs. 4-2	Rs. 3-10	Rs. 3-4	Rs. 3

Represent the above on a graph and find the expenses per head corresponding to 175, 225 and 375 inmates

Solution : Units : 100 inmates $= 1''$ on x-axis.
Rs. 1-4 $= 1''$ on y-axis.

Let the origin O represent (150, 6) and P, Q, R and S, (200, Rs, 4-14), (250, Rs. 4-2), (300, Rs. 3-10), (330, Rs. 3-4) and (400, Rs. 3) respectively, Join P, Q, R, S by means of a smooth curve.

From the graph it is clear that points A, B, C represent 175, 225, and 375 inmates and their corresponding expenses per head are given by the ordinates of D, E and F respectively. The expenses are therefore Rs. 5-7, Rs 4-8 and Rs. 3-2 respectively.

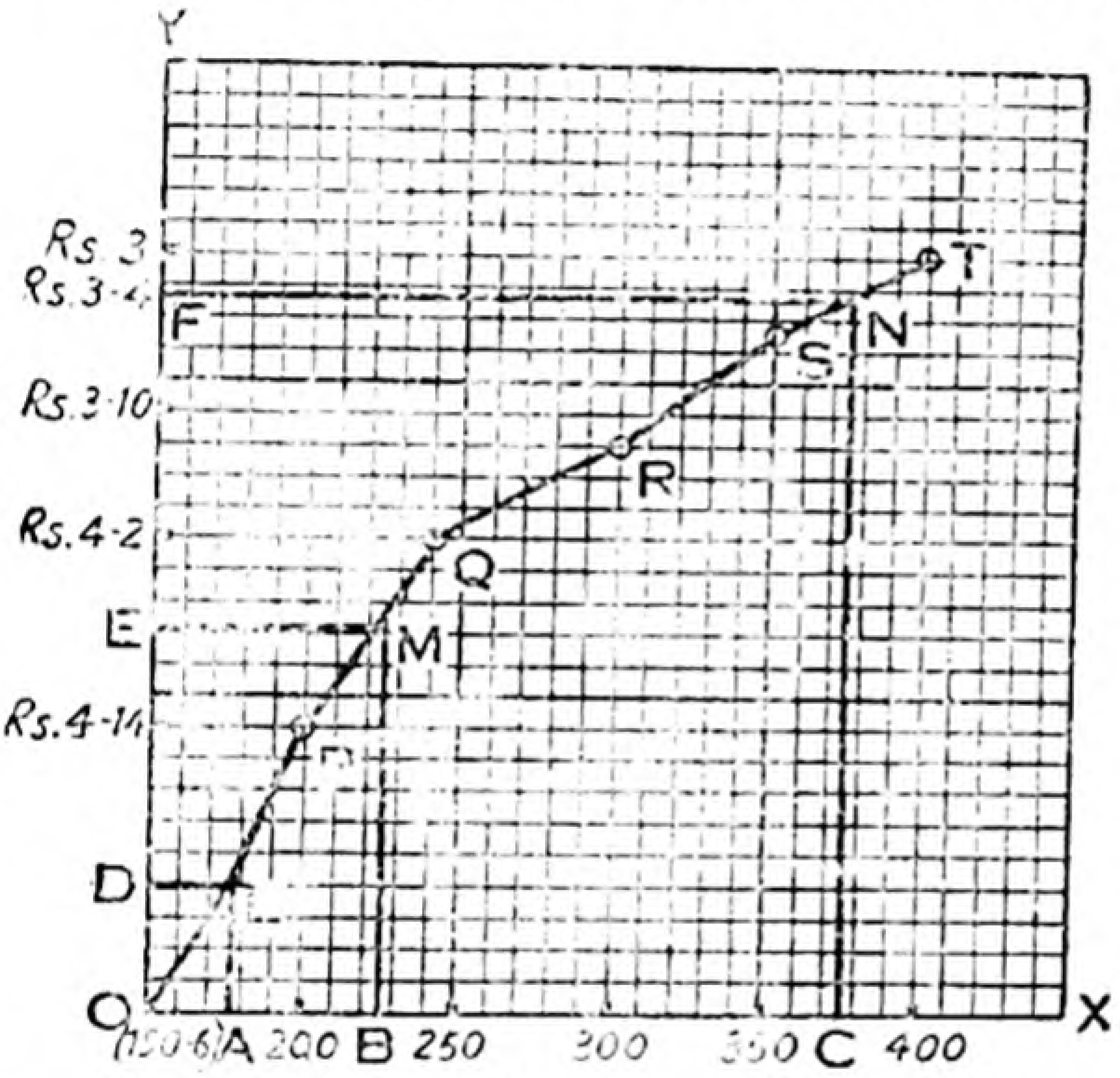


Fig. 20.

EXERCISE 43

(1) The temperature of a room from 8 A. M. to 4 P. M is given by the following table.

8 A.M.	9 A.M.	10 A.M.	11 A.M	12 A.M.	1 P.M.	2 P.M.	3 P.M.	4 P.M
66°	68°	70°	72°	74°	76°	78°	80°	82°

Represent the above graphically and read off the temperature at 10-30 A. M.

(2) The following table shows degrees F corresponding to degrees C Draw a graph showing the relation between F and C degrees and read off from the graph degrees F corresponding to 25°C and degree C corresponding to 50°F.

C°	40	—10	80	—40
F°	104	14	176	– 40

(3) The following tables gives seven readings of the thermometer on a certain day. Draw a graph by plotting them; and on the graph find the probable temperature at 1-20 P. M. and 2-10 P. M.

Time ...	11 A.M.	11.30 A.M.	12 noon	1 P.M.	2 P.M.	2.30 P. M.	3 P.M.
Temperature in degrees.	80.0	82 7	85.5	88.6	89.4	89	87 8

(Mad. 1920.)

(4) The table below gives the expenses and receipts for a publication for various number of copies produced.

Number of copies.	1000	2000	3000	4000
Expenses	Rs.218.12a	Rs.312.8a.	Rs. 406. 4a.	Rs.500
Receipts	Rs. 175	Rs. 300	Rs. 425	Rs. 540

Draw a graph showing the relation between the number of copies and the expenses; also another in the same diagram the relation between number and receipts, taking 8 m. m. to represent Rs. 25/- and 10 m. m. to represent 200 copies.

Find from the graph the amount of receipts from advertisement and estimate the smallest number of copies that must be produced to make the Publication pay. (A. U. 1918.)

(5) The following table gives the population (in million) of a country for the years specified. Illustrate these by means of graph and from it find the year when the population was (*a*) 21 millions (*b*) 23·7 millions and what was the population in (*c*) 1862 and (*d*) 1888 ?

Year ...	1850	1855	1860	1865	1870	1875	1880	1885
Population in millions	18	19.2	20	22·1	22·7	23·2	24·0	24·7

(6) The following table gives the population of two countries A and B. Draw two graphs to show the growth of population of each and from it find the probable date when the populations were equal.

Year ...	1811	1821	1831	1841	1851
A ...	17·7	24·8	30 4	36·6	40·7
B ...	23·1	27·3	31·2	35·1	39 3

(7) Draw graphs to show the rates of growth of two boys S and Y from the following particulars, and find when they were of the same height.

Jany. 1st	1930	1932	1934	1936	1938	1940	1942	1944
S	3′ 5″	3′ 9″	4′ 1″	4′ 4″	4′ 6″	4′ 7½″	4′ 8″	4′ 9½″
Y	3′ 3″	3′ 7½″	4′ 0″	4′ 3″	4′ 7½″	4′ 9″	4′ 11½″	5′ 2″

(8) The following table shows the increase in population in millions (for period of 10 years) for two countries A and B. Illustrate by means of graphs.

Assuming these increases occur gradually, find the increase in 1898 and 1933

Year	...	1865	1875	1885	1895	1905	1915	1925	1935	1945
A	...	1·25	1·85	1·9	2·1	2·3	2·0	2·2	2·3	2·4
B	...	.7	·9	1·2	1·1	1·3	1·5	1·7	1 0	2·2

(9) The following are the maximum and minimum (shade) temperatures for June. Plot these on the same page, and find from your graphs on what dates were (*a*) the greatest, (*b*) the least, ranges of temperature.

June	1	5	9	13	17	21	25	30
Maximum	108°	109°	113°	118°	119°	117°	116°	11p°
Minimum	101°	105°	108°	106°	110°	106°	103°	100°

(10) Draw a graph to show the weekly amounts of rain fall in inches for June and July from the following particulars and from it find (i) the driest week and (ii) the wettest week.

	June				July			
Week ending	8	15	22	29	6	13	20	27
Rainfall ...	3·5	6·0	4·1	5·8	7 2	8·0	5·2	8·5

(11) The following scores were made by a student in a series of 10 innings at cricket.

4, 20, 0, 13, 4, 19, 15, 3, 11, 7. Exhibit these graphically.

(12) The average monthly rainfall (in inches) of a town in U. P is given in the following table :

Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct	Nov.	Dec.
2·9	3·1	3·0	2·7	5·0	8·3	1·79	15·3	7·2	2 0	1 6	·7

Draw a graph to represent variations.

(13) The premium (in rupees) for life insurance for Rs. 1000

payable after 20 years or at death if earlier are given for various ages in the following table :

Age ...	25	30	35	40	45	50	55
Premium...	60	63	67	72	77	94	112

Exhibit these graphically.

(14) The following table gives the average weight (in pounds) of children at different ages :

Age in years...	6	8	10	12	14
Weight ...	46	54	65	81	100

Exhibit these graphically and find the average weight of a child at ages 9 and 13.

(15) Exhibit graphically the following data showing the daily receipts and expenses per head of a hotel as quoted by the manager.

Guests ...	100	150	200	250	300	350	400
Receipts per head.	Rs.5-13a	Rs. 5-1 a	Rs. 4-6	Rs.4-11a	Rs. 4-1	Rs. 3-13	Rs. 3
Expenses per head.	Rs. 5	Rs. 4-8a	Rs.3-14a	Rs. 3-6a	Rs. 3 1a	Rs. 3	Rs.2-12a

(16) A boy was measured on his 12th, 14th, 16th, 18th, 20th, 22nd and 24th birth days, when his height was found to be $4\frac{1}{3}$ ft., 4 ft. $10\frac{3}{4}$ in , $5\frac{1}{4}$ ft., 5 ft. $6\frac{1}{4}$ in., 5 ft. $8\frac{3}{4}$ in , 5 ft. $10\frac{1}{4}$ in.,

and 5 ft $11\frac{3}{4}$ in. respectively. Exhibit his growth graphically, and estimate his height at $13\frac{1}{2}$ years of age. At what age was he just 5 feet high? (Ajmer, 1935.)

(17) Of 10,000 children who passed the age of 9, 490 died at 10 years of age, 272 died at 14 years of age, 650 at 22, 617 at 25, 757 at 35, 950 at 45, 1399 at 55, 2141 at 65, 2578 at 74, 1138 at 85, 86 at 95. Draw a graph to illustrate this and estimate from it the number of deaths among these 10,000 children at the ages of 18 and 40 (U. P. 1921)

(18) The following table gives the population (in millions) of two countries A and B for the years specified :—

Year ...	1861	1871	1881	1891	1901	1911	1921
A	3·1	3·4	3·7	4·0	4·57	4·7	5·0
B	5·8	5·4	5·2	4·7	4·45	4·2	4·0

Plot the graphs on the same diagram. Estimate approximately the population when it was the same in each country and the year in which this happened. (U. P. 1929).

76. GRAPHICAL SOLUTION OF PROBLEMS

Solved Examples :

(1) Given that the price of a seer of milk is six annas, draw a graph from which the price of any number of seers may be read off. Find the price of $6\frac{1}{2}$ seers and the number of seers that can be had for 27 as.

Solution : Units : Take 5 small divisions on x-axis to represent 1 seer and 1 small division on y-axis to represent 1 anna. Let O represent 1 seer and 6 annas.

Since the price of 1 seer is 6 annas, measure the length of 5 small divisions on x-axis to represent 1 seer and then measure parallel to OY the length of 6 small divisions to represent 6 annas ann mark a point A there. Join OA Then OA produced represents the required graph.

To find the price of $6\frac{1}{2}$ seers, take B along OX to represent $6\frac{1}{2}$ seers and draw BP parallel to OY to cut the graph at P. From P draw PQ parallel to OX cutting OY at Q.

OQ, therefore, represents the price of $6\frac{1}{2}$ seers, which is annas 39.

To get the number of seers that can be had for annas 27, find the point C along OY representing annas 27. From C draw CT parallel to OX cutting the graph at T. From T draw TS parallel to OY to meet OX at S.

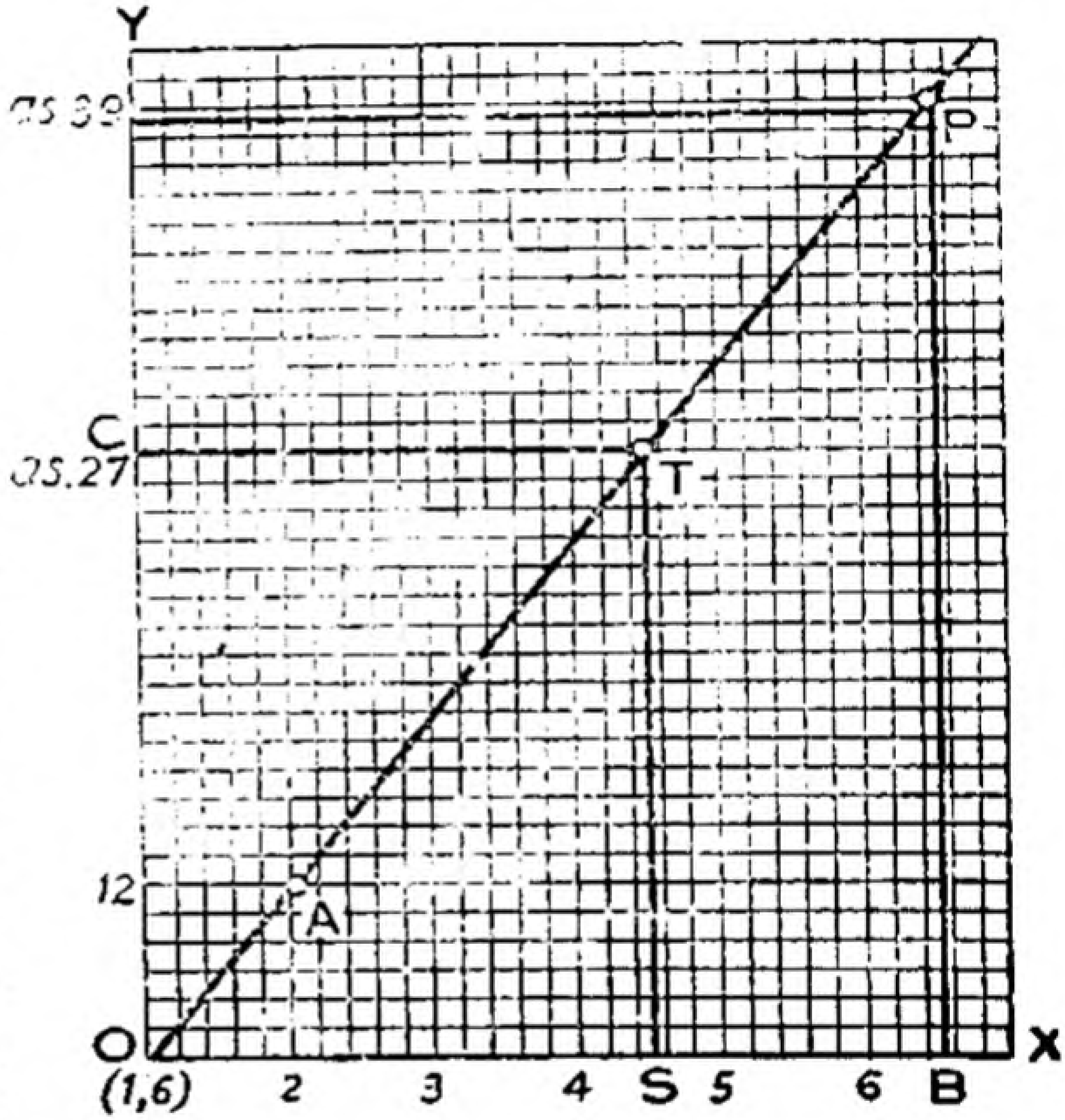


Fig. 21.

OS re presents the number of seers that can be had for annas 27. It is $4\frac{1}{2}$ seers.

EXERCISE 44

(1) If 1 md. of rice costs Rs. 16, constuct a graph which the price of any number of maunds can be found and from the

graph find the price of $3\frac{1}{2}$ maunds and the number of maunds that can be had for Rs. 28.

(2) If $2\frac{1}{2}$ seers of sugar-crystal cost Re. 1, construct a graph from which you can read off the price of 12 seers of sugar—crystal aud the quantity of crystal that can be had for Rs. 5 - 4 as.

(3) If one tola of gold is worth, Rs. 75, draw a graph to show the price of any number of tolas of the metal and from the graph read off the quantity of gold worth Rs. 1,025 and the price of $10\frac{1}{2}$ tolas of the metal.

(4) Given 1 inch equal to 2·5 cm., draw a graph to show the number of cm. in any number of inches and *vice versa*.

Read off from the graph (i) the number of cm. in 7 inches and (ii) the number of inches in 23 cms.

Solution: Scale. Take the length of 5 small divisions equal to 1″ on x-axis and the length of 2 small divisions equal to 1 cm. along OY.

Then as before let A represent 1″ and 2·5 cm. Join OA.

OA produced is the required graph

From the graph it is clear that 7″ correspond to 17·5 cm. and 23 cm., to 9·2″.

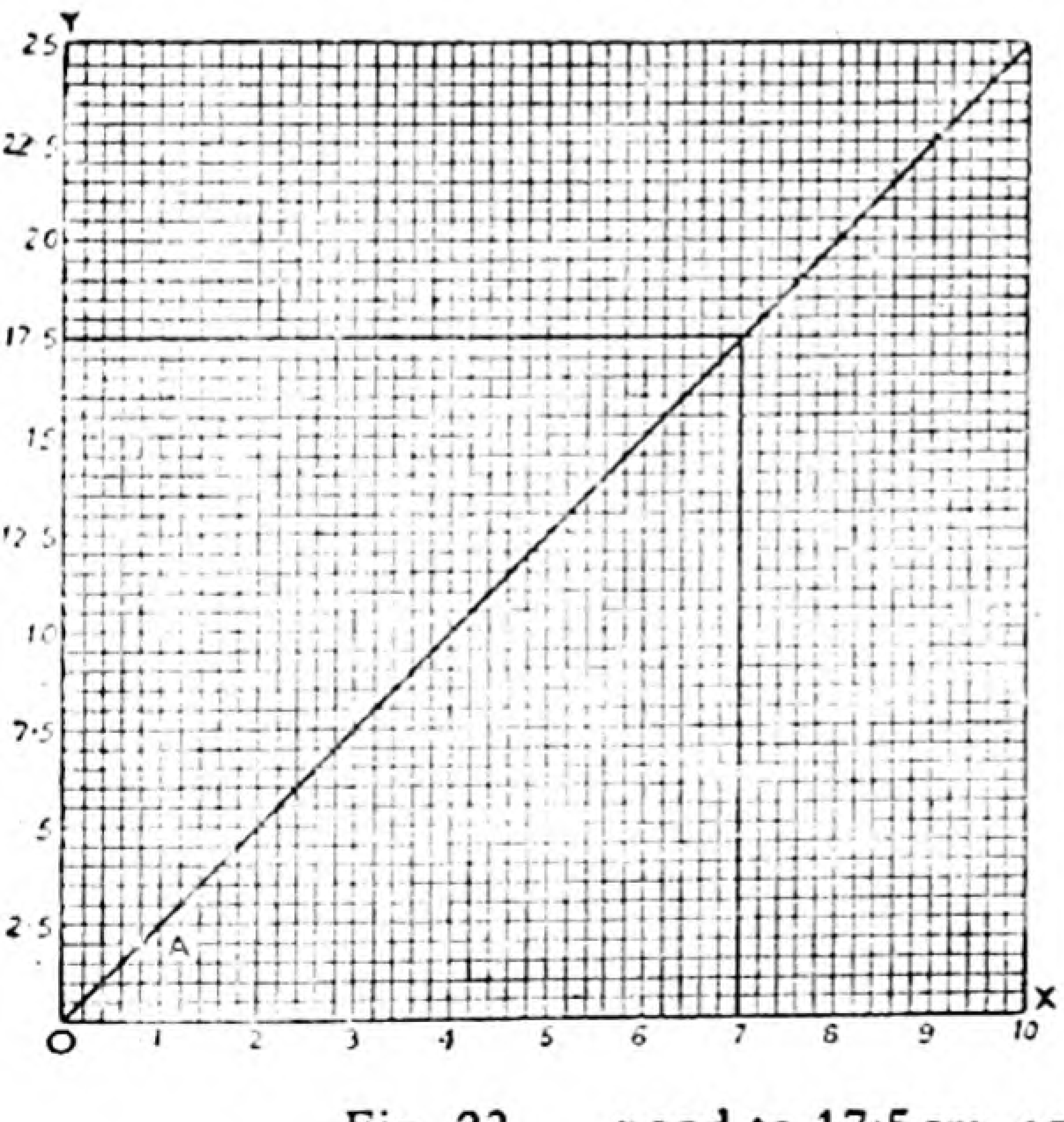


Fig. 22

(5) Given that 1 Kilogram $=2 \cdot 2$ lb. Construct a graph to show the relation between Kilogrammes and lbs. and from the graph read off the number of lbs. in 5 Kilogrammes and the number of Kilogrammes in 22 lb.

(6) Given that 1 cubic ft. of water weighs $62 \cdot 5$ lbs., draw a graph to represent the quantity of water in any number of cubic feet and from the graph read off the volume of water weighing 300 lb. and the quantity of water occupying $4 \cdot 3$ cubic ft.

(7) A cyclist travels uniformly at the rate of 15 miles an hour, draw a graph of his motion and find from the graph how far he will travel in $2\frac{1}{2}$ hours and how much time he will take in covering 40 miles.

(8) A train travels uniformly at the rate of 35 miles an hour. Draw a graph of its motion From the graph find (i) the time it will take to cover 200 miles and (ii) the distance it will cover in $3\frac{1}{2}$ hours.

(9) A walks from P to Q a distance of $12\frac{1}{2}$ miles at the rate of 3 miles an hour. At the same time B starts from Q to walk upto P at the rate of 2 miles per hour. Draw, with the same axes, the graphs of their motions and from it determine at what point they cross each other.

(10) A walks at the rate of 3 miles per hour. After three hours B starts from the same place and cycles in the same direction at the rate of 9 miles an hour. Find when and where B over takes A.

(11) A man walks uniformly at the rate of 3 miles an hour. But after two hours' journey he takes rest for 1 hour Draw a graph of his motion and read off from it (i) how far he goes in 10 hours and (ii) at what time he is 18 miles off from his starting point.

Solution: Scale : Length of 5 small divisions along OX to represent 1 hour.

Length of 2 small divisions along OY to represent 1 mile.

Since he walks at the rate of 3 miles an hour starting from O he reaches a point A in two hours. OA is his motion graph for the first two hours. He then takes rest for 1 hour. AB represents his rest. He resumes his journey and reaches C in two hours from B. Then he takes rest which is represented by CD and so on. His graph is therefore represented by OABCDEFG.

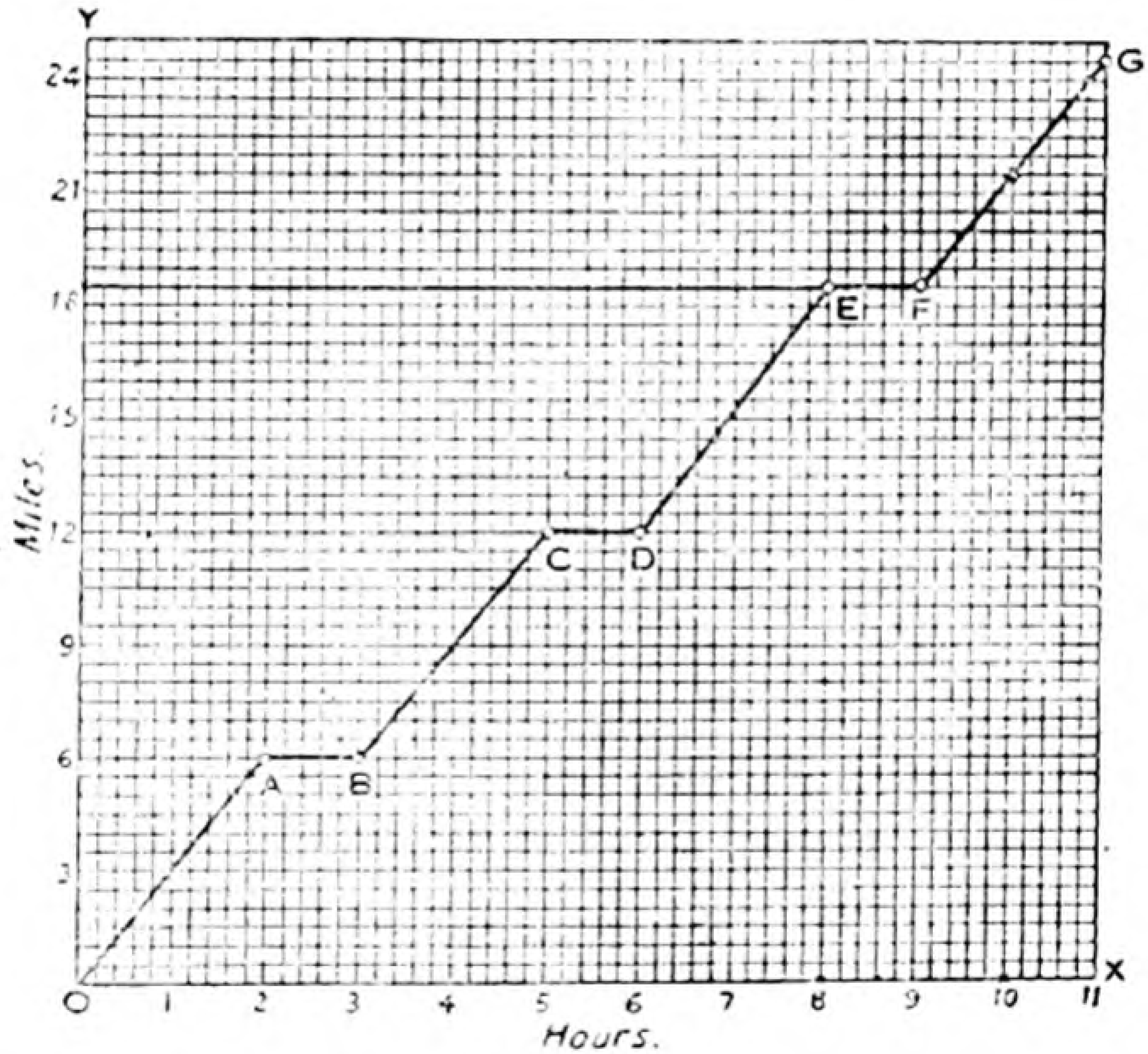


Fig. 23

From the graph it is clear that he covers 21·5 miles in 10 hours and takes 8 hours to cover 18 miles

(12) *A* walks at the rate of 4 miles an hour and rests for 18 minutes at the end of every hour Two hours later *B* runs

at the rate of 6 miles an hour. Find graphically when and where they will meet. Find the equation of the inclined portion of the graph between the first and second haltage. (A. B. 1934.)

(13) A man starts from a place P to walk towards Q at the rate of 3 miles an hour. After 4 hours he changes his mind and walks back towards P at the rate of 4 miles an hour. At the end of 3 hours he again changes his mind and runs towards Q at the rate of 6 miles an hour. Draw his motion graph.

(14) A starts on a cycle at the rate of 15 miles an hour stopping for half an hour at the end of every two hours. After 3 hours B starts from the same place and motors after him without stopping at the rate of 35 miles an hour. Draw the graphs of their motion and find when and where B will overtake A.

(15) The following table shows the timings of two trains running between Agra and Tundla and the distances of the stations from Agra. Draw their motion graphs and find when and at what distance from Tundla they cross each other assuming that all runs are made at a constant speed.

Distance.	↓			↑
15	12·30 depart	Tundla	arrive	1 10
12		Etmadpur	depart arrive	1·02 12 58
8		Kuberpur	depart arrive	12·50 12 48
2		Jamna Bridge	depart arrive	12·16 12·12
0	1·07 arrive	Agra City	depart	12·4

Solution: Scale : 1 mile = length of two small divisions along OX.

2 minutes = length of one small division along OY.

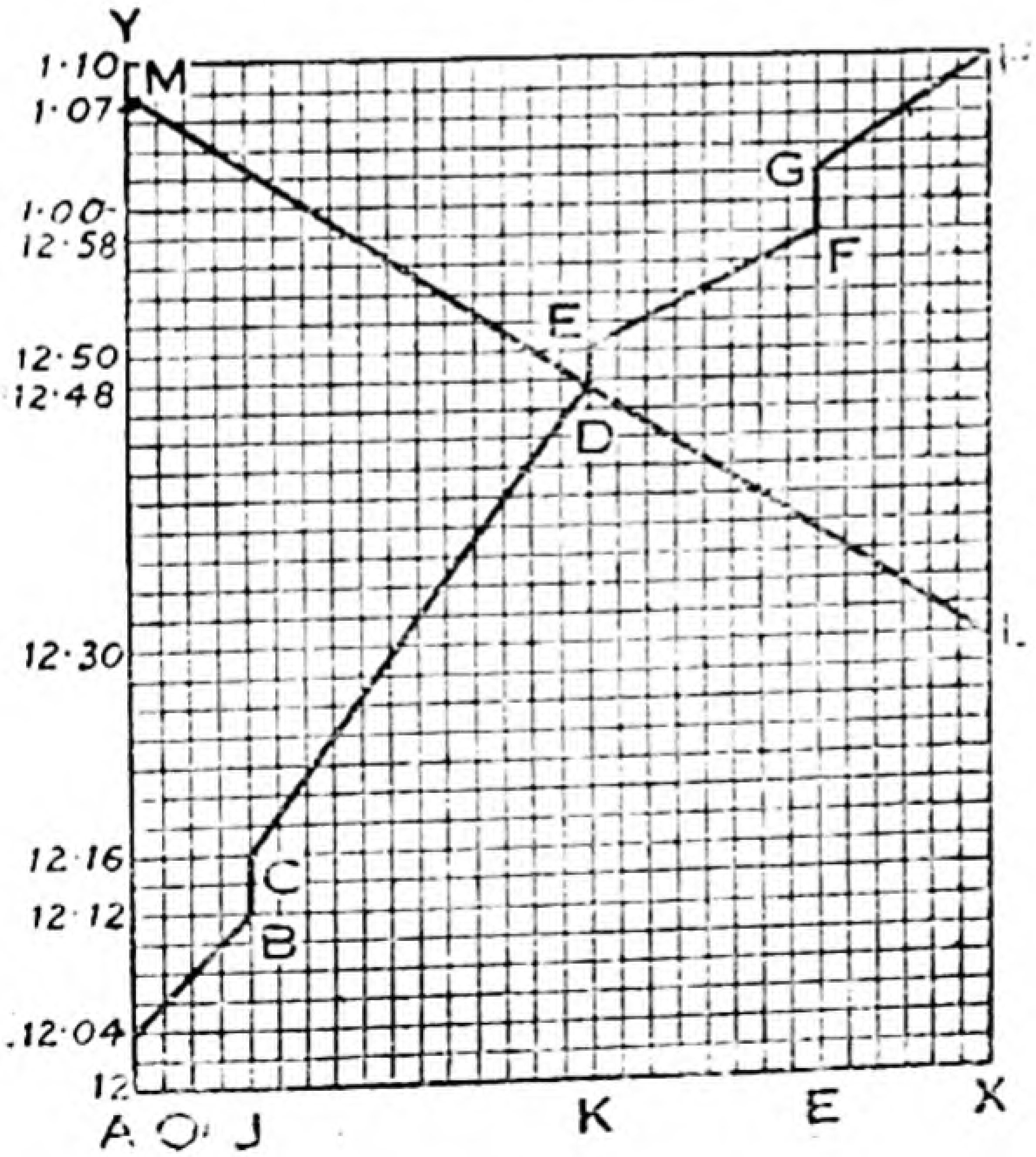
Since the direct train starts from Tundla at 12·30. L is its starting point. It reaches Agra City at 1·07. M indicates its reaching the destination.

LM is therefore its motion graph.

The other train starts at 12·04 from Agra City and reaches Jamna Bridge at 12·12. Its motion graph for that part of the journey is represented by AB It halts for 4 minutes and then reaches Kubairpur at 12·48.

Its motion graph for the second journey is represented by CD.

Similarly; the third and the final motion-graphs are represented by EF and GH respectively.



The figure ABCDEFGH represents the total graph of its motion.

From the figure it is clear that they cross each other at 12-48 at a distance of 7 miles from Tundla.

Fig. 24.

(16) The following table shows the timings of two trains between Allahabad and Cawnpore and the distances of stations from Allahabad. Assuming that the trains run with a uniform speed draw their motion graphs.

Find from the graphs when and at what distance from Allahabad they meet.

	Distance in miles			
18-47 arrive ↑	0	Allahabad	depart ↓	15·05
	81	Fatehpur	arrive	17-17
			depart	17-30
16-35 depart	120	Cawnpore	arrive	18-47

(17) The following table shows the distance from Khurja Junction of certain stations, and the timings of two trains one up and one down. Suppose the trains travel with a constant speed. Draw their motion graphs and from them find at what time and at what distance from Khurja Junction they pass each other.

	Distance in miles			
12·53 arrive ↑	0	Khurja Junction	depart ↓	12.10
	4	Khurja City	arrive	12.18
			depart	12.21
	10	Maman	arrive	12 31
			depart	12.33
12.20 depart	15	Bulandshahr	arrive	12.45

(18) The following table shows the distances from Calcutta of certain stations, and the timings of two trains, one up and one down. Assuming the speed of the train to be uniform

draw their motion graphs. From the graphs show at what distance and at the what time they cross each other.

		Distance in miles			
3-33	arrive	0	Calcutta	depart	2-00
		10	Sodepur	arrive	2-20
				depart	2-24
	↑	20	Shamnager	arrive ↓	2·44
				depart	2-48
		25	Naihati	arrive	2-54
				depart	2-58
2-25	depart	45	Ranighat	arrive	3·38

(19) *A* can do a piece of work in 40 days, *B*, in 20 days, and *A*, *B* and *C* together, in 10 days. In how many days can *C* alone do it ?

Solution: Scale: Length of 1 small division along OX=1 day
Length of 20 small divisions along OY=1 work.

Then OA represents the work-graph of A; OB; of B and OC, of A+B+C

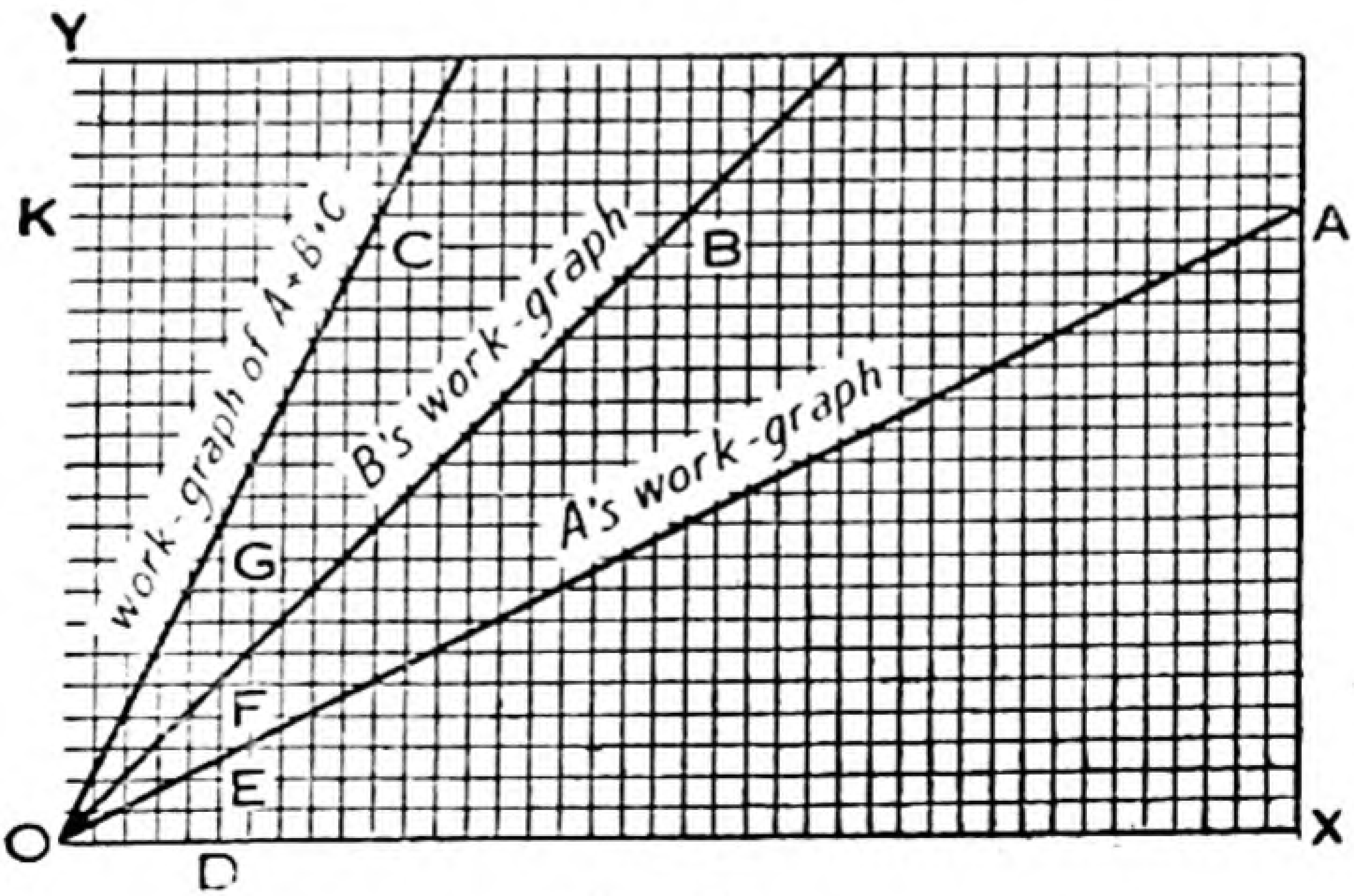


Fig. 25.

In this figure OD represents 5 days; DE, A's work in 5 days, DF, B's work in 5 days and DG, A+B+C's work in 5 days. The amounts of work are $\frac{5}{2}$, 5 and 10 small divisions respectively. Therefore the amount of work done by C alone is represented by $(10-\frac{15}{2})=\frac{5}{2}$ small divisions.

From D measure DE equal to $\frac{5}{2}$ divisions. Join OE and produce OE to meet KA in A.

Then OA represents the work-graph of C alone.

From the graph it is clear that C alone does the work in 40 days.

(20) A and B can do a piece of work separately in 15 and 45 days respectively How long will they take, working together, to do it?

(21) A cistern can be filled by one pipe in 6 hours and by a second pipe in 4 hours. It can be emptied by a third pipe in 5 hours. Find graphically in what time it can be filled if all the three pipes are opened together.

(22) A can do a piece of work in 10 days. B in 15. They work together for 2 days and then A goes away. In how many days will B finish it ?

(Delhi, 1931).

(23) A and B can do a piece of work in 8 days, B and C in 10 days and C and A in 12 days respectively, All the three work for two days and then A alone goes on. Find how long does he take now to finish it alone.

(24) Two pipes A and B can fill a cistern in 30 minutes and 40 minutes respectively. They are opened together and after 8 minutes B is stopped. How long will it take A to fill the cistern?

(25) The expenses of a family are Rs. 72 a month when rice is at 10 seers a rupee and Rs. 75 when rice is at 8 seers a rupee, if the other expenses remain constant, find these; the quantity

of rice consumed every month being supposed constant. (U. P. 1926).

(26) The annual expenses of a hospital are partly constant and partly proportional to the number of patients. The expenses were Rs. 7,680 for 12 patients and Rs. 8,640 for 16. Draw a graph to show the expenses for any number of patients and find from it the cost of maintaining 15.

In a rival establishment the expenses were Rs. 7,500 for 5 and Rs. 8,900 for 15 patients. Find graphically for what number of patients the cost would be the same in the two institutions.

Miscellaneous Examples III

(1)

(1) Resolve into factors:

$$x^2+\frac{1}{x^2}+2\left(x-\frac{1}{x}\right)-5$$

(2) Find the L. C. M. and H. C. F. of :—

$64x^6-729y^6$, $16x^4+36x^2y^2+81y^4$, $8x^3+27y^3$.

(3) Simplify:

$$\frac{1}{a(a-b)(a-c)}+\frac{1}{b(b-c)(b-a)}+\frac{1}{c(c-a)(c-b)}.$$

(4) Solve:

$$\frac{x}{11}+\frac{23}{x+4}=\tfrac{1}{3}(x+5).$$

(5) A person rowed 12 miles down a river and back again in 8 hours and found that it took thrice as long to row against the stream as to row with it. Find the rate of the stream and of the boat in still water.

(6) Find the square root of:

$(x+1)(x+2)(x+3)(x+4)+1$

(7) If $a:b=3:4$; find the value of $5a+6b:5a+7b$.

(8) Show that

$$\frac{1}{1-x}-\frac{1}{1+x}-\frac{2x}{1+x^2}-\frac{4x^3}{1+x^4}-\frac{8x^7}{1+x^8}-\frac{16x^{15}}{1-x^{16}}$$

is equal to zero.

(9) Show that the graphs represented by the following equations have a common point, and find the co-ordinates of that point

$$3x+4y=10,\ 4x+y=9,\ 5x-2y=8.$$

(10) Find the value of a and b for which x^2+x-6 may be a factor of the expression x^3-ax^2-bx-6.

(2)

(1) Resolve into elementary factors :
$3a^2-10ab+8b^2$

(2) Find the H. C. F. of :
$24x^4-2x^3-60x^2-32x$ and $18x^4-6x^3+4x$

(3) Find the square root of :
$4x^4+12x^3-11x^2-30x+25.$

(4) Simplify :

$$\frac{a^2}{(a-b)(a-c)}+\frac{b^2}{(b-c)(b-a)}+\frac{c^2}{(c-a)(c-b)}.$$

(5) Solve :

$$\frac{1}{x-1}+\frac{2}{x}=\frac{3}{2-x}.$$

(6) If $a:b::c:d::e:f$, show that

$$\frac{3a-4c-5e}{3b-4d-5f}=\sqrt[3]{\frac{ace}{bdf}}.$$

(7) Find the value of $x^2+\frac{1}{x^2}$ when $x+\frac{1}{x}=2.$

(8) Find two numbers one of which is greater by four than three-fifth of the other so that the difference of their squares may be equal to 24.

(9) Solve graphically :

$3x+y=14x-2y=0.$

(10) Fill in the blanks

$(x-a)(x+b)(x+c) \equiv x^3+x^2(\quad\quad)+x(\quad\quad)-abc$

(3)

(1) Find the continued product of :

$(x+y)(x^2+y^2)(x^4+y^4)(x-y).$

(2) Solve :

$(x+a)(y-b)=(x-a)(y+b).$

$x-y=1$

(3) Resolve into factors :

$x^4-16y^4.$

(4) Simplify :

$$\frac{1}{x^2-3x+2}+\frac{1}{x^2-5x+6}-\frac{1}{x^2-4x+3}.$$

(5) What number must be added to

$x^4+4x^3+10x^2+12x+3$ to make it a perfect square ?

(6) Solve graphically :

$4y=3x$

$4x-3y=14.$

(7) If $2s=a+b+c$, prove that :

$(s-a)^2+(s-b)^2+(s-c)^2+s^2=a^2+b^2+c^2.$

(8) Find the H. C. F. of

$27a^5-45a^4-16$ and $18a^5-45a^4-5a-14.$

(9) A colonel wishing to form his men into a solid square finds he has 55 men over If he increases the side of the square by 1, he has 40 men too few. How many men are there in the regiment ?

(10) If $f(x)=x^3-3x^2+3x-1$, find the value of $f(0)$, $f(1)$, $f(x+1)-f(x-1)$

(4)

(1) Resolve into factors :

$x^2y^2-6xyz-72z^2$.

(2) Find the L. C. M of :

$4x^3+16x^2-3x-45$ and $10x^3+63x^2+119x+60$.

(3) Solve

$$\frac{x}{x+1}+\frac{x+1}{x}=2\frac{1}{6}.$$

(4) Find the continued product of :

x^2+x+1, x^2-x+1, x^4-x^2+1.

(5) If $x+\frac{1}{x}=5$, find the value of $x^3+\frac{1}{x^3}$.

(6) A man distributed Rs 100 equally among his friends; if there had been 5 more friends each would have received Re. 1 less. How many friends had he ?

(7) Simplify :

$$\frac{a}{(a-b)(a-c)}+\frac{b}{(b-c)(b-a)}+\frac{c}{(c-a)(c-b)}.$$

(8) If $\frac{2x^2-3y^2}{x^2+y^2}=\frac{2}{41}$, find $x : y$.

(9) Show that $x-1$, $x-2$ and $x+3$ are the factors of $x^4+x^3-7x^2-x+6$.

(10) Find the square root of :

$x^4+y^4+(x+y)^4$.

(5)

(1) Resolve into factors :

$a^2-a+b^2+b-c^2+c-2ab$.

(2) Solve :

$$\frac{3}{x+3}+\frac{4}{x+5}=\frac{7}{x+11}.$$

(3) Tin appears to lose one-seventh of its weight when weighed in water. Lead appears to lose one twelfth of its weight under the same condition. An alloy of tin and lead which weighs 270 lbs. in air, appears to weigh only 240 lbs. when weighed in water How much of each metal does it contain?

(4) Find the L. C. M. of:

$3x^2-10x-8$, $4x^2-20x+9$. and $6x^2+x-2$.

(5) Simplify:

$$\frac{1}{a^2-1}+\frac{1}{a^2-2}-\frac{1}{a^2+1}-\frac{1}{a^2+2}.$$

(6) Find the value of:

$$\frac{5x+2a}{5x-2a}+\frac{5x+2b}{5x-2b},\text{ when } x=\frac{4ab}{5(a+b)}.$$

(7) If $x:a=y:b=z:c$, show that

$$\sqrt[3]{\frac{x^3+y^3+z^3}{a^3+b^3+c^3}}=\frac{ax^2+by^2+cz^2}{a^2x+b^2y+c^2z}.$$

(8) Find the equation of the straight line passing through the points:

(4, 5), (6, 8).

(9) Find the value of:

$xy+yz+zx$, when $x+y+z=17$, $x^2+y^2+z^2=129$.

(10) Determine the value of x which makes $4x^6+4x^5+9x^4+16x^3+10x^2+13x+7$ a perfect square.

(6)

(1) Resolve into factors:

$(a+2b-c)^2-(a+b)^2-(b-c)^2$.

(2) Simplify:

$$\frac{bc}{(a-b)(a-c)}+\frac{ca}{(b-c)(b-a)}+\frac{ab}{(c-a)(c-b)}.$$

(3) If $a : b :: c : d$, prove that :

$$\frac{1}{ma}+\frac{1}{nb}+\frac{1}{pc}+\frac{1}{qd}=\frac{1}{bc}\left(\frac{a}{q}+\frac{b}{p}+\frac{c}{n}+\frac{d}{m}\right).$$

(4) A man after travelling 12 miles east and a certain distance due north, finds himself 13 miles from his starting point Find, with the help of a squared paper, how far north did he travel ?

(5) Find the square root of :

$$\left(x+\frac{1}{x}\right)^2-4\left(x-\frac{1}{x}\right)$$

(6) Solve :

$$x+y=25.$$
$$y+z=27.$$
$$z+x=32$$

(7) If $\left(a+\frac{1}{a}\right)^2=3$, prove that $a^3+\frac{1}{a^3}=0$.

(8) Find the H.C.F. of :

x^3+x^2+x+1 and x^3+3x^2+3x+1.

(9) If $\frac{2x^2-x-3}{(x-1)^2(x-2)} \equiv \frac{A}{x-2}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$, find the values of A, B and C.

(10) The denominator of a fraction exceeds the numerator by 3, and if the numerator be increased by 7, the fraction is increased by unity. Find the fraction.

(7)

(1) Solve :

$$\frac{1}{1+\frac{1}{1+\frac{1}{x}}}=5.$$

(2) Simplify :

$$\frac{2}{x^2-1}+\frac{3}{x^2+x-2}+\frac{2}{x^2+3x+2}.$$

(3) Find the H.C.F. of :

x^3-4x^2+7x-6 and $2x^3-7x^2+2x+8$.

(4) Draw the graphs of $3x-2y=6$ and $2x+3y=0$ and measure their angle of intersection.

(5) Find the factors of :

$(1-ab)^2(a+b)^2-(1+ab)^2(a-b)^2$.

(6) The area of a rectangular field is 3 acres ; and the ratio of the length to the breadth is 6:5. Find the length and breadth.

(7) What must be added to the terms of $a:b$ so that the resulting ratio may be in the duplicate ratio of $a:b$?

(8) If $f(x)=x^2-9x+3$, find the value of $f(x-1)-f(x+1)-f(0)$.

(9) If $\frac{a}{b}=\frac{b}{c}$ prove that

$(a+b+c)(a-b+c)=a^2+b^2+c^2$.

(10) Find the square root of :

$$\frac{9x^2}{4y^2}+\frac{5}{2}+\frac{y^2}{4x^2}-\frac{3x}{y}-\frac{y}{x}.$$

(8)

(1) Find the factors of :

$8x^3-1-y^3-6xy$.

(2) Reduce to its simplest form the expression :

$$\frac{a}{a-b}-\frac{b}{a+b}-\frac{b^2}{a^2-b^2}-\frac{a^2}{a^2+b^2}.$$

(3) Solve :

$$\frac{10}{x-4}-\frac{5}{x-3}=\frac{4}{x-6}+\frac{1}{x-1}.$$

(4) Simplify :

$$\frac{(x+a)^2}{(a-b)(a-c)}+\frac{(x+b)^2}{(b-c)(b-a)}+\frac{(x+c)^2}{(c-a)(c-b)}.$$

(5) In an examination a candidate takes 5 compulsory and 2 optional papers. All the papers have the same maximum and the candidate scores equal marks in all the papers but fails by 45 marks in the aggregate. On a second attempt his marks in each paper are better in the ratio 36 : 25 and he omits one of the optionals, thus securing 54 marks more than the minimum. Find the number of marks required for a pass.

(6) Draw the graph of $y+3x+2=0$ for values of x from $x=0$ to $x=-4$ and, by the aid of your graph, obtain the value of x when $y=3$.

(7) Find for what value of p the expression $x^5-61x+p$ is exactly divisible by $x+1$.

(8) The H. C. F. of two expressions is x^2+3x+2 and their L. C. M. is $(x^2+3x+2)\ (x-3)\ (x+5)$; one expression is $x^3+8x^2+17x+10$. Find the other.

(9) If a, b, c, d are in continued proportion then $\sqrt{(a+b+c)\ (b+c+d)}=\sqrt{ab}+\sqrt{bc}+\sqrt{cd}$

(10) A boat's crew can row at the rate of 8 miles an hour in still water. What is the speed of the river's current, if it takes them 2 hours and 40 minutes to row 8 miles up and 8 miles down

(9)

(1) Resolve into factors :

$$x^3(y-z)+y^3(z\ \ x)+z^3(x-y).$$

(2) Solve :

(i) $3y+4x=3xy$

$$\frac{9}{x}+\frac{8}{y}=7$$

(ii) $(x+4)(y-3)=xy+11.$

$(x-3)(y-4)=xy+1.$

(3) The sum of the squares of two consecutive even numbers is 100. Find the numbers.

(4) Explain the method of completing the square to solve the equation $ax^2+bx+c=0$.

(5) Let the two expressions whose H. C. F. is $x-a$ be $(x-a)p$ and $(x-a)q$ where p and q are prime to each other. Then their L. C. M. is $(x-a)pq$

(6) If a, b, c are in continued proportion and if $a(b-c)=2b$, prove that $a-c=2\left(\frac{a+b}{a}\right)$.

(7) Draw the triangle whose vertices are (3, 4), (—5,—6), and (3, 0). What are the mid-points of the sides ?

(8) Find the H. C F. of :

$6x^4+x^3-6x^2-5x-2$ and $2x^4+3x^3+2x^2-7x-6$.

(9) Find the square root of :

$$\left(x-\frac{1}{x}\right)^2-16\left(x+\frac{1}{x}\right)+68.$$

(10) Find the value of :

$$\frac{1}{2x(y-x)(a-x)}+\frac{1}{2x(y+x)(a+x)}+\frac{1}{(y^2-x^2)(a+x)}.$$

(10)

(1) Simplify :

$$\frac{x^3+8y^3}{x^2-4y^2}\times\frac{x^2-xy-2y^2}{x^4+4x^2y^2+16y^4}\div\frac{x^2+xy}{x^3+2x^2y+4xy^2}.$$

(2) Solve :

$$\frac{7x-1}{4}-\frac{1}{3}\left(2x-\frac{1-x}{2}\right)=6\frac{1}{3}$$

(3) If a, b, c, d are in continued proportion show that $(a-d)^2=(b-c)^2+(b-d)^2+(c-a)^2$.

(4) A labourer is engaged for 40 days on the condition that he would get Rs 2. 8a. for each day he works and would lose 10a. for each day he is idle. After the expiry of the time he receives nothing Find how many days does he work ?

(5) If $x+y+z=0$, prove that

$$3+\frac{y^2+z^2-x^2}{2yz}+\frac{z^2+x^2-y^2}{2zx}+\frac{x^2+y^2-z^2}{2xy}=0$$

(6) Find the factors of :

(i) x^6-64

(ii) $x^3-17x+26$.

(7) The salary of a clerk is increased each year by a fixed sum. After six years service his salary is raised to £ 128 and after 15 years to £ 200. Draw a graph from which his salary may be read off for any year and determine from it

(i) his initial salary and

(ii) the salary he should receive for his 21st year of service.

(8) If y is a mean proportional between x and z, show that $xy+yz$ is a mean proportional between x^2+y^2 and y^2+z^2.

(9) Prove that the product of any four consecutive integers increased by unity is a perfect square.

(10) Find two expressions both of the second degree in x whose H. C. F. is $x-2$ and whose L. C. M. is

$$x^3-8x^2+17x-10.$$

CHAPTER XVII

LAWS OF INDICES

77. *Definition*: In the expression x^m, m is called the *index of the power of* x and x is the *base*. x^m means the product of m factors each equal to x.

78. We are familiar with :

(1) $x^3 \times x^4 = x \times x \times x \times x \times x \times x \times x. = x^7$

(2) $x^5 \div x^2 = \dfrac{x \times x \times x \times x \times x}{x \times x}$.

$= x \times x \times x. \quad = x^3$

(3) $(x^2)^3 \quad = x^2 \times x^2 \times x^2. \quad = x^6.$

(4) $(xy)^3 \quad = xy \times xy \times xy.$

$= x \times x \times x \times y \times y \times y \quad = x^3 y^3.$

LAWS OF INDICES :

In (1) $x^a \times x^b = x^{a+b}$.

(2) $x^a \div x^b = x^{a-b}$.

(3) $(x^a)^b = x^{ab}$.

(4) $(xy)^a = x^a y^a$ etc., etc., the first, *i. e.* $x^a \times x^b = x^{a+b}$ is the Laws of Indices : others are its extensions.

79. General proofs of the Laws of Indices :

To prove that.

(1) $x^a \times x^b = x^{a+b}$.

Since $x^a \quad = x \times x \times x \times x$..............to a factor.

and $x^b \quad = x \times x \times x$...........to b factors.

$$\therefore x^a \times x^b = (x \times x \times x \times x \ldots\ldots\ldots \text{to } a \text{ factors}).$$
$$\times (x \times x \times x \times \ldots\ldots\ldots \text{to } b \text{ factors.})$$
$$= x \times x \times x \times x \times x \times x \times x \times x \ldots\ldots\ldots \text{to } a+b \text{ factors:}$$
$$= x^{a+b}.$$

Similarly, $x^a \times x^b \times x^c = x^{a+b+c}$.

Hence, *the product of any mumber of powers of a given quantity (base) is equal to the sum of the indices of the factors.*

(2) (*i*) $x^a \div x^b = x^{a-b}$ when $a > b$.

Proof : $x^a = x \times x \times x \times x \ldots\ldots\ldots$ to a factors.

and $x^b = x \times x \times x \ldots\ldots\ldots$ to b factors.

$$\therefore x^a \div x^b = \frac{x \times x \times x \times x \ldots\ldots \text{to } a \text{ factors}}{x \times x \times x \ldots\ldots\ldots \text{to } b \text{ factors}}.$$

Cancel b factors.

$$\therefore x^a \div x^b = x \times x \ldots\ldots \text{to } (a-b) \text{ factors.}$$
$$= x^{a-b}.$$

Second proof :

Since $x^{a-b} \times x^b = x^{(a-b)+b} = x^a$.

$\therefore x^a \div x^b = x^{a-b}$ (by transposition.)

(ii) $x^a \div x^b = \dfrac{1}{x^{b-a}}$, if $b > a$.

$$\text{Proof : } x^a \div x^b = \frac{x \times x \times x \times \ldots\ldots \text{to } a \text{ factors}}{x \times x \times x \times x \ldots\ldots \text{to } b \text{ factors}}.$$

$$= \frac{1}{x \times x \times x \ldots\ldots \text{to } b-a \text{ factors}}.$$

$$= \frac{1}{x^{b-a}}.$$

Hence, when one power of a quantity is divided by another power of the same quantity, the index of the quotient is obtained by subtracting the index of the divisor from that of the quotient. This is in the case when the power of the dividend is greater than that of the divisor.

If however the power of the divisor, say b, is greater than that of the dividend say a, a factors of the numerator will cancel with a factors of the denominator. Then the required index is obtained by subtracting the index of the dividend from the index of the divisor, *i.e*, $b-a$, in this case.

80. Solved examples.

Find the value of :

(1) $16^{\frac{1}{2}}$.

Solution :

Since $a^{\frac{1}{2}} \times a^{\frac{1}{2}} = a^{\frac{1}{2}+\frac{1}{2}} = a$.

also $\sqrt{a} \times \sqrt{a} = a$.

$\therefore\ a^{\frac{1}{2}} = \sqrt{a}$.

Hence $a^{\frac{1}{2}}$ *is the square root of* a.

Similarly $16^{\frac{1}{2}} = \sqrt{16}$. $\quad = 4$.

(2) $64^{\frac{1}{3}}$.

Solution :

Since $a^{\frac{1}{3}} \times a^{\frac{1}{3}} \times a^{\frac{1}{3}} = a$.

also $\sqrt[3]{a} \times \sqrt[3]{a} \times \sqrt[3]{a} = a$.

$\therefore\ a^{\frac{1}{3}} = \sqrt[3]{a}$.

Hence, $a^{\frac{1}{3}}$ *is the cube root of* a.

Similarly, $64^{\frac{1}{3}} = \sqrt[3]{64}. \quad = 4.$

(3) 2^0.

Solution :

Since $a^m \times a^n = a^{m+n}$ for all values of m and n
Put $m = 0$.

$\therefore \ a^0 \times a^n = a^{0+n}$.

$= a^n$.

$\therefore \ a^0 = \dfrac{a^n}{a^n} \quad = 1.$

or, $a^0 = 1$.

Hence, any quantity raised to the power zero is equivalent to 1.

Similarly, $2^0 = 1$.

(4) $4^{\frac{3}{2}}$

Solution :

Since $a^m \times a^n = a^{m+n}$, for all values of m and n, put $\dfrac{p}{q}$ for each of them.

Then $a^{\frac{p}{q}} \times a^{\frac{p}{q}} = a^{\frac{p}{q}+\frac{p}{q}}. \quad = a^{\frac{2p}{q}}.$

Similarly, $a^{\frac{p}{q}} \times a^{\frac{p}{q}} \times a^{\frac{p}{q}}. \quad = a^{\frac{3p}{q}}.$

or $a^{\frac{p}{q}} \times a^{\frac{p}{q}} \times a^{\frac{p}{q}}$ ....to q factors.

$= a^{\frac{p}{q}+\frac{p}{q}+\frac{p}{q}+\ldots\ldots}$ to q factors :

$= a^{\frac{pq}{q}}$. $\qquad = a^{p}$.

Hence $a^{\frac{p}{q}}$ is the q^{th} root of a^{p}.

Similarly, $4^{\frac{3}{2}}$ is the square root of 4^{3}.

or $4^{\frac{3}{2}} = \sqrt{4\times4\times4}$

$= \sqrt{2\times2\times2\times2\times2\times2}$

$= 2\times2\times2$. $\qquad = 8$:

$\frac{1}{3^{2}}$ and $\frac{1}{3^{-2}}$.

Solution :

Since $\frac{1}{x^{m}} = \frac{x^{0}}{x^{m}}$ (for $x^{0}=1$).

$= x^{0-m}$. $\qquad = x^{-m}$.

or, since, $x^{m} \times x^{n} = x^{m+n}$, put $m=-n$.

$\therefore x^{-n} \times x^{n} = x^{n-n}$.

$= x^{0}$. $\qquad = 1$.

$\therefore x^{-n} = \frac{1}{x^{n}}$ (dividing both sides by x^{n}).

Hence, when a quantity raised to a certain power is *removed* from the *numerator* to the *denominator*, or *vice versa*, the sign of the index is changed.

Similarly, $\frac{1}{3^2} = 3^{-2}$.

and $\frac{1}{3^{-2}} = 3^2 = 9$.

EXERCISE 45

Simplify:

(1) $x^m \times x^n$.

(2) $x^m \div x^n$.

(3) $3^2 \times 3^2$.

(4) $3^3 \div 3^2$.

(5) $2^5 \div 2^0$.

(6) $2^6 \times 2^5$.

(7) $(2^3)^2$.

(8) $(x^m)^2$.

(9) $(y^m)^n$.

(10) x^0.

(11) 3^0.

(12) $x^a \times x^{-a}$.

(13) $3^2 \times 3^{-2}$.

(14) $x^0 a^0$.

(15) $3a^0$.

(16) $27^{\frac{2}{3}}$.

(17) $16^{\frac{2}{3}}$.

(18) $\frac{1}{5^{-3}}$.

(19) $x^p \div x^{p+q}$.

(20) $2^3 \div 2^{-5}$.

(21) $2^3 \times 2^{-5}$.

(22) $\frac{1}{a^{-3}}$.

(23) $\frac{1}{6^{-2}}$.

(24) $(5^{-1} \times 2^2 \times 4^{-3})^3$.

(25) $(2^{-2} \times 2^3 \times 4^{-2})$.

(26) $\sqrt[3]{x^4} \div \sqrt[0]{x^3}$.

(27) $\dfrac{1}{\sqrt[3]{x^2}}$. (28) $(a^{2}b^{3}c^{4})^{3}$

(29) $(a^2b^{-3}c^4)^{-2}$. (30) $(3p)^{2r} \div (3p)^{5r}$.

(31) $\dfrac{x^{m+2n}x^{3m-8n}}{x^{5m-6n}}$ (C. U. 1874.)

(32) $\sqrt{a^{-3}b^{4}} \times \sqrt[4]{a^{4}b^{-4}}$

(33) $\sqrt{a^{\frac{2}{3}}b^4c^{-\frac{1}{3}}} \div \sqrt[3]{a^2b^4c^{-2}}$.

(34) $\left(\dfrac{a^{-1}b^8}{a^2b^3}\right)^{\frac{1}{4}} \div \left(\dfrac{a^3b^6}{a^5b^8}\right)^{-2}$.

(35) $\dfrac{2^{m+1}.\,3^{2m-n}.\,5^{m+n}.\,6^n}{6^m.\,10^{n+2}.\,15^n}$

Solution :

$$\text{Exp.} = \frac{2^{m+1}\,3^{2m-n}.\,5^{m+n}.\,2^n.\,3^n}{3^m.\,2^m.\,2^{n+2}.\,5^{n+2}.\,5^n.\,3^n}.$$

$$= \frac{2^{m+n+1}.\,3^{2m-n+n}.}{2^{m+n+2}.\,3^{m+n}.}\,\frac{5^{m+n}}{5^{2n+2}}$$

$$= 2^{-1}.\,3^{m-n}\,5^{m-n-2}$$

$$= \frac{3^{m-n}}{2}.\,5^{m-n-2}$$

$$= \frac{3^{m-n}}{2}.\,\frac{5^{m-n}}{5^2}$$

$$=\frac{15^{m-n}}{2\times 25} \qquad =\frac{15^{m-n}}{50}$$

(36) $\sqrt{x^{3}y^{-\frac{2}{3}}z^{-\frac{7}{6}}}\div\sqrt[3]{x^{4}y^{-1}z^{\frac{5}{4}}}$

(37) $\dfrac{2^{n}\cdot 6^{m+1}.10^{m-n}.15^{m+n-2}}{4^{m}.3^{2m+n}.25^{m-1}}$

(38) $\dfrac{6^{n}.2^{2n}.3^{3n}}{30^{n}.3^{2n}.2^{3x}}$ and find its numerical value correct to two places of decimal when $n=\frac{1}{2}$

(39) $2^{3^{2}}\div\left(2^{3}\right)^{2}$.

(40) $\left(\dfrac{a-b}{a^{\frac{1}{2}}-b^{\frac{1}{2}}}-\dfrac{a^{\frac{3}{2}}-b^{\frac{3}{2}}}{a-b}\right)^{-1}$.

(41) $2^{3^{2}}\div 2^{2^{3}}$.

(42) $x^{a(b-c)}\times x^{b(c-a)}\times x^{c(a-b)}$.

Find the value of :

(43) $\sqrt[3]{\left(\frac{1}{32}\right)^{-3}}$.

(44) $\dfrac{2^{6}\times 16^{-2}\times 256}{4^{-9}\times 2^{23}}$.

(45) $\left(729\right)^{-\frac{2}{3}}\times\sqrt[3]{\left(\frac{64}{125}\right)^{2}}\div\left(\frac{121}{144}\right)^{-\frac{1}{2}}$.

(46) Show that

$$\frac{3^{n}+3^{n-1}}{3^{n+1}-3^{n}}=\frac{2}{3}.$$

(47) Prove that

$$\frac{y^{-1}}{x^{-1}+y^{-1}}+\frac{y^{-1}}{x^{-1}-y^{-1}}=\frac{2xy}{y^2-x^2}\ .$$

Simplify :

(48) $$\frac{2^{\frac{1}{3}}.12^{\frac{1}{2}}.27^{\frac{1}{3}}.5^{\frac{1}{3}}}{10^{\frac{1}{3}}.4^{\frac{1}{3}}.18^{\frac{1}{2}}.81^{\frac{1}{8}}}\ .$$

(49) $$\left(\frac{x^l}{x^m}\right)^{l+m}\times\left(\frac{x^m}{x^n}\right)^{m+n}\times\left(\frac{x^n}{x^l}\right)^{n+l}.$$

(Dacca 1925).

(50) $$\frac{5^{3}.2^{\frac{1}{4}}.10^{-\frac{1}{4}}}{15^{\frac{3}{4}}.6^{-\frac{3}{4}}.4^{\frac{3}{4}}}\ .$$

81. *Multiplication and Division, etc. of quantities with integral. fractional, positive and negative indices :*

Solved example :

Multiply :

(1) $x-x^{\frac{1}{2}}y^{\frac{1}{2}}+y$ by $x^{\frac{1}{2}}+y^{\frac{1}{2}}$.

Solution :

$$\begin{array}{l}
x-x^{\frac{1}{2}}y^{\frac{1}{2}}+y \\
\qquad x^{\frac{1}{2}}+y^{\frac{1}{2}} \\
\hline
x^{\frac{3}{2}}-xy^{\frac{1}{2}}+x^{\frac{1}{2}}y \\
\qquad +xy^{\frac{1}{2}}-x^{\frac{1}{2}}y+y^{\frac{3}{2}} \\
\hline
x^{\frac{3}{2}} \qquad\qquad\qquad +y^{\frac{3}{2}}
\end{array}$$

EXERCISE 46

Multiply :

(1) $x-x^{\frac{1}{2}}y^{\frac{1}{2}}+y$ by $x^{\frac{1}{2}}-y^{\frac{1}{2}}$.

(2) $x^{\frac{1}{4}}y+y^{\frac{2}{3}}$ by $x^{\frac{1}{3}}-y^{\frac{1}{3}}$.

(3) $x^{\frac{2}{3}}+x^{\frac{1}{3}}y^{\frac{1}{3}}+y^{\frac{2}{3}}$ by $x^{\frac{1}{3}}-y^{\frac{1}{3}}$.

(4) $x+y+z-x^{\frac{1}{2}}y^{\frac{1}{2}}-y^{\frac{1}{2}}z^{\frac{1}{2}}-z^{\frac{1}{2}}x^{\frac{1}{2}}$ by $x^{\frac{1}{2}}+y^{\frac{1}{2}}+z^{\frac{1}{2}}$.

(5) Find the continued product of :

$x^{\frac{1}{4}}-y^{\frac{1}{4}}, x^{\frac{1}{4}}+y^{\frac{1}{4}}, x^{\frac{1}{2}}+y^{\frac{1}{2}}$.

Divide :

(6) $x^{\frac{4}{3}}+a^{\frac{2}{3}}x^{\frac{2}{3}}+a^{\frac{4}{3}}$ by $x^{\frac{2}{3}}+a^{\frac{1}{3}}x^{\frac{1}{3}}+a^{\frac{2}{3}}$.

Solution :

$$x^{\frac{2}{3}}+a^{\frac{1}{3}}x^{\frac{1}{3}}+a^{\frac{2}{3}}\ \Big)\ x^{\frac{4}{3}}+a^{\frac{2}{3}}x^{\frac{2}{3}}+a^{\frac{4}{3}}\ \Big(\ x^{\frac{2}{3}}-a^{\frac{1}{3}}x^{\frac{1}{3}}+a^{\frac{2}{3}}$$

$$\begin{array}{l}
x^{\frac{4}{3}}+a^{\frac{1}{3}}x+a^{\frac{2}{3}}x^{\frac{2}{3}} \\
\hline
-a^{\frac{1}{3}}x \\
-a^{\frac{1}{3}}x-a^{\frac{2}{3}}x^{\frac{2}{3}}-ax^{\frac{1}{3}} \\
\hline
a^{\frac{2}{3}}x^{\frac{2}{3}}+ax^{\frac{1}{3}}+a^{\frac{4}{3}} \\
a^{\frac{2}{3}}x^{\frac{2}{3}}+ax^{\frac{1}{3}}+a^{\frac{4}{3}}. \\
\hline
\times
\end{array}$$

$\therefore$ Quotient $=x^{\frac{2}{3}}-a^{\frac{1}{3}}x^{\frac{1}{3}}+a^{\frac{2}{3}}$.

(7) $a-b$ by $a^{\frac{1}{2}}-b^{\frac{1}{2}}$. (8) $a-b$ by $a^{\frac{1}{3}}-b^{\frac{1}{3}}$.

(9) $a^{-2}-16b^{2}$ by $a^{-\frac{1}{2}}-2b^{\frac{1}{2}}$.

(10) $x^{\frac{4}{3}}+a^{\frac{2}{3}}x^{\frac{2}{3}}+a^{\frac{4}{3}}$ by $x^{\frac{2}{3}}-a^{\frac{1}{3}}x^{\frac{1}{3}}+a^{\frac{2}{3}}$.

(11) $x^2y^{-2}+x^{-2}y^2+2$ by $x^{\frac{2}{3}}y^{-\frac{2}{3}}+x^{-\frac{2}{3}}y^{\frac{2}{3}}-1$.

(12) $a^{5m}+b^{5m}$ by a^m+b^m.

Find the H. C F. of :—

(13) $e^{2x}x^3+e^{2x}-x^3-1$ and $e^{2x}x^2+2e^xx^2$.

$$-e^{2x}+x^2-2e^x-1.$$

Solution :

$$e^{2x}x^3+e^{2x}-x^3-1=e^{2x}(x^3+1)-1\,(x^3+1)$$
$$=(e^{2x}-1)(x^3+1)$$
$$=(e^x+1)\ (e^x-1)\ (x+1)\ (x^2-x+1).$$

$$e^{2x}x^2+2e^xx^2-e^{2x}+x^2-2e^x-1$$
$$=e^{2x}(x^2-1)+2e^x(x^2-1)+1(x^2-1)$$
$$=(x^2-1)\ (e^{2x}+2e^x+1)$$
$$=(x+1)\ (x-1)\ (e^x+1)\ (e^x+1).$$

Therefore, since the first expression,

$=(e^x+1)\ (e^x-1)\ (x+1)\ (x^2-x+1)$ and the second expression $=(e^x+1)\ (e^x+1)\ (x+1)\ (x-1)$

$\therefore$ H. C. F. $=(e^x+1)\ (x+1)$.

(14) $2-7x^{-1}-46x^{-2}-21x^{-3}$ and

$2x+11-13x^{-1}-99x^{-2}-45x^{-3}$.

(15) $5x^{-2}-3x^{-3}+64x^{-6}$ and

$x^{-2}-3x^{-5}+20x^{-6}$.

Find the L. C. M. of :

(16) $3x^{-2}-12x^{-3}-8x^{-4}$, $4x^{-2}-20x^{-3}+9x^{-4}$

and $6x^{-2}+x^{-3}-2x^{-4}$.

Solution :

Since $3x^{-2}-10x^{-3}-8x^{-4}=(x^{-1}-4x^{-2})(3x^{-1}+2x^{-2})$,

$4x^{-2}-20x^{-3}+9x^{-4}=(2x^{-1}-x^{-2})(2x^{-1}-9x^{-2})$,

and $6x^{-2}+x^{-3}-2x^{-4} \quad =(2x^{-1}-x^{-2})(3x^{-1}+2x^{-2})$,

$\therefore$ L. C. M. $=(x^{-1}-4x^{-2})(3x^{-1}+2x^{-2})(2x^{-1}-9x^{-2})$.

$(2x^{-1}-x^{-2})$.

(17) $8x^{\frac{2}{3}}+27, 16x^{\frac{8}{9}}+36x^{\frac{4}{9}}+81$ and $6x^{\frac{4}{9}}-5x^{\frac{2}{9}}-6$.

(18) $x^{4}-2x^{\frac{4}{3}}+1$ and $x^{4}+2x^{\frac{8}{3}}-1$.

Find the square root of :—

(19) $x^{\frac{8}{5}}-2a^{-\frac{3}{5}}x^{\frac{11}{5}}-2a^{\frac{4}{5}}x^{\frac{4}{5}}+a^{-\frac{6}{5}}x^{\frac{14}{5}}+2a^{\frac{1}{5}}x^{\frac{7}{5}}+a^{\frac{8}{5}}$

(C. E 1880).

Solution : Arrange the expression in the descending powers of x and ascending powers of a

Then the expression

$$=a^{-\frac{6}{5}}x^{\frac{14}{5}}-2a^{-\frac{3}{5}}x^{\frac{11}{5}}+x^{\frac{8}{5}}+2a^{\frac{1}{5}}x^{\frac{7}{5}}-2a^{\frac{4}{5}}x^{\frac{4}{5}}+a^{\frac{8}{5}}$$

Extract the square root in the ordinary way.

$$\pm\left(a^{-\frac{3}{5}}x^{\frac{7}{5}}-x^{\frac{4}{5}}-a^{\frac{4}{5}}\right)$$

$$a^{-\frac{3}{5}}x^{\frac{7}{5}}\Big)\ a^{-\frac{6}{5}}x^{\frac{14}{5}}-2a^{-\frac{3}{5}}x^{\frac{11}{5}}+x^{\frac{8}{5}}+2a^{\frac{1}{5}}x^{\frac{7}{5}}-2a^{\frac{4}{5}}x^{\frac{4}{5}}+a^{\frac{8}{5}}$$

$$
\begin{array}{rl}
 & a^{-\frac{6}{5}}x^{\frac{14}{5}}. \\
2a^{-\frac{3}{5}}x^{\frac{7}{5}}-x^{\frac{4}{5}}) & -2a^{-\frac{8}{5}}x^{\frac{11}{5}}+x^{\frac{8}{5}} \\
 & -2a^{-\frac{8}{5}}x^{\frac{11}{5}}+x^{\frac{8}{5}} \\
\hline
2a^{-\frac{3}{5}}x^{\frac{7}{5}}-2x^{\frac{4}{5}}-a^{\frac{4}{5}}\ ) & -2a^{\frac{1}{5}}x^{\frac{7}{5}}+2a^{\frac{4}{5}}x^{\frac{4}{5}}+a^{\frac{8}{5}} \\
 & -2a^{\frac{1}{5}}x^{\frac{7}{5}}+2a^{\frac{4}{5}}x^{\frac{4}{5}}+a^{\frac{8}{5}} \\
\hline
 & \times
\end{array}
$$

the required square root $=\pm(a^{-\frac{3}{5}}x^{\frac{7}{5}}-x^{\frac{4}{5}}-a^{\frac{4}{5}})$.

(20) $a+2\sqrt{2ab}+2b+4\sqrt{2ac}+8\sqrt{bc}+8c$.

(M. U. 1881)

CHAPTER XVIII

ELEMENTARY SURDS

82. Examine the quantities :

(i) $\sqrt{4}$, $\sqrt{9}$, $\sqrt{a^2}$, $\sqrt[3]{x^3}$

and (ii) $\sqrt{2}$, $\sqrt{3}$, $\sqrt{a^3}$, $\sqrt[3]{x^4}$

In the first case the values of the quantities can be *exactly* found.

Hence such quantities as $\sqrt{4}$, $\sqrt{9}$, $\sqrt{a^2}$ $\sqrt[3]{x^3}$ are said to be *Rational Quantities*

In the second case the values of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{a^3}$, $\sqrt[3]{x^4}$ cannot be *exactly* found. They are called *Irrational Quantities.*

It is clear that in order to know whether an algebraic quantity say $\sqrt{a}$ is rational or not we must know its arithmetical value.

83. *Definition.*

The *Order* or *Degree* of the surds is indicated by the symbol of its root.

As for example, $\sqrt{x}$, $\sqrt[3]{y}$, $\sqrt[4]{z}$, and $\sqrt[n]{p}$ are the surds of the *second* (*quadratic*), *third* (*cubic*), *fourth* and n^{th} order or degree respectively.

84. Since $\sqrt{a}=a^{\frac{1}{2}}$, $\sqrt[3]{x^2}=x^{\frac{2}{3}}$ etc, surds can be expressed as quantities with fractional indices. They are therefore governed by the law of indices.

85. Consider the following examples :

$$\sqrt{5} = 5^{\frac{1}{2}} = 5^{\frac{3}{6}} = \sqrt[6]{5^3}.$$

$$\sqrt[3]{5} = 5^{\frac{1}{3}} = 5^{\frac{2}{6}} = \sqrt[6]{5^2}$$

$$\sqrt[6]{5} = 5^{\frac{1}{6}}. \qquad = \sqrt[6]{5}.$$

$$\sqrt[n]{5} = 5^{\frac{6}{n}\cdot\frac{1}{6}} \qquad = \sqrt[6]{5^{\frac{6}{n}}}$$

That is, the surds of different orders, *e. g.*, $\sqrt{5}$ (second order), $\sqrt[3]{5}$ (third order) $\sqrt[6]{5}$ (sixth order) and $\sqrt[n]{5}$ (n^{th} order) are transformed into surds of the same order *i. e.*, of the sixth order.

In order to change the surds of different orders to those of the same lowest order, express them as the surds of the order of their L. C. M.

As for example, $\sqrt{x^3}$, $\sqrt[3]{x^2}$, $\sqrt[4]{x}$, $\sqrt[6]{x^3}$ can be represented as :

$$\sqrt{x^3} = x^{\frac{3}{2}} = x^{\frac{18}{12}} = \sqrt[12]{x^{18}}$$

$$\sqrt[3]{x^2} = x^{\frac{2}{3}} = x^{\frac{8}{12}} = \sqrt[12]{x^8}$$

$$\sqrt[4]{x} = x^{\frac{1}{4}} = x^{\frac{3}{12}} = \sqrt[12]{x^3} \text{ and}$$

$$\sqrt[6]{x^3} = x^{\frac{3}{6}} = x^{\frac{6}{12}} = \sqrt[12]{x^6}.$$

All of them are thus transferred to the twelfth order, 12 being the L. C. M. of 2, 3, 4, 6.

86. Consider the following examples.

$$3 = (3^3)^{\frac{1}{3}} = 27^{\frac{1}{3}} = \sqrt[3]{27}.$$

$$a^2 = (a^8)^{\frac{1}{4}} = \sqrt[4]{a^8}.$$

$$x^2y^3 = (x^{2n}y^{3n})^{\frac{1}{n}} = \sqrt[n]{x^{2n}y^{3n}}$$

That is, a quantity which is not a surd can be changed into a surd of any order.

87. Consider the following examples:—

$$17\sqrt{2} = (17^2)^{\frac{1}{2}}(2)^{\frac{1}{2}} = \sqrt{17\times17\times2} = \sqrt{578}.$$

$$7\sqrt[3]{5} = (7^3)^{\frac{1}{3}}(5)^{\frac{1}{3}} = (7\times7\times7\times5)^{\frac{1}{3}} = \sqrt[3]{1715}.$$

$$2a^2b^3\sqrt{x^3} = (2^2)^{\frac{1}{2}}(a^4)^{\frac{1}{2}}(b^6)^{\frac{1}{2}}(x^3)^{\frac{1}{2}}$$

$$= (4a^4b^6x^3)^{\frac{1}{2}} = \sqrt{4a^4b^6x^3}.$$

$$x\sqrt[n]{y} = (x^n)^{\frac{1}{n}}(y)^{\frac{1}{n}} = (x^ny)^{\frac{1}{n}} = \sqrt[n]{x^ny}.$$

Thus a mixed surd (product of a rational quantity and a surd) can be expressed as a surd.

88. Consider the following examples:—

$$\sqrt{1000} = \sqrt{10\times10\times10} = 10\sqrt{10}$$

$$\sqrt[3]{864} = \sqrt[3]{6\times6\times6\times4} = 6\sqrt[3]{4}$$

$$\sqrt[4]{405} = \sqrt[4]{3\times3\times3\times3\times5} = 3\sqrt[4]{5}.$$

$$\sqrt[4]{81a^4b^8c^{10}} = (3^4a^4b^8c^8c^2)^{\frac{1}{4}}$$

$$= 3ab^2c^2\sqrt[4]{c^2}.$$

That is, entire surds are transformed into mixed surds if possible.

This can be done by the help of factors

EXERCISE 47

Express the following as surds of the second order :

(1) 2, (2) 5. (3) 7. (4) a (5) c

(6) a^4. (7) d^5. (8) a^3b^5. (9) $a+b$.

(10) x^2-y^5. (11) $5x^3y^2$. (12) $3x^4y^6z^5$,

Express the following as the surds of the fifth order.:

(13) 2. (14) 3. (15) 5. (16) 9.

(17) a^2. (18) a^2b^2. (19) $ab\sqrt{x}$. (20) $a^2b^3\sqrt[3]{c}$.

(21) a^5b^7. (22) $2a^2x^3z^5$.

Express the following as the surds of n^{th} degree.

(23) 3. (24) 5. (25) a^2. (26) c^5.

(27) a^3b^4. (28) $x^2y^3z^5$. (29) $\sqrt{a}\sqrt[3]{b}\sqrt[5]{c}$.

(30 $\sqrt{xy^2z^3}$ (31) $\sqrt[3]{p^3q^4r^2}$.

(32) a^2-x^2. (33) $\sqrt{a^8-x^8}$.

(34) $a\sqrt[l]{x^a y^b z^c}$. (35) $a^2+x^2y^3$.

Express the following as the surds of the same lowest order :

(36) $\sqrt{x}, \sqrt[3]{a^2}$. (37) $\sqrt{x^3}; \sqrt[3]{y^4}$.

(38) $\sqrt[3]{a^4}, \sqrt[4]{x^5}$, (39) $\sqrt{y}, \sqrt[5]{z}$.

(40) $ab^2, \sqrt[3]{a^3c}, \sqrt[4]{b^2c^3}$.

(41) $\sqrt{a-b}, \sqrt[3]{a^2-b^2}$.

(42) $\sqrt{xy}, \sqrt[4]{x^3y^3}. \sqrt[6]{x^2y^2}$

(43) $\sqrt{ab^2}, \sqrt[3]{x^2y^3}, \sqrt[4]{y^3z^4}$.

(44) $\sqrt[3]{p^4q^4}, \sqrt[6]{p^2q^6}, \sqrt{p^6q^4}$.

(45) $\sqrt{x^2+y^2}, \sqrt[3]{x^3-y^3}, \sqrt{x^2+xy+y^2}$.

Express the following as the entire surds :

(46) $2\sqrt{3}$. (47) $3\sqrt{5}$. (48) $5\sqrt{7}$.

(49) $a^2\sqrt{b}$. (50) $a^2.\sqrt[3]{b}$. (51) $a^3.\sqrt[4]{c}$.

(52) $7x^2.\sqrt{3x}$. (53) $2x^3.\sqrt[3]{x^2}$. (54) $3x^2y\sqrt[3]{2x^2y}$.

Express the following in their simplest form:

(55) $\sqrt{20}$ (56) $\sqrt{63}$. (57) $\sqrt[3]{81}$.

(58) $\sqrt{a^3}$. (59) $\sqrt{a^5}$. (60) $\sqrt[3]{-6000}$.

(61) $\sqrt[3]{128a^4x^6}$. (62) $\sqrt[5]{160x^6y^5}$.,

Arrange the following according to the descending order of magnitude :

(63) $\sqrt[3]{2}, \sqrt[4]{5}, \sqrt[6]{6}$.

Solution :

Transform them into surds of the same order and then arrange them according to the magnitude of the numbers under the roots.

We have $\sqrt[3]{2} = \sqrt[12]{2^4} = \sqrt[12]{16}$

$\sqrt[4]{5} = \sqrt[12]{5^3} = \sqrt[12]{125}$

$\sqrt[6]{6} = \sqrt[12]{6^2} = \sqrt[12]{36}$

$\therefore$ The reqd. arrangement is $\sqrt[4]{5}, \sqrt[6]{6}, \sqrt[3]{2}$.

(64) $\sqrt{3}, \sqrt[3]{4}, \sqrt[4]{5}$. (65) $\sqrt{2}, \sqrt[4]{3}, \sqrt[3]{4}$.

(66) $\sqrt{3}, \sqrt[3]{6}, \sqrt[4]{5}$. (67) $\sqrt{10}, \sqrt[3]{25}, \sqrt[4]{30}$

(68) $\sqrt{a}, \sqrt[3]{a}, \sqrt[3]{a^2}$. (69) $\sqrt[3]{2x^2}, \sqrt{4x^3}$ and $\sqrt[6]{5x^3}$.

(70) $\sqrt{x}, \sqrt[3]{2x^2}, \sqrt[3]{3x^4}$.

89. Consider the following examples :

(i) $\sqrt{2}, \sqrt[3]{2}, \sqrt[5]{2}$. They are *like or similar surds*. Similarly, $\sqrt{2}, \sqrt{8}, \sqrt[3]{16}$ are *like or similar surds* because they can be reduced to $\sqrt{2}, 2\sqrt{2}, 2^3\sqrt{2}$. respectively.

(ii) $\sqrt{2}, 3\sqrt{3}, \sqrt[3]{5}$ are *unlike or dissimilar* surds.

90. ADDITION AND SUBTRACTION OF SURDS

Solved Example

$\sqrt{32}+\sqrt{98}-\sqrt{50}$.

Solution : $\sqrt{32} = 4\sqrt{2}$

$\sqrt{98} = 7\sqrt{2}$

$\sqrt{50} = 5\sqrt{2}$.

$\therefore$ Exp. $= 4\sqrt{2}+7\sqrt{2}-5\sqrt{2}$

$= 6\sqrt{2}$

$= \sqrt{72}$.

Therefore, *transform each term to the same surd factor and then add or subtract their rational factors, and affix the result to the surd factor.*

[Dissimilar surds cannot be treated as above.]

91. MULTIPLICATION AND DIVISION OF SURDS

Solved Examples :

(i) Multiply : $3\sqrt{5}$ by $5\sqrt{3}$.

Solution:

The product $= 3\times5\times\sqrt{5}\times\sqrt{3}$

$= 15\sqrt{15}$.

(ii) Find the continued product of :

$5\sqrt[3]{2}$, $2\sqrt[3]{5}$ and $3\sqrt{3}$.

Solution :

$5\sqrt[3]{2} = 5\sqrt[6]{4}$

$2\sqrt[3]{5} = 2\sqrt[6]{25}$

$3\sqrt{3} = 3\sqrt[6]{27}$.

$\therefore$ the product $= 30\sqrt[6]{2700}$.

(iii) $x\sqrt{a^2-b^2}$, $y\sqrt{a^2+b^2}$, $z\sqrt{a^4-b^4}$.

Solution : The product $= xyz\sqrt{(a^2-b^2)(a^2+b^2)(a^4-b^4)}$

$= xyz\sqrt{(a^4-b^4)^2}$

$= xyz\,(a^4-b^4)$

Therefore, *reduce them to their simplest form and multiply the rational and irrational factors separately.*

92. RATIONALIZATION OF SURDS

Solved Examples :

Find the value of :

(1) $\dfrac{35}{3\sqrt{5}}$

In order to find the required value it is convenient to multiply the numerator and denominator by $\sqrt{5}$.

$$\text{Then } \frac{35}{3\sqrt{5}} = \frac{35\times\sqrt{5}}{3\times\sqrt{5}\times\sqrt{5}}.$$

$$=\frac{35\sqrt{5}}{15}.$$

$$=\frac{7\sqrt{5}}{3}.$$

$$=\frac{7\times 2\cdot 236\ldots}{3}.$$

$$=5\cdot 217.$$

(ii) $\dfrac{1}{3\sqrt{2}-2\sqrt{3}}$.

Multiply numerator and denominator by $3\sqrt{2}+2\sqrt{3}$.

$$\text{Then the exp.} = \frac{3\sqrt{2}+2\sqrt{3}}{(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}+2\sqrt{3})}.$$

$$=\frac{3\sqrt{2}+2\sqrt{3}}{18-12}$$

$$=\frac{3\sqrt{2}+2\sqrt{3}}{6}.$$

In the above examples the surd in the denominator has been removed. This process is called the *Rationalization of the Demominators.*

93. COMPLEMENTARY OR CONJUGATE SURDS

In the above example (*ii*) $3\sqrt{2}+2\sqrt{3}$ *is the complementary or conjugate surd of the compound surd* $3\sqrt{2}-2\sqrt{3}$. *They differ in sign only, and their product is rational*. Conjugate surds are used to rationalize a binomial surd.

Solved Examples :

Rationalize the denominator of :

(i) $\dfrac{2\sqrt{5}+3\sqrt{2}}{3\sqrt{5}+2\sqrt{3}}$.

Solution : Multiply the numerator and denominator by the conjugate surd of the denominator, *i. e.*, by $3\sqrt{5}-2\sqrt{3}$.

$$\text{The exp.}=\frac{(2\sqrt{5}+3\sqrt{2})(3\sqrt{5}-2\sqrt{3})}{(3\sqrt{5}+2\sqrt{3})(3\sqrt{5}-2\sqrt{3})}.$$

$$=\frac{6\times5+9\sqrt{10}-4\sqrt{15}-6\sqrt{6}}{45-12}.$$

$$=\frac{30+9\sqrt{10}-4\sqrt{15}-6\sqrt{6}}{33}.$$

(ii) $\dfrac{a\sqrt{x}+x\sqrt{a}}{a\sqrt{x}-x\sqrt{a}}$.

Solution : Multiply the numerator and denominator by $a\sqrt{x}+x\sqrt{a}$.

$$\text{Then the exp.}=\frac{(a\sqrt{x}+x\sqrt{a})\,(a\sqrt{x}+x\sqrt{a})}{(a\sqrt{x}-x\sqrt{a})\,(a\sqrt{x}+x\sqrt{a})}.$$

$$=\frac{(a\sqrt{x}+x\sqrt{a})^2}{a^2x-x^2a}.$$

$$=\frac{a^2x+2ax\sqrt{ax}+x^2a}{ax(a-x)}.$$

$$=\frac{ax(a+2\sqrt{ax}+x)}{ax(a-x)}.$$

$$=\frac{a+2\sqrt{ax}+x}{a-x}.$$

(iii) Divide : $5\sqrt{3}+4\sqrt{2}$ by $5\sqrt{3}-4\sqrt{2}$.

The quotient $=\dfrac{5\sqrt{3}+4\sqrt{2}}{5\sqrt{3}-4\sqrt{2}}$

$$=\frac{(5\sqrt{3}+4\sqrt{2})(5\sqrt{3}+4\sqrt{2})}{(5\sqrt{3}-4\sqrt{2})(5\sqrt{3}+4\sqrt{2})}$$

$$=\frac{(5\sqrt{3}+4\sqrt{2})^2}{75-32}$$

$$=\frac{75+40\sqrt{6}+32}{43}$$

$$=\frac{107+40\sqrt{6}}{43}$$

EXERCISE 48

Simplify :

(1) $\sqrt{18}+\sqrt{72}+\sqrt{162}$.

(2) $\sqrt{50}+\sqrt{98}+\sqrt{128}$.

(3) $\sqrt{75}-\sqrt{108}+\sqrt{147}$.

(4) $\sqrt[3]{16}+\sqrt[3]{54}+\sqrt[3]{128}$.

(5) $5\sqrt[3]{2}+2\sqrt[3]{16}-3\sqrt[3]{2}$.

(6) $\sqrt{3}(\sqrt{2}-2\sqrt{2})+\sqrt{2}(\sqrt{3}+3\sqrt{3})$.

(7) $\sqrt{5}(4\sqrt{2}+3\sqrt{3})+2\sqrt{5}(3\sqrt{5}-2\sqrt{3})$.

(8) $2\sqrt{7}(3\sqrt{2}-7\sqrt{3})-3\sqrt{7}(4\sqrt{3}-5\sqrt{2})$.

(9) $3\sqrt{5}\times4\sqrt{8}$

(10) $3\sqrt[3]{50}\times4\sqrt[3]{16}\times2\sqrt[3]{10}$.

(11) $(4\sqrt{5}+3\sqrt{3})(4\sqrt{5}-3\sqrt{3})$.

(12) $(7\sqrt{3}-4\sqrt{2})(4\sqrt{2}+7\sqrt{3})$.

(13) $\left(\sqrt{2}+\frac{1}{\sqrt{2}}\right)\left(\sqrt{2}-\frac{1}{\sqrt{2}}\right)$.

(14) $\left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)^2$.

(15) $\left(3\sqrt{5}+\frac{1}{3\sqrt{5}}\right)^2$.

(16) $\left(2\sqrt{2}+\frac{1}{2\sqrt{2}}\right)\left(2\sqrt{2}-\frac{1}{2\sqrt{2}}\right)$

(17) Find the quotient of :

$a^3-1+3a\sqrt[3]{2}\div a+1-\sqrt[3]{2}$.

(18) Find the value of :

$\sqrt{x}+\frac{1}{\sqrt{x}}$ When $x=7+4\sqrt{3}$.

(19) Find the value of x and y from

$\sqrt{22-4\sqrt{30}}=\sqrt{x}-\sqrt{y}$.

(20) If $x=7-4\sqrt{3}$, find the value of :

$\sqrt{x}+\frac{1}{\sqrt{x}}$.

(21) Find the value of $\sqrt{a}-\frac{1}{\sqrt{a}}$ when $a=5+2\sqrt{6}$.

(22) Find the value of $\sqrt{5\frac{9}{16}}-\frac{3}{2}\sqrt{5}$

(23) Find the value of $a^3+\frac{1}{a^3}$ when $a=2+\sqrt{3}$.

(24) Simplify :

$$\frac{x+\sqrt{x^2-1}}{x-\sqrt{x^2-1}}-\frac{x-\sqrt{x^2-1}}{x+\sqrt{x^2-1}}$$ (B. U. 1863)

(25) $\dfrac{1}{x+\sqrt{x^2-1}}+\dfrac{1}{x-\sqrt{x^2-1}}\cdot$

(26) Find the value of :

$$\sqrt{x}-\frac{1}{\sqrt{x}} \text{ when } x=3+2\sqrt{2}.$$

(27) Find the value of :

$$x^4+\frac{1}{x^4} \text{ when } x=3+\sqrt{8}$$

Rationalize the denominator and reduce to simplest term :

(28) $\dfrac{1}{3\sqrt{2}-2\sqrt{3}}\cdot$

(29) $\dfrac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}.$

(30) $\dfrac{\sqrt{x+y}-\sqrt{x-y}}{\sqrt{x+y}+\sqrt{x-y}}.$

(31) $\dfrac{\sqrt{x^2+y^2}+\sqrt{x^2-y^2}}{\sqrt{x^2+y^2}-\sqrt{x^2-y^2}}\cdot$

(32) $\dfrac{\sqrt{x+y^2}+\sqrt{x-y^2}}{\sqrt{x+y^2}-\sqrt{x-y^2}}.$

94. If $\sqrt{(a+\sqrt{b})}=\sqrt{x}+\sqrt{y}$, then

$$\sqrt{(a-\sqrt{b})}=\sqrt{x}-\sqrt{y}$$

We have $\sqrt{(a+\sqrt{b})}=\sqrt{x}+\sqrt{y}\cdot$

Square both sides.

Then $a+\sqrt{b}=x+2\sqrt{xy}+y.$

Equate the rational and irrational parts separately.

Then $a=x+y$

$$\sqrt{b}=2\sqrt{xy}.$$

$\therefore a-\sqrt{b}=x+y-2\sqrt{xy}$

or $\quad = (\sqrt{x} - \sqrt{y})^2$

$\therefore \sqrt{(a - \sqrt{b})} = \sqrt{x} - \sqrt{y}.$

95. Find the square root of $a + \sqrt{b}$

Suppose $\sqrt{a + \sqrt{b}} = \sqrt{x} + \sqrt{y}$

Then $a + \sqrt{b} = x + y + 2\sqrt{xy}$

and $a - \sqrt{b} = x + y - 2\sqrt{xy}$ as in § 94.

$\therefore \quad a = x + y \ldots\ldots\ldots\ldots\ldots\ldots(\text{i})$

and $\sqrt{b} = 2\sqrt{xy}$

$\therefore a^2 - b = (x+y)^2 - 4xy$

$\quad\quad = (x - y)^2$

$\therefore \sqrt{a^2 - b} = x - y \ldots\ldots\ldots\ldots(\text{ii})$

From (i) and (ii)

$2x = a + \sqrt{a^2 - b}$

and $2y = a - \sqrt{a^2 - b}$

or $x = \frac{1}{2}(a + \sqrt{a^2 - b})$

and $y = \frac{1}{2}(a - \sqrt{a^2 - b}).$

$$\therefore \sqrt{a + \sqrt{b}} = \sqrt{\tfrac{1}{2}(a + \sqrt{a^2 - b})} + \sqrt{\tfrac{1}{2}(a - \sqrt{a^2 - b})}$$

NOTE: The square root is of little value if $a^2 - b$ is not a perfect square.

96. *Solved Examples.*

Find the square root of :

(i) $8 + 2\sqrt{15}$

Suppose $\sqrt{8 + 2\sqrt{15}} = \sqrt{x} + \sqrt{y}$

Square both sides.

$8 + 2\sqrt{15} = x + y + 2\sqrt{xy}$

Compare the rational and irrational quantities.

Then $x+y=8$

$$2\sqrt{xy}=2\sqrt{15}$$

$$\therefore xy=15$$

Now $(x-y)^2=(x+y)^2-4xy$

$$=64-60$$

$$=4$$

$\therefore \quad x-y=\pm 2$

Now $x+y=8$

and $x-y=2$

$\therefore \quad x=5$

and $\quad y=3$

Again $x+y=8$

$$x-y=-2$$

$\therefore \quad x=3$

and $\quad y=5$

$\therefore \sqrt{8+2\sqrt{15}}=\sqrt{5}+\sqrt{3}.$

(ii) $2\sqrt{8}+2\sqrt{6}$

Solution :

$2\sqrt{8}+2\sqrt{6} \qquad =2\sqrt{2}(2+\sqrt{3})$

$$\therefore \sqrt{2\sqrt{8}+2\sqrt{6}}=\sqrt{2\sqrt{2}(2+\sqrt{3})}$$

$$=2^{\frac{3}{4}}\sqrt{[\tfrac{3}{2}+2\sqrt{\tfrac{3}{2}}\sqrt{\tfrac{1}{2}}+\tfrac{1}{2}]}$$

$$=2^{\frac{3}{4}}\sqrt{[(\sqrt{\tfrac{3}{2}}+\sqrt{\tfrac{1}{2}})^2]}$$

$$=2^{\frac{3}{4}}[\sqrt{\tfrac{3}{2}}+\sqrt{\tfrac{1}{2}}].$$

EXERCISE 49

Find the square root of :

(1) $7+2\sqrt{5}.$ (2) $4+4\sqrt{2}$

(3) $11+6\sqrt{3}.$ (4) $3\sqrt{5}+2\sqrt{20}.$

(5) $2\sqrt{8}-2\sqrt{6}$.

(6) $7-4\sqrt{3}$

(7) $6+\sqrt{35}$.

(8) $\sqrt{27}-\sqrt{24}$.

(9) $\sqrt{32}+\sqrt{24}$.

(10) $9+2\sqrt{14}$.

(11) $x+\sqrt{x^2+y^2}$.

(12) $a+b+\sqrt{a^2-b^2}$.

(13) $2a+2\sqrt{a^2+b^2}$.

(14) $x^2+2y\sqrt{x^2+1}$

(15) $x+4a-4\sqrt{2ax}$.

97. EQUATIONS INVOLVING SURDS

Solved examples :

(i) $\sqrt{x-1}=1-\sqrt{x}$.

Solution :

Square both sides.

Then $x-1=1-2\sqrt{x}+x$.

Transpose

$$2\sqrt{x}=2$$

or $4x=4$

$\therefore\ x=1$.

(ii) $2\sqrt{x+3}+\sqrt{3x+13}=8$.

Solution :

Transpose $2\sqrt{x+3}=8-\sqrt{3x+13}$.

Square both sides

$$4(x+3)=64+3x+13-16\sqrt{3x+13}$$

or $4x+12=77+3x-16\sqrt{3x+13}$

or $x-65=-16\sqrt{3x+13}$.

Square both sides

$$x^2-130x+4225=256(3x+13)$$

or $\quad x^2-130x+4225=768x+3328$

or $\quad x^2-130x-768x=3328-4225$

or $\qquad x^2-898x=-897$

or $\qquad x^2-898x+897=0$

or $\qquad x^2-897x-x+897=0$

or $x(x-807)-1(x-897)=0$

or $\qquad (x-1)(x-897)=0.$

or $x=1$, or 897.

EXERCISE 50

Solve :

(1) $\sqrt{x-3}-2=0$, (2) $3+\sqrt{x+3}=6$.

(3) $10+\sqrt{x+7}=13$ (4) $x+1+\sqrt{x-1}=2$.

(5) $\sqrt{x+5}+\sqrt{x}=5$. (6) $\sqrt{6x-5}+\sqrt{5x}=10$.

(7) $\sqrt{3x}-\sqrt{2x+1}=1$. (8) $\sqrt{3x-1}+\sqrt{3x+14}=5$.

(9) $\sqrt{7x-10}+\sqrt{7x+1}=11$.

(10) $\sqrt{x+4}+\sqrt{3x+13}=3$.

(11) $\sqrt{2a+x}+\sqrt{7a+x}=5\sqrt{a}$.

(12) $\dfrac{ax-1}{\sqrt{ax}+1}=4+\dfrac{\sqrt{ax}-1}{2}$

(13) $\sqrt{x^2+7a^2}+\sqrt{x^2-5a^2}=6a$.

(14) $\dfrac{\sqrt{1-x}+\sqrt{1+x}}{\sqrt{1-x}-\sqrt{1+x}}=\dfrac{2}{3}$.

Solution :

Apply Componendo and dividendo.

Then $\dfrac{\sqrt{1-x}+\sqrt{1+x}+\sqrt{1-x}-\sqrt{1+x}}{\sqrt{1-x}+\sqrt{1+x}-\sqrt{1-x}+\sqrt{1+x}}=\dfrac{2+3}{2-3}=-5$

or $\dfrac{2\sqrt{1-x}}{2\sqrt{1+x}}=-5$

or $\dfrac{\sqrt{1-x}}{\sqrt{1+x}}=-5.$

Multiply across

Then $\sqrt{1-x} = -5\sqrt{1+x}$

Square both sides.

$$1-x=25(1+x)$$
$$=25+25x$$

or $-26x=24$

$$\therefore x=-\frac{24}{26}=-\frac{12}{13}.$$

(15) $\dfrac{\sqrt{x^2+a^2}-\sqrt{x^2-a^2}}{\sqrt{x^2+a^2}+\sqrt{x^2-a^2}}=\dfrac{3}{2}.$

(16) $\dfrac{a+x-\sqrt{x^2-a^2}}{a+x+\sqrt{x^2-a^2}}=\dfrac{3}{4}.$

(17) $\sqrt[3]{1+x}+\sqrt[3]{1-x}=\sqrt[3]{2}.$ (C. U. 1885)

(18) $2(x+2)=1+\sqrt{4x^2+9x+14}.$ (C. U. 1877)

(19) $\sqrt{x^2+11x+20}-\sqrt{x^2+5x-1}=3.$ (C. U. 1881)

(20) $\dfrac{1}{x+\sqrt{x^2-1}}+\dfrac{1}{x-\sqrt{x^2-1}}=4.$

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१. गुणनखण्डों में विभक्त करो:—

(क) $य^3 + ८$,

(ख) $य^४ + य^२र^२ + र^४$,

(ग) $य^२ + २६य + १५३$ ।

२. (क) $य^3 + ४य^२ — ५$ और $य^3 - ३य^२ + २$ का महत्तम समापवर्तक निकालो ।

(ख) $य^२ — ४$ और $य^२ — य — २$ का लघुतमापवर्त्य ज्ञात करो ।

३. निम्न को सरल करो:—

(क) $$\frac{ख—ग}{क^२—(ख - ग)^२} + \frac{ग}{ख^२-(ग-क)^२} + \frac{क—ख}{ग^२—(क-ख)^२}$$

(ख) $$\frac{१}{क(क—ख)(क—ग)} + \frac{१}{ख(ख - क)(ख—ग)} + \frac{१}{ग(ग—क)(ग—ख)}$$

४. (क) $४क^४ + ६\left[क^२ + \frac{१}{क^२}\right] + १२ क^२\left[क + \frac{१}{क}\right] + १=$ का वर्गमूल निकालो ।

(ख) $\frac{क}{ख} = \frac{ख}{ग}$

तो सिद्ध करो कि $\frac{क}{ग} = \frac{क^२ + ख^२}{ख^२ + ग^२}$

५. निम्न समीकरणों को हल करो: —

(क) $\frac{य+१}{य}+\frac{य}{य+१}=२\frac{१}{६}$

(ख) $\frac{१}{य-१}-\frac{१}{य-४}+\frac{१}{य}=\frac{१}{य+३}$

६. एक गाय ६० रुपये में घाटे पर बिकी, परन्तु यदि वह ८८ रुपये में बिकी होती तो लाभ पहले घाटे का दो पँचमांश होता। गाय का मूल्य बताओ।

खण्ड ख

७. निम्न को गुणनखन्डों में विभक्त करो :—

(क) (य+१) (य+३) (य+५) (य+७) + १५

(ख) $य^३+र^३+ल^३-३$ य र ल

८. (क) म का मान निकालो जिससे

$$१६य^४-२४य^३+मय^२-२४य+१६$$

पूर्ण वर्ग हो जाय।

(ख) यदि $य-\frac{१}{य}=१$, तो $य^३-\frac{१}{य^३}$ का मान निकालो।

९. किसी भिन्न का हर अन्श से १ अधिक है। इस भिन्न से दो अन्य भिन्न बनाये जाते हैं, एक हर में ९ जोड़कर और दूसरा अन्श से ६ घटा कर। यदि इस प्रकार प्राप्त दोनों भिन्न बराबर हों तो दिये हुये भिन्न को ज्ञात करो।

१०. रेखाचित्र द्वारा उस त्रिभुज के शीर्षों के नियामक ज्ञात करो जो निम्न सरल रेखाओं से बनता है।

(१) २ र+य+८ = ०, (२) य = ०, (३) ८ र = ३ (य+१)

खण्ड ग

११. [क] यदि क प = ख फ = ग ब, तो सिद्ध करो कि

$$\frac{प^२}{बफ}+\frac{फ^२}{बप}+\frac{ब^२}{पफ}=\frac{खग}{क^२}+\frac{गक}{ख^२}+\frac{कख}{ग^२}$$

(ख) $\dfrac{र+ल-य}{क}=\dfrac{ल+य-र}{ख}=\dfrac{य+र-ल}{ग}$

तो सिद्ध करो कि $\dfrac{क+ख+ग}{य+र+ल}=\dfrac{क र+ख ल+ग य}{य^२+र^२+ल^२}$

१२. (क) सिद्ध करो कि—

$लघु_{१०}\dfrac{प}{फ}=लघु_{१०} प-लघु_{१०} फ$

(ख) सिद्ध करो कि

$७ लघु_{१०}\dfrac{१०}{९}-२ लघु_{१०}\dfrac{२५}{२४}+लघु_{१०}\dfrac{८१}{८०}=लघु_{१०} २$

१३. यदि स्कूल के सभाभवन में बेंचों की संख्या १२ बढ़ा दी जाय तो प्रत्येक बेंच पर एक लड़का कम बैठ सकता है। और यदि बेंचों की संख्या ८ कम करदी जाय तो प्रत्येक बेंच पर एक लड़के को और बैठना पड़ेगा यदि प्रत्येक बार उतने ही लड़के थे तो बताओ सभाभवन में कितने लड़के बैठ सकते हैं ?

१४. डाकगाड़ी ५० मील प्रति घन्टे के वेग से जा रही है। रास्ते में यह मालगाड़ी को पार करती है जो इस से दुगुनी लम्बी है और समानान्तर पटरी पर उसी दिशा में २० मील प्रति घन्टे के वेग से चल रही है। मालगाड़ी को पूरा पार करने में डाकगाड़ी को १८ सेकण्ड लगता है। यदि डाकगाड़ी किसी स्टेशन के प्लेटफार्म को पार करने में ९ सेकण्ड समय लेती है तो इस प्लेटफार्म की लम्बाई बताओ।

1950

खण्ड क

१. गुणनखण्डों में विभक्त करो :—

(क) $(य+२)^२-(य-२)^२$,

(ख) $४य^२+८य-५$,

(ग) $य^३+१२५$ ।

२. (क) $य^४ — य$ और $य^३ + य^२ + य$ का महत्तम समापवर्तक निकालो।

(ख) $य^३ र + य र^३$ और $य^४ र^२ - य^२ र^४$ का लघुत्तम समापवर्तक (लघुत्तम समापवर्त्य) निकालो।

३. सरल करो: --

(क) $\frac{क—ख}{कख} + \frac{ख—ग}{खग} + \frac{ग—क}{गक}$,

(ख) $\frac{खग}{(क-ख)(क-ग)} + \frac{गक}{(ख-क)(ख-ग)} + \frac{कख}{(ग-क)(ग-ख)}$

४. (क) $य^४ + \frac{१}{य^४} - ६\left(य^२ — \frac{१}{य^२}\right) + ७$

का वर्गमूल निकालो।

(ख) यदि $\frac{क}{र+ल} = \frac{ख}{ल+य} = \frac{ग}{य+र}$

तो सिद्ध करो कि

$क (र—ल) + ख(ल—य) + ग (य—र) = ०$।

५. हल करो

(क) $\frac{६}{य+१} - \frac{४}{य—३} = \frac{५}{य+४}$

(ख) $३य + ४र = २५$ और $५य - ६ र = —६$।

६. दो संख्याओं का योग १६ और गुणनफल ६३ है। संख्याएं ज्ञात करो।

खंड—ख

७. गुणनखण्डों में विभक्त करो: —

(क) $६४—क^६$

(ख) $४य^२ — ७ य + ३$।

८: (क) दो पदों के लघुत्तम समापवर्त्य और महत्तम समापवर्तक

क्रमशः $६य^५ + १८य^४ — य^३ — ३य^२ — य — ३$ और $२ य^२ — १$ हैं। यदि उनमें से एक पद $२ य^३ + ६ य^२ - य - ३$ हैं तो दूसरा पद निकालो।

(ख) यदि लघु य = लघु ०. ३ + ३ लघु २ — $\frac{३}{५}$ लघु ३२, तो य का मान बिना सारणी का प्रयोग किये हुए ज्ञात करो।

९. एक सख्या में तीन अङ्क हैं जिनमें से बीच का अङ्क शून्य है और अङ्कों का योग १३ है। किनारे के अंकों को आपस में बदल देने से जो संख्या बनती है वह दी हुई संख्या से २९७ अधिक है। संख्या ज्ञात करो।

१०. निम्न सरल रेखाओं के लेखा चित्र खींचो और उनमें बने हुए त्रिभुज के शीर्षों के नियामक पढ़ो :—

(१) $य - ७ र + २५ = ०$; (२) $५ य + ३ र + ११ = ०$,

(३) $३ य - २ र — १ = ०$।

खंड—ग

११. (क) १५, ११, २१ और १५ में से कौनसी संख्या घटाई जाय कि शेषफल समानुपात में हों ?

(ख) यदि $य + \frac{१}{य} = ३$ तो $य^३ — \frac{१}{य^३}$ का मान निकालो।

१२. ८०० रुपये का मिश्रधन दो वर्ष में चक्रवृद्धि ब्याज की रीति से ८८२ रुपया किसी ब्याज-दर से हो जाता है। तो ब्याज-दर प्रतिशत ज्ञात करो।

१३. एक मल्लाह १२$\frac{१}{२}$ घण्टे में नाव को धार के प्रतिकूल २० मील ले जाकर फिर लौट आता है। मल्लाह को ज्ञात होता है कि वह जितने समय में धार के अनुकूल ४ मील खेता है उतने ही समय में धार के अनुकूल १ मील। नदी की धार की गति ज्ञात करो।

१४. एक वर्ग की परिमित दूसरे वर्ग की परिमित से ४० गज अधिक है। बड़े वर्ग का क्षेत्रफल छोटे वर्ग के क्षेत्रफल के तीन गुने से ५० वर्ग गज़ कम है। वर्गों की भुजाएं ज्ञात करो।

1951

१. गुणनखन्डों में विभक्त करो:—

(क) $य^४ - र^४$,

(ख) $१३ अ^२ + २७ अ - ६$,

(ग) $२७ य^३ — ६४ र^३$ ।

२. (क) $य^३ + ४य^२ + ६य + ६$ और $य^३ + ५ य^२ + १२ य + १२$ का महत्तम समापवर्तक निकालो ।

(ख) $३य^२ + ५य - २$, $१०य + ५$ और $२य^२ + ५य + २$ का लघुत्तम समापवर्त्य (लघुतमापवर्त्य) निकालो ।

३. सरल करो:—

(क) $$\frac{क^२ (ख - ग)}{(क + ख)(क + ग)} + \frac{ख^२ (ग - क)}{(ख + क)(ख + ग)} + \frac{ग^२ (क — ख)}{(ग + क)(ग + ख)}$$

(ख) $$\frac{\dfrac{क^३ - ख^३}{क - ख} - \dfrac{क^३ + ख^३}{क + ख}}{(क + ख)^२ — (क - ख)^२}$$ ।

४. (क) $४\left(य^२ + \frac{१}{य^२}\right) — १२\left(य - \frac{१}{य}\right) + १$ का वर्ग निकालो

(ख) यदि $\frac{र + ल}{क} = \frac{ल + य}{ख} = \frac{य + र}{ग}$ तो सिद्ध करो कि

$य : र : ल = ख + ग - क : ग + क - ख : क + ख — ग$ ।

५. सरल करो :—

(क) $$\frac{३य + ४}{४य + ५} = \frac{६य — ११}{८य — १५}.$$

(ख) $३य + ४र = २$ और $२१ य — ३६र = — २$

६. (क) $६य + \frac{१}{य} = ६$ को हल करो ।

(ख) $६य^२ - य - १ = ०$ को हल करो :

७. गुणनखन्डों में विभक्त करो : —

(क) $क^२ य^२ + २८ क ख य र — ८१ख^२ र^२$,

(ख) $क^३ + \frac{ख^३}{२७} + ग^३ —$ कखग ।

८. एक धन कुछ आदमियों में बराबर बराबर बांटना है। यदि ४ आदमी अधिक होते तो प्रत्येक को १ रुपया कम मिलता और यदि ५ आदमी कम होते तो प्रत्येक को २ रुपया अधिक मिलता । तो बांटने के लिये कितना धन है और उसे कितने आदमियों में बांटना है ?

९. (क) यदि $य^४ + ४य^३ + २य^२ +$ अय $+ १$ पूर्ण वर्ग हो तो अ का मान ज्ञात करो ।

(ख) व्यंजकों $यप + २य^२ — ३$ और $य^४ — पय + ४$ को $य - २$ से भाग देने से शेष वही बचता है तो प का मान ज्ञात करो :—

१०. (क) (४,—३) और (०,३) को अंकित करो और इनके मिलाने से जो सरल रेखा बनती है उसके मध्य विन्दु के नियामक निकालो ।

(ख) $२य — ३र = १२$ और $य = १$ के चित्र खींचो । इन दोनों रेखाओं और य—अक्ष से बने हुए त्रिभुज के शीर्षों के नियामक पढ़ो ।

११. ४ प्रतिशत वार्षिक चक्रवृद्धि व्याज की दर से १०० रु०, कितने वर्ष में १००० रु० हो जायेंगे ? उत्तर दशमलव के २ अंकों तक ठीक निकालो । (दिया है कि लघु १०४ = २·०१७०)

१२. एक रेलगाड़ी के ३० मील चलने के पश्चात् एक घटना हो जाती है जिसके कारण अब उसकी चाल पहले की $\frac{४}{५}$ हो जाती है । रेलगाड़ी नियत स्थान पर ४५ मिनट देर से पहुंचती है । यदि घटना १० मील और आगे जाने पर हुई होती, तो रेलगाड़ी ३० मिनट पहले पहुंच गई होती तो गाड़ी की चाल और यात्रा की लम्बाई ज्ञात करो ।

१३. $य(र+ल)=४८$, $र(ल+य)=५०$, और $ल(य+र)=५४$ को हल करो।

१४. दो व्यक्ति क और ख दो स्थानों प और फ से एक ही समय एक दूसरे की ओर चलते हैं। २ घंटे के पश्चात् वे प से २४ मील की दूरी पर मिलते हैं। जब ख, प पर पहुँच जाता है तो क को फ तक जाने में २० मील रह जाते हैं। तो प और फ के बीच की दूरी ज्ञात करो।

1952

१. गुणनखंडों (factors) में विभक्त (break up) करो :—

(क) $१२य^२-य-१$, (ख) $८१क^२-६४ख^२$, (ग) $य^३-१$।

२. (क) $य^3-य^२-य-१५$ और $य^3-३य^२-३य+६$ का महत्तम समापवर्तक (H. C. F.) निकालो।

(ख) $य^२+३य+२$, $य^२+य-६$ और $य^२-य-२$ का लघुत्तम समापवर्त्य (L. C. M.) निकालो।

३. सरल करो (simplify) :—

(क) $$\frac{\dfrac{क}{क-ख}+\dfrac{ख}{क+ख}}{\dfrac{क}{क-ख}+\dfrac{ख}{क+ख}},$$

(ख) $$\frac{क+ख}{(ग-क)(ग-ख}+\frac{ख+ग}{(क-ख)(क-ग)}+\frac{ग+क}{(ख-क)(ख-ग)}$$

४. (क) $य^४+\frac{१}{य^४}-६\left(य^२+\frac{१}{य^२}\right)+११$ का वर्गमूल (square root) निकालो।

(ख) यदि $\frac{य}{ख+ग-क} = \frac{र}{ग+क-ख} = \frac{ल}{क+ख-ग}$

तो सिद्ध (prove) करो कि

$$(ख-ग)\,य + (ग-क)\,र + (क-ख)\,ल = ०$$

५. हल करो (solve) :—

(क) $\frac{२य-७}{२य-३} = \frac{य-२}{य+२}$,

(ख) ५ य—१५ र = २२ और ७ य + १० र = ३७

६. हल करो :—

(क) $२य^{२} - ५य - १२ = ०$,

(ख) $क\,य^{२} + ख\,य + ग = ०$ ।

७. गुणनखंडों (factors) में विभक्त (break up) करो।

(क) य (य+५) (य+३) (य+८) + ५०,

(ख) $य^{३} - ८य^{२} + १७य - १०$ ।

८. १० वर्ष पहले दो भाइयों की अवस्थाओं (ages) का योग (sum) उनके पिता की अवस्था का तिहाई था। बड़ा लड़का छोटे से २ वर्ष बड़ा है। इस समय उन दोनों की अवस्थाओं का योग पिता की अवस्था से १४ वर्ष कम है। तीनों की वर्तमान अवस्थाएँ (Present ages) क्या हैं?

९. (क) ४ प्रति सैंकड़ा प्रति वर्ष चक्रवृद्धि व्याज (compound interest) की दर (rate) से ७५० रु० का ३ वर्ष का व्याज निकटतम पाई तक (to the nearest pie) ज्ञात करो।

(ख) यदि लघु २ = ०·३०१०, और लघु ३ = ०·४७७१, तो $७२^{२०}$ में अंकों (digits) की संख्या (number) ज्ञात करो। [लघु = log]

१०. (क) लेखा-चित्र द्वारा हल (solve by a graph) करो

३य − २र = ६, और य—र = १।

(ख) विन्दु [points] [१, २], [—३, —२], [३, ४] और [०, १]

को अंकित करो [plot]। इन विन्दुओं से होकर जाने वाली रेखा का समीकरण [equation] निकालो।

११. [क] सिद्ध [prove] करो कि

$$\text{लघु}_{\text{ग}}\ \text{क} = \text{लघु}_{\text{ख}}\ \text{क} \times \text{लघु}_{\text{ग}}\ \text{ख}$$ । [लघु = log]

[ख] य र = १२, र ल = २० और ल य = १५ को हल [solve] करो।

१२. एक रेलगाड़ी एक ही चाल (speed) से चलकर उसी दिशा में ५ मील प्रति घण्टा की चाल से जाने वाले एक आदमी को ९ सेकंड में पार करती है। यदि वह स्टेशन के ६६ गज लम्बे प्लेटफार्म को १३$\frac{1}{2}$ सेकंड में पार करे तो उसकी चाल और लम्बाई ज्ञात करो।

१३. एक आयताकार (rectangular) बाग की लम्बाई और चौड़ाई में ५ : ४ का अनुपात (ratio) है और उसका क्षेत्रफल २००० वर्ग गज (square yds.) है। उसके भीतर चारों ओर एक ही चोड़ाई की सड़क बनी है। यदि सड़क के बनवाने का खर्चा ३ आ० ६ पा० प्रति वर्गगज की दर से ७५ रु० ४ आ० हो, तो सड़क की चौड़ाई ज्ञात (find) करो।

१४. एक आयताकार पिंड (rectangular solid) की लम्बाई उसकी चौड़ाई से ४ फुट और ऊंचाई से ६ फुट अधिक (ज्यादा) है। यदि लम्बाई ३ फुट और चौड़ाई ४ फुट बढ़ा दी जाय और ऊंचाई २ फुट कम कर दी जाय तो उसका आयतन (volume) ५१२ घनफ़ीट (cubic feet) बढ़ जाता है तो पिंड का पहले का विस्तार (dimensions) बताओ।

1953

१. गुणनखंडों (factors) में विभक्त (break up) करो:—

(क) $१६क^{४} — ख^{४}$,

(ख) $२य^{२} + ११य — २१$,

(ग) $(य + १)^{३} — १$ ।

२. (क) $य^3 - ४य^2 + ७य - ६$

और $२य^3 - ७य^2 + २य + ८$

का महत्तम समापवर्तक (H.C.F.) निकालो।

(ख) $य^2 - ७य + १२$, $३य^2 - ६य - ९$

और $२य^3 - ६य^2 - ८य$

का लघुत्तम समापवर्त्य (L.C.M.) निकालो।

३. सरल करो (simplify):—

$$(क) \frac{\frac{१ + य}{१ - य} - \frac{२य}{१ + य}}{\frac{१ - य}{१ + य} + \frac{२य}{१ - य}}$$

$$(ख) \quad \frac{१}{क(क - ख)(क - ग)} + \frac{१}{ख(ख - ग)(ख - क)} + \frac{१}{ग(ग - क)(ग - ख)},$$

४. (क) $य^४ + \frac{१}{य^४} - ८\left(य^२ + \frac{१}{य^२}\right) + १८$

का वर्गमूल (square root) निकालो।

(ख) यदि

$$\frac{य}{२क + ख} = \frac{र}{क + ख + ग} = \frac{ल}{ख + २ग}$$

तो सिद्ध (prove) करो कि

$$य - २र + ल = ०,$$

५. हल करो (solve):—

(क) $\frac{य + ५}{२य + ७} = \frac{४य - ४}{८य - ११}$,

(ख) $३र + य = १४$ और $३य - ४र = ३$

६. हल करो (solve):—

(क) $४य^२ — ८य + ३ = ०$,

(ख) $१०य — \frac{१}{य} = ३$

७. गुणनखंडों (factors) में विभक्त (break up) करो :—

(क) $(य+१)\,(य+३)\,(य+५)\,(य+७)+१५$;

(ख) $क^२ + ३ख^२ — ४ग^२ — ४क\,ख + ४ख\,ग$ ।

८. वह कौन-सी भिन्न (fraction) है जिसके अंश (numerator) में से १ घटाने और हर (denominator) में १ जोड़ने से $\frac{१}{२}$ मिलता है और यदि अंश और हर में क्रमानुसार १ और २ जोड़ें तो भिन्न $\frac{२}{३}$ हो जाती है ?

९. (क) $७ \text{ लघु } \frac{१०}{९} - २ \text{ लघु } \frac{२५}{२४} + ३ \text{ लघु } \frac{८१}{८०}$ को सरल (simplify) करो । [लघु=log]

(ख) यदि लघु ३=०·४७७१ और लघु ७=०·८४५१, तो $(२१)^{२१}$ में अंकों (digits) की संख्या (number) ज्ञात करो (find) ।

१०. निम्न सरल रेखाओं (straight lines) के लेखा-चित्र (graph) खींचो और उनसे बने हुए त्रिभुज (triangle) के शीर्षों (vertices) के नियामक (co-ordinates) पढ़ो:—

(क) $२य+र=५$,

(ख) $य+२र=७$,

(ग) $य-र=१$ ।

खंड (ग)

११. (क) यदि

$$य + \frac{१}{य} = \sqrt{३}, \text{ तो } य^३ + \frac{१}{य^३}$$

का मान (value) निकालो ।

(ख) एक आयत (rectangle) का विकर्ण (diagonal) ५० गज का और उसकी परिमित (perimeter) १२४ गज की है, तो आयत की लम्बाई और चौड़ाई ज्ञात करो।

१२. चार आदमियों और दस औरतों ने एक काम को २ दिन में समाप्त किया। यदि एक औरत पूरा काम करने में एक आदमी से ६ दिन अधिक लगाये, तो एक आदमी उस काम को कितने दिन में कर सकता है ?

१३. ५ प्रति सैकड़ा प्रति वर्ष चक्रवृद्धि ब्याज (compound interest) की दर (rate) से कितने वर्षों में ८,००० रु० के ९,२६१ रु० हो जायेंगे ?

१४. एक आयताकार (rectangular) बाग का क्षेत्रफल ४, ८०० वर्ग गज हैं। उसके बाहर चारों ओर ५ गज की दर से तार लगाने का व्यय ४२१ रु० १४ आ० है। तो बाग की लम्बाई और चौड़ाई ज्ञात करो।

RAJPUTANA UNIVERSITY

1948

1. (i) Factorise $3(a+b)+(a+b)-2$.

   (ii) $x^3-y^3-1-3xy$

2. (i) Solve—

$$\frac{x}{4}+\frac{2}{y}=2$$

$$\frac{2x}{5}+\frac{3}{2y}=2\tfrac{7}{20}$$

   (ii) Solve—

$$\frac{7x^2-4}{56x-47}+\frac{7x-11}{6}=\frac{31x-41}{24}$$

   (iii) Solve—

$$\frac{x-1}{x-2}+\frac{x-3}{x-4}=3\tfrac{1}{3}$$

3. (a) Simplify :—

$$\frac{bc}{(a-b)(a-c)}+\frac{ca}{(b-c)(b-a)}+\frac{ab}{(c-a)(c-b)}$$

(b) Extract the sq. root of :—

$$x^2+\frac{1}{x^2}+4\left(x+\frac{1}{x}\right)+6$$

4. (a) What number must be subtracted from each of the numbers 6, 8, 7, and 11, so that the remainders shall be in the proportion.

(b) A certain sum of money is to be divided in a certain number of persons. If there were 3 less each would have obtained 150 more and if 6 more, each man would have obtained 120 less. Find men and sum.

5. $3x-4y=-4.$

If $x=$	4	0	12	16
Then $y=$	4	1	10	13

Draw the graph of this function

6. Or the following table gives the population of two countries A & B for the yrs. specified (in millions).

Year	1861	1871	1881	1891	1901	1911	1921
A	3·1	3·4	3·7	4·0	4·47	4·7	5·0
B	5·8	5·4	5·2	4·7	4·45	4·2	4·0

Draw the graphs of both and estimate from it the year in which the population of two countries were equal approximately.

1949

1. Resolve in factors :—

(i) $x^2-y^2-4x-6y-5$

(ii) $(x+2)(x+4)(x+6)(x+8)+15$

(iii) $a(a-1)-b(b-1)$

2 (a) Find the Value of $x^2+\frac{1}{x^2}$ when $x-\frac{1}{x}=8$

(b) Find the H. C. F. of x^3+x^2-2 and x^3+2x^2-3

(c) If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$, show that

$$\frac{2a^3-3c^3+4e^3}{2b^3-3d^3+4f^3}=\frac{a^2e}{b^2f}$$

3. Solve the equations :—

(i) $$\frac{x+4}{x-4}+\frac{x-4}{x+4}=\frac{10}{3}$$

(ii) $$\frac{3}{x}+\frac{5}{y}=1$$

$$\frac{1}{x}+\frac{2}{y}=\frac{11}{36}$$

4. (a) What number must be subtracted from each of the numbers 6, 8, 7, and 11 so that the remainder shall be in proportion ?

(b) The length and breadth of a room are such that if the former were increased and the latter diminished by 3 yds. each, the area of the room would be diminished by 18 sq. yds; while if both were increased by 3 yds., the area would be increased by 60 sq. yds Find the length and breadth of the room.

1950

1. Resolve into Factors :

   (i) $x^2-y^2-z^2-2yz+x+y+z$

   (ii) $(a-b)^3+(b-c)^3+(c-a)^3$

   (iii) $x^3-6x^2+11x-6$

2. (a) If $xy+yz+zx=1$ prove that

$$\frac{1+x^2}{(x+y)(x+z)}+\frac{1+y^2}{(y+z)(z+y)}+\frac{1+z^2}{(z+x)(z+y)}=3$$

   (b) Find the square-root of

$$\left(x+\frac{1}{x}\right)^2-4\left(x-\frac{1}{x}\right)$$

3. $2y+z=11\ldots(1)$

   $2z+x=12\ldots(2)$

   $2x+y=13\ldots(3)$

   (ii) $\frac{x-2}{x-1}+\frac{x-2}{x}+\frac{x}{x+1}=3$

1951

1. Resolve into factors –

   (i) $x^3-19x-30$

   (ii) $(x-2)(2x-1)(2x-5)(x-4)-7$

   (iii) $8x^3-1-y^3-6xy$

2. Solve the equations :—

   (i) $\frac{3}{x}+\frac{5}{y}=1;\ \frac{1}{x}+\frac{2}{y}=\frac{11}{30}$.

   (ii) $\frac{x-5}{x-7}-\frac{x-6}{x-7}=\frac{x-1}{x-2}-\frac{x-2}{x-3}$

3. (a) If a, b, c, be in the continued proportion show that

$$a^2\ b^2\ c^2\left(\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}\right)=a^3+b^3+c^3$$

(b) Find the square root of :

$$\left(\frac{x^2}{y^2}+\frac{y^2}{x^2}\right)-\frac{x}{y}+\frac{y}{x}-1$$

4. (a) A man buys two horses for Rs. 765. By selling one for $\frac{4}{5}$ of its cost price and the other at $\frac{5}{4}$ of its cost price, he makes a profit of Rs. 76 on the whole transaction. Find the cost price of each horse ?

(b) One cyclist rides 2 miles an hour faster than another and takes half an hour less for a journey of 30 miles. Find the rate at which they travel.

5. In a Fahrenheit thermometer the freezing point stands at 32° and the boiling point at 212°; and in the Centigrade thermometer the Freezing and boiling points stand at 0° and 100° respectively.

Construct a graph to convert F degrees into c. (centigrades) and vice versa. Read off 108° F. in c. and 20 c. in F. degrees.

1952

(1) Factorise any two of the following

(i) $4a^2-4ab-8b^2+6bc-c^2$

(ii) $(x-y)^3+(y-z)^3+(z-x)^3$

(iii) $a^2(b+c)+b^2(c+a)+c^2(a+b)+3abc$

(2) Solve any two of the following equations

(i) $\frac{x+2}{x+1}+\frac{3x-1}{3x+1}=\frac{8x+9}{4x+4}$

(ii) $3xy=4(x+y), 2xz=3(x+z), 5yz,=12(y+z)$

(iii) $\frac{a}{a-x}+\frac{a}{a+x}=4$

3. (*a*) What value of p will make

$$4x^4+12x^3+13x^2+2px+1$$

a perfect square ?

(*b*) If $\frac{x}{a+b-c}=\frac{y}{b+c-a}=\frac{z}{c+a-b}$, prove that each ratio is equal to $\frac{x+y+z}{a+b+c}$.

4. (*a*) An aeroplane flies from Delhi to Calcutta, a distance of 900 miles with a uniform speed. If its speed had been 20 miles an hour less, it would have reached Calcutta 30 minutes later. Find its speed.

(*b*) A man says to his son, 'Seven years ago I was seven times as old as you were and three years hence I shall be three times as old as you. Find their ages.

5. (*a*) With the same axes and unit, draw the graphs of the straight lines represented by the equations (i) $2x+y=5$, (ii) $x-2y=0$ and (iii) $x=0$. Read the co-ordinates of the vertices of the triangle formed by the lines.

Or

(*b*) The relation between the age and weight of a boy is given in the following table:—

Age in years	8	10	12	14	16	18
Weight in lbs....	60	70	82	95	110	127

Represent the above graphically and read the weights of the boy at the age of 9 and 15 years and the age of the boy when he is 120 lbs.

ANSWERS

EXERCISE 1

1. $a(b+c)$.
2. $a(b+2)$.
3. $2(x+2y)$.
4. $ab(a+b)$.
5. $x(y^2-xz)$.
6. $3x^2(1-3x)$.
7. $b(a+2c+3ac)$.
8. $3xy(1+3x-9z)$.
9. $xyz(1+xyz+xy)$.
10. $q^3(p^3+r^3+p^3r^3)$
11. $4m^2(3nq^2r^2+7n^2p^2-m^2+4n^2)$.
12. $-7bc(3d^2+a^3b+2cd)$.
14. $(p+q)(x+y)$.
15. $(l+n)(m-n)$.
16. $(b-c)(a^2+b^2)$.
17. $(b-c)(a^2+b^2+c^2)$.
18. $(b+c)(b-c)(a+b+c)$
19. $a(ab-cd)(b+a+a^2)$.
20. $3(x+y)\{2+x+y-3(x+y)^2\}$

EXERCISE 2

1. $(y+z)(x+a)$.
2. $(3a-4b)(1+4p)$.
3. $(3+pr)(2-3q)$.
4. $(x+1)(x+y)$.
5. $(x-1)(x^2+1)$.
6. $x^2(x+1)^2(x-1)$.
7. $ab(a^2+b^2)(2a-3b)$.
8. $(3p-2)(7p^2+11)$
9. $(b-c)(a^2+p)$.
10. $(x^2+y^2)(2a-3b)$.
11. $(p+q)(ab+cd)$.
12. $(x+yz)(y+zx)$.
13. $x(x-y)(xy+yz+zx)$.
14. $(x+y+z)(a+b)$.
15. $(x+y+z)(a-b)$.
16. $(a+b)(ab+b-1)$.
17. $(x+2y)(3xy+5yz-7zx)$.
18. $(p^2+q^2)(5ab+4cd+3)$.
19. $(y+z)(xy+yz+zx)$.
20. $(3-4q)(x^2+y^2+z^2)$.

EXERCISE 3

1. $(x+1)^2$.
2. $(x+2)^2$.
3. $(2x-1)^2$.
4. $(3a+4b)^2$.
5. $(5p-6q)^2$.
6. $(5p+7q)^2$.
8. $(a+b+3c)^2$.
9. $(2x+2y-6z)^2$.
10. $(5p-5q+4r)^2$.
11. $(6a+6b+5c)^2$.
13. $(x-y+a+b)^2$.
14. $16a^2$.
15. $(3l+13m)^2$.
17. $(2x^2+9y^2)^2$.
18. $(6p^3+1)^2$.
19. $(5a^4-6b^4)^2$.
20. $16(2a^2+ab+2b^2)^2$.
22. 361.
23. 324
24. 1.
25. 100.
26. 144.

EXERCISE 4

1. $(a+2b+3c)^2$.
2. $(x-2y+5z)^2$.
3. $(2x+3y-4z)^2$.
4. $(p-3q-5r)^2$.

EXERCISE 5

1. $(x+2y)(x-2y)$.
2. $(2x+3y)(2x-3y)$.
3. $(2x+3)(2x-3)$.
4. $(3x+5y)(3x-5y)$
5. $(7-a)(7+a)$.
6. $(a+b)(a-b)$.
7. $(a^2+b)(a^2-b)$.
8. $(a^2+b^2)(a+b)(a-b)$.
9. $(lx+p)(lx-p)$.
10. $(10+p)(10-p)$
11. $a(2a+5x)(2a-5x)$.
12. $(4x^2+1)(2x+1)(2x-1)$.
13. $(8a^2+7b^2)(8a^2-7b^2)$.
14. $x^2(1+9x^2)(1+3x)(1-3x)$.
15. $2ax(7ax^2-8)(7ax^2+8)$.
16. $3a^5(8a^2+9x^2)(8a^2-9x^2)$.
18. $(2x^3+3y^3)(2x^3-3y^3)$.
19. $a^3(5a^3+8b)(5a^3-8b)$.
21. $(a+2b-3c)(a-2b+3c)$.
22. $(4p+5q+3c)(4p+5q-3c)$.
23. $(3x^2-4y^2+5z^2)(3x^2-4y^2-5z^2)$.
25. $60ab$.
26. $160xy$,

27. $4b(a+c)$.

28. $-(7a+7b+2c)(3a+b+12c)$.

30. 199. 31. 230. 32. 999996.

33. 3·9984.

EXERCISE 6

1. $(z+6)(z+4)$. 2. $(x+5)(x+3)$.

3. $(x+11)(x+6)$. 4. $(x-1)(x-7)$.

5. $(p+8)(p+12)$. 6. $(x-6)(x-4)$.

7. $(x-11)(x-6)$. 8. $(x+23)(x-3)$.

9. $(x+28)(x-4)$. 10. $(x-28)(x+4)$

12. $(x+7y)(x+3y)$. 13. $(x-11y)(x-5y)$.

14. $(x+12y)(x-20y)$. 16. $(x^2+11)(x^2+1)$.

17. $(a^2+13)(a^2+5)$. 18. $(a^2+18)(a^2+7)$

19. $(a^4-16)(a^4+5)$. 20. $(a^3-8)(a^3+1)$.

21. $(x^3+2)(x^3-2)(x^6-2)$. 23. $(y+2a)(y+3b)$.

24. $(x^2+m^2)(x^2+n^2)$.

25. $(y-2a)(y-b)$. 26. $(x-a)(x-5b)$.

28. $(x+1)(x+2)(x^2+3x+1)$. 27. $(a^2+2a-2)(a^2+2a+1)$.

29. $(a-2)(a+2)(a+9)(a+5)$

30. $(x-1)^2(x+1)(x-3)$.

EXERCISE 7

1. $(2x+1)(x+1)$. 2. $(2x-1)(x+1)$.

3. $(2p+1)(p+9)$. 4. $(3x-2)(x+3)$.

5. $(6x+11)(x+4)$. 6. $(2x+1)(x-1)$

7. $(2p-1)(2p-3)$. 8. $(7a+1)(7a+2)$.

9. $(1-3a)(3-2a)$. 10. $(8-7x)(1+x)$.

11. $(5a-9b)(2a+b)$. 12. $(5a+3b)(3a-2b)$.

13. $(9c-5d)(3c-d)$. 14. $(a+9b)(6a-5b)$

15. $(3x+7y)(4x-11y)$. 16. $(4a-3x)(3a+5x)$.

18. $(3x^2-2y^2)(2x^2-y^2)$. 19. $(4x^2y^2-15)(8x^2y^2+9)$.

20. $(3x^2-11)(7x^2-8)$. 21. $(3-x^2y^2)(5+2x^2y^2)$.

23. $(x-y)^2(2x^2+2y^2+xy)$.

24. $(a+b)^2(2a^2+2b^2+ab)$.

25. $(x-4y)(4x-y)(x^2+y^2)$.

26. $(a^2-5)(2a+3)(2a-3)$.

27. $(x+7y+5z)(x+7y-2z)$.

28. $(2a-x-7)(6a-3x-2)$.

EXERCISE 8

1. 4. 2. 16. 3. $\frac{289}{4}$. 4. $\frac{1}{144}$.

5. b^2. 6. 1. 7. $10yz$. 8. $12xy$.

9. $70xy$. 10. $2x$. 11. $20xy$.

13. $(x+12)(x-5)$. 14. $(4x-3y)(4x+5y)$.

15. $(2a+1)(2a+3)$. 16. $(3x+1)(3x-2)$.

17. $(p+3)(13p+2)$. 18. $(2a-3)(13a-1)$.

19. $(5p-2q)(7p+11q)$. 21. $(x^2+2x+2)(x^2-2x+2)$.

22. $(2x^2+2xy+y^2)(2x^2-2xy+y^2)$.

23. $(x^2+4x+8)(x^2-4x+8)$.

24. $(a^2b^2+2ab+2)(a^2b^2-2ab+2)$.

25. $(2x^2+6x+9)(2x^2-6x+9)$.

26. $(x^4+4x^2+8)(x^4-4x^2+8)$.

28. $(2x^2+xy+7y^2)(2x^2-xy+7y^2)$.

29. $(2x^2+x-9)(2x^2-x-9)$.

30. $(3a^2+ab-3b^2)(3a^2-ab-3b^2)$.

31. $(7x^2+13ax+11a^2)(7x^2-13ax+11a^2)$.

32. $(x^2+3xy+4y^2)(x^2-3xy+4y^2)$.

34. $(x^4+3x^2y^2-2y^4)(x^4-3x^2y^2-2y^4)$.

35. $(a^2+a+1)(a^2-a+1)(a^4-a^2+1)$.

36. $(1+a^2)(1-a^2)(5-3a^2)(5+3a^2)$.

37. $(5a^2+2ab+b^2)(a^2+2ab+5b^2)$.

38. $(2x^2+2y^2+4xy+10x+10y+25)(2x^2+2y^2+4xy-10x-10y+25)$.

EXERCISE 9

1. $(a+2b)(a^2-2ab+4b^2)$.
2. $(x+5y)(x^2-5xy+25y^2)$.
3. $(a-2b)(a^2+2ab+4b^2)$.
4. $(x-3y)(x^2+3xy+9y^2)$.
5. $(2p-3q)(4p^2+6pq+9q^2)$.
6. $(5p-2q)(25p^2+10pq+4q^2)$.
7. $(10-4y)(100+40y+16y^2)$.
8. $(6a-10b)(36a^2+60ab+100b^2)$.
10. $(x+y)(x-y)(x^2+xy+y^2)(x^2-xy+y^2)$.
11. $(x^2+5y^2)(x^4-5x^2y^2+25y^4)$.
12. $(x^2-2y^2)(x^4+2x^2y^2+4y^4)$.
13. $\left(x+\frac{1}{3}\right)\left(x^2-\frac{x}{3}+\frac{1}{9}\right)$.
14. $\left(2a+\frac{b}{2}\right)\left(4a^2-ab+\frac{b^2}{4}\right)$.
15. $\left(\frac{2}{3x}-\frac{3y}{2}\right)\left(\frac{4}{9x^2}+\frac{y}{x}+\frac{9y^2}{4}\right)$.
16. $(x+a+b)\{x^2-x(a+b)+(a+b)^2\}$.
17. $(a+2x+2y)(a^2-2ax-2ay+4x^2+8xy+4y^2)$.
18. $(a+b-x-y)(a^2+2ab+b^2+ax+ay+bx+by+x^2+2xy+y^2)$.
19. $-(a+5b)(19a^2+10ab+7b^2)$.
20. $(p+18q)(61p^2+36pq+84q^2)$.

EXERCISE 10

1. $(a-b+c)(a^2+b^2+c^2+ab+bc-ca)$.
2. $(a+b-c)(a^2+b^2+c^2-ab+bc+ca)$.
3. $(a-b-c)(a^2+b^2+c^2+ab-bc+ca)$.
4. $(2a+3b+c)(4a^2+9b^2+c^2-6ab-3bc-2ca)$.
5. $(2a+3b-4c)(4a^2+9b^2+16c^2-6ab+12bc+8ca)$.

6. $(3x-2y-z)(9x^2+4y^2+z^2+6xy-2yz+3xz)$.
7. $(x+y+1)(x^2+y^2+1-xy-y-x)$.
8. $(x+2y-2)(x^2+4y^2+4-2xy+4y+2x)$.
9. $\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)\left(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}-\frac{1}{xy}-\frac{1}{yz}-\frac{1}{zx}\right)$
10. $\left(x+2y-\frac{z}{3}\right)\left(x^2+4y^2+\frac{z^2}{9}-2xy+\frac{2yz}{3}+\frac{zx}{3}\right)$.
11. $(x^2+y^2+z^2)(x^4+y^4+z^4-x^2y^2-y^2z^2-z^2x^2)$
12. $(x^2+2x-4)(x^4-2x^3+8x^2+8x+16)$.
13. $2(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$.
14. $x^2+9+25y^2-3x+15y+5xy$.
15. $2a-3b-c$.

EXERCISE 11

1. $(x+1)(x+6)(x^2+7x+16)$
2. $(x^2+8x+20)(x^2+8x+2)$.
3. $(x^2+8x+10)(x+6)(x+2)$
4. $(x^2-3x-5)(x^2-3x-17)$.
5. $(x^2-5x+7)(x^2-5x-3)$.
6. $(x-3)(2x+3)(2x^2-3x+7)$.
7. $(x^2+3x-12)(x^2+2x-12)$
8. $(x^2-18)(x^2+10x-18)$

EXERCISE 12

1. 9.	2. 0.	3. $-8\frac{3}{4}$.	4. -140.
5. -9.	6. 0.	7. 0.	8. 10.
17. -5.	18. ·132.	20. 52.	21. $-17, -30$.
22. $a=-4$, $b=6$.			

EXERCISE 13

1. $(x-1)(x^2-3x-2)$.
2. $(x-1)(x+3)(2x+1)$.
3. $(x-1)(3x^2+11x+3)$,
4. $(x+1)(x+3)(x-4)$.
5. $(x-5)(x+5)^2$.
6. $(x+1)(x-1)(x+2)$.

7. $(a-3)(a^2+3)$

8. $(x-1)(x+2)(x+3)(x-4)$.

10. $(x-y)(x+2y)(x-3y)$.

11. $(x-1)(x+1)(x-a)$.

12. $(x+y)(x^2+3xy-2y^2)$.

13. $(x-2y)(2x^2+5xy+3y^2)$.

14. $(a+2b)(a^2-2ab-5b^2)$.

EXERCISE 14

1. $-(x-y)(y-z)(z-x)$.
2. $-(x-y)(y-z)(z-x)$.
4. $(a+b+c)(ab+bc+ca)$.
5. $(a-b)(b-c)(c-a)(a+b+c)$.
6. $-(a-b)(b-c)(c-a)(a^2+b^2+c^2+ab+bc+ca)$.
7. $(x+y)(y+z)(z+x)$.
8. $-(x-y)(y-z)(z-x)(x+y+z)$.
9. $(a-b)(b-c)(c-a)(a+b+c)$.
11. $3(2a+b+c)(a+2b+c)(a+b+2c)$.
12. $(a+b)(b+c)(c+a)$.

Miscellaneous Exercise

I

1. $(a^2+b^2)(x^2+y^2)$
2. $3(2x-3y)(3y-z)(z-2x)$.
3. $(x-a)\left(x-\dfrac{1}{a}\right)$.
4. $x^2(x^2+1)(x+1)$.

5. $(x+2)(x-3)(x+5)$.

6. $(x+2)(2x+1)(2x-5)$.
7. $(x^2+11x+40)(x^2+11x-2)$.
8. $(a-1)(a^2-a-4)$.

9. $(x^2+1)(x^2+x+1)$.

10. $(3x+7y)(7x-3y)$.

11. $(p+q)(p+q-1)$.

12. $(x+3)(x-3)(x^2+3x+9)(x^2-3x+9)$.
13. $(x^2+xy+y^2)(x^2-xy+y^2)$.
14. $(1+x)(1-y)(1+y)(x-1)$.
15. $(3x+4y+1)(2x-3y-2)$.
16. $(x^2+4xy+y^2)(x^2-4xy+y^2)$.

17. $(4x-3)(x-8)$.
18. $(y-3)(x-y+2)$.
19. $\{(a-b)x+(a+b)y\}\{(a+b)x-(a-b)y\}$.
20. $(a+b+c)(b+c-a)(c+a-b)(a+b-c)$.
21. $(x+1)(x-6)(x^2-5x+16)$.
22. $(x-y+1)(x^2+xy-x+y^2-2y+1)$..
23. $(bc-a)(ca-b)$.
24. $(x-5)(x+3)(x+2)$.
25. $(a+2)(a^2-a+4)$.
26. $(x^4+y^4)(x^2+y^2)(x+y)(x-y)$
27. $(x+y)(x-2y+1)$.
28. x^2+x+1.
29. $(x+1)(x-1)(x+4)(x+6)$.
30. $(x-1)^2(2x^2-x+2)$.
31. $3(x-y)(y-z)(z-x)$.
32. $(b-c)(ab+bc+ca)$.
33. $(a-1)(a+3)(a^2+2a+2)$.
34 $3(x-5)(5x+1)$.
35. $(x-2y)(x^2+xy+y^2)$.
36. $(a+b-4c)(a-b+4c)$.
37. $(x-2y+3z)(x^2+4y^2+9z^2+2xy+6yz-3zx)$.
38. $(x^2+x+1)^2$.
39. $(x^2+4x+12)(x^2-4x+12)$.
40. $(2x-y)(3x+y-3)$.
41. $x(x+2)(x^2+2x+2)$.
42. $(x-y)(2x+3y+1)$.
43. $4x(x-1)$.
44. $(1-a+b)(2-a+b)$
45. $(a-2b)(a-3b)(a+5b)$.
47. $(x^2-2x-6)(x^2+2x-6)$.
48. $(x-2y)(x-3y-3)$.
49. $(x-2)(x^2+2x-13)$.
50. $-3(a+b)(b+c)(a+2b+c)$.
51 $9(x-y)(x^2-xy+y^2)$.
52. $(x-1)(x-3)(x+1)$.
53. $\left(x+\frac{2}{y}-\frac{1}{3}\right)\left(x^2+\frac{4}{y^2}+\frac{1}{9}-\frac{2x}{y}+\frac{2}{3y}+\frac{x}{3}\right)$.
54. $\left(9x^2+5+\frac{4}{x^2}\right)\left(9x^2-5+\frac{4}{x^2}\right)$.
55. $(xy+1)(xy-1)(x-y+1)$.

EXERCISE 15

1. ab.
2. ab.
3. x^2y^2.
4. a^2.
5. $5y^2z^2$.
6. x^2.
7. $7xy$.
8. ab.
9. $3a^3b^3$.

10. xy.
11. ab^2.
12. $3x^2y^2z^3$.
13. $12a^3b^2c^4$.
14. $5x^2y^2z^2$.
16. $a+b$.
17. $(a+b)$.
18. $(a+b)^2$.
19. $a+b$.
20. x^2-y^2.
22. $3(a-1)$.
23. x^2-y^2.
24. $x-2$
25. $x-13$.
26. $x+15$.
27. $x^2(x+2a)$.
28. $a-2x$,
29. $3x+4$.
30. $5x-6$.
31. $x-2$.
32. $x+1$.
33. $x-1$
35. $x-y$
36 x^2+xy+y^2.
37 $x-y$.
38. $x+a$.
39. x^2-5x+6.
40. x^2+1.

EXERCISE 16

1. $(x-1)$.
2. x^2-x-2.
3. x^3-4x^2+5x-2.
4. $2x^2-x-2$.
5. x^2+x+2.
6. $2x+5$.
8. x^2+x-4.
9. x^2+x+1.
10. x^2+2x+3
11. $2x^2+3x+2$.
12. $2x^2+7x+3$.
13. $x-3$.
14. x^2+1.
15. $x-2$.
16. x^2-3x+2.
17. $x-2$.
18. x^2-3x+2.
19. $a-1$.
20. x^2-2x+3.
21. $4x^2-6x+9$.
22. $x+7$.
23 $x-1$.
24. $x+3y$.
25 $x+4$

EXERCISE 17

1. $a^2b_j^2$.
2. $6x^2y^3$.
3. $20abx^2y^4$.
4. $ab^2x^4y^2$.
5. $12a^2b^2p^2q^2$.
6. $21x^5y^4z^3$.
7. $15a^2b^3c^3$.
8 $75a^3b^3x^3y^4$.
10. $(a+b)(a-b)^3(a^2+ab+b^2)$
11. $(a^2-b^2)(a^3-b^3)$.
12. $x^2y^2z^2(a+b)^3(a-b)^2$.
14. $(x+2)(x-1)^2$.
15. $(x-1)(x+3)(2x-5)$.
16. $(x+2)(x+3)(x-3)^2$.

17. $(2x+5)(3x+2)(2x+1)(3x-2)$.

18. $(x-3)(x+5)(x+4)(x+7)$.

19. $6(3a-x)^2(a^2-4x^2)$. 20. $(x+2)(2x-1)(3x+1)$.

21. $(2a-3b)(3a+2b)(a-b)(4a^2+6ab+9b^2)$.

22. $(x+1)(x+3)(3x+2)(x-1)$.

23. $(a-b)(3a+b)(a+b)(3a-b)$.

24. $(x-1)(x+1)(x^2+x-1)$.

25. $(2x+3)(2x-3)(4x^2-6x+9)(4x^2+6x-9)(3x+2)$.

EXERCISE 18

1. $(a-4)(2a-1)(3a+5)$.
2. $(5x+4)(4x^3+16x^2-3x-45)$.
3. $x^4-2x^3-4x^2+5x-6$.
4. $(a^2-b^2)(a^2-4b^2)$.
5. $(3a^2-a+1)(2a-3)(3a-2)$.
6. $x(9x^4-36x^3-x^2+2x+8)$.
7. $6(x+3p)(x-3p)(x-5q)$
8. $12x^4-14x^3-94x^2+63x+180$.
9. $(5x+3)(x-1)(x+2)(x+3)$.
10. $(x+3)(x-2)$, $(x-1)(x+3)$.

EXERCISE 19

1. $\pm xy$, $\pm 4xy$, $\pm 5x^2y$, $\pm 6x^3y^2$, $\pm 3x^4y^4$
2. $\pm 7abc$, $\pm 8a^2bc^3$, $\pm 9a^4b^3c^2$, $\pm a^3b^4pq^2$
3. $\pm 1a^3b \pm 6x^4y^5$, $\pm \dfrac{8a^5}{7b^4}$, $\pm \dfrac{14x^2y^3}{15x^2y^2}$.

5. $\pm(x+5)$. 6. $\pm(x-3)$.

7. $\pm(2a+3b)$. 8. $\pm(6x^2+10)$.

9. $\pm(7a^3-5)$. 10. $\pm(7x^2+6y^2)$.

12. $\pm\left(\dfrac{xy}{3}+\dfrac{yz}{6}\right)$ 13. $\pm\left(\dfrac{a^2b^3}{5}-\dfrac{xy^2}{10}\right)$

15. $\pm(2x+3y+5z)$. 16. $\pm(3a-4b+5c)$.

17. $\pm(x^2-y^2-z^2)$. 18. $\pm\left(\frac{a}{2}+\frac{b}{3}+\frac{c}{5}\right)$.

EXERCISE 20

1. $\pm(2x^2-3x-1)$. 2. $\pm(2x^2-2xy^2-y^4)$.
3. $\pm(2x^2-3xa-5a^2)$. 4. $\pm(x^2-2x+3)$.
5. $\pm(x^2-2x+1)$. 6. $\pm(2x^2+3x-5)$.
7 $\pm(4x^2-3x+2)$. 8. $\pm(x^2-ax+2a^2)$.
9. $\pm\left(2x^2+\frac{x}{2}-2\right)$. 10. $\pm(3x^2-5x-9)$.

EXERCISE 21

1. $\pm(x^2+3x+1)$. 2. $\pm(x^2+7x+11)$.
3. $\pm(x^2+9x+19)$. 4. $\pm(4x^2+8x-1)$.
5. 1·6. 6. 16.
8. $\pm(x+3)(2x+1)(x-1)$. 9. $\pm(x+3)(2x+1)(2x-3)$.
10. $\pm(x+a)(x-a)(x+2a)$ 12. $\pm\left(x+\frac{1}{x}+2\right)$.
13. $\pm\left(x-\frac{1}{x}-\frac{5}{2}\right)$. 14. $\pm\left(x+\frac{1}{x}+3\right)$.
15. $\pm\left(x-7+\frac{1}{2x}\right)$. 16. $\pm\left(x^2-2+\frac{1}{x^2}\right)$.

EXERCISE 22

1. $\pm(x+y+z)$. 2. $\pm(2a-3b+4c)$.
3. $\pm(2x-5y+1)$. 4. $\pm(x^2+x+1)$.
5. $\pm(4a^2-3a-5)$. 6. $\pm(2x^2+3x+5)$.
7 $\pm(x^2+2x+3)$. 8. $\pm(2x^2+19x+2)$.
10. 2. 11. -5.
12. $\pm(3x^2+2x+1)$. 13. $\pm(3x^3-2x^2+5x-7)$.

14. $\pm(5x^3+3x^2-2.)$

15. $\pm(x^3-x+1.)$

16. $\pm(x^3-x^2+2x+3.)$

17. $\pm\left(\frac{a}{b}-\frac{3}{2}+\frac{b}{a}.\right)$

18. $\pm\left(\frac{2a-3b+c}{2a-3c}\right)$

19. $\pm\left(x+\frac{a}{3}-\frac{b}{2}.\right)$

20. $\pm\left(\frac{x^2}{2}-2x+\frac{a}{3}.\right)$

21. $\pm\left(x^2-x-\frac{1}{4}.\right)$

22. $\pm\left(x-3-\frac{2}{x}\right)$

23. $-\frac{3b}{a}-\frac{3b^2}{4a^2}.$

24. $-4.$

25. $\frac{1}{4}.$

EXERCISE 23

1. $\frac{a^2}{4}$

2. $\frac{y}{x}.$

3. $\frac{x}{4yz}$

4. $\frac{1}{4xy^2z}$

5. $\frac{b}{5acd}$

6. $\frac{ay^3z}{2b^2}.$

7. $\frac{c}{5a^3b^2d^2}.$

8. $-\frac{a}{3xyb}.$

9. $-\frac{2}{7pqz^2}$

10. $-\frac{abcd}{3}.$

11. $\frac{b+c}{a^2+b}.$

12. $\frac{a}{b+c}.$

13. $\frac{ab}{a-b}.$

14. $\frac{a-b}{a^2+ab+b^2}.$

15. $\dfrac{x+y}{3b(x^2-xy+y^2)}$.

16. $\dfrac{x^2-xy+y^2}{(x+y)^2}$.

17. $\dfrac{(x-y)}{3xyz(x+y)}$.

18. $\dfrac{x+y}{(x^2+xy+y^2)}$.

19. $\dfrac{x^2-2a^2}{yz}$.

20. $\dfrac{x^2+y^2}{x^4+x^2y^2+y^4}$.

21. $\dfrac{a(a-b)}{3b(a^2-ab+b^2)}$.

22. $\dfrac{x+4}{x+1}$.

23. $\dfrac{x+1}{x-1}$.

24. $\dfrac{x^2-xy+y^2}{x^2+xy+y^2}$.

25. $\dfrac{x+3}{5(x-4)}$.

26. $\dfrac{a+b-x-y}{a-b+x-y}$.

27. $\dfrac{(x-y)(x^3+y^3)}{x^2+xy+y^2}$.

28. $\dfrac{x+7}{x-7}$.

29. $\dfrac{x-1}{x+3}$.

30. $\dfrac{2x+3y}{2x-5y}$.

31. $\dfrac{a+b-c}{a-b-c}$.

32. $\dfrac{a+1}{a-1}$.

33. $\dfrac{x^2+1}{x^2-1}$.

34. $\dfrac{ab}{xy}$.

35. $abxy$.

36. $\dfrac{5abc}{2xyz}$.

37. $\dfrac{z}{x}$.

38. $\dfrac{x}{y}$.

39. a^3.

40. $\dfrac{y^2}{3}$.

41. 1.

42. $\dfrac{1}{x+y}$.

43. $\dfrac{1}{x^2+x+1}$.

44. $\dfrac{(x-5)(x-2)}{(x+5)^2}$.

45. $(x+y+z)$.

46. $\dfrac{(a+b-c)(a-b+c)}{(a+b+c)(a-b-c)}$.

47. $\dfrac{a^2+ab+b^2}{(a+b)^2}$.

48. $\dfrac{a^2+b^2}{(a-b)^2}$.

49. $\dfrac{9a^2-8b^2}{9a^2+8b^2}$.

50. 1.

51. 1.

52. $\dfrac{x(x+1)}{x-1}$.

53. $x-2y$.

54. $(75)^3$.

56. $\dfrac{x^2+3x+5}{x^2+3x-5}$.

57. $\dfrac{1-x-2x^2}{2+x+x^2}$.

58. $\dfrac{x^2-2x-3}{x^2-x-20}$.

59. $\dfrac{2x-3}{3x-2}$.

60. $\dfrac{4x^2+1}{5x^2+x+1}$.

61. $\dfrac{3(x^2-7ax+12a^2)}{2(x^2+7ax+12a^2)}$.

62. $\dfrac{2x^2+3x-5}{7x-5}$.

63. $\dfrac{7x-2y}{5x^2-3xy+2y^2}$.

64. $\dfrac{x+1}{x+2}$.

65. $x-3$.

EXERCISE 24

1. $\dfrac{24a}{120}, \dfrac{20b}{120}, \dfrac{15c}{120}$.

2. $\dfrac{ca}{abc}, \dfrac{ab}{abc}, \dfrac{bc}{abc}$.

3. $\dfrac{a(x-1)}{x^2-1}, \dfrac{b(x+1)}{x^2-1}$.

4. $\dfrac{x-9}{(x+1)(x+2)(x-9)}, \dfrac{x+1}{(x+1)(x+2)(x-9)}$.

5. $\dfrac{25(5y+4x)}{20(25y^2-16x^2)}, \dfrac{16(5y-4x)}{20(25y^2-16x^2)}$.

6. $\dfrac{a(5a-6b)}{25a^2-36b^2}, \dfrac{b(5a+6b)}{25a^2-36b^2}$.

7. $\dfrac{(a-b)^2}{a^2-b^2}, \dfrac{(a+b)^2}{a^2-b^2}$.

8. $\dfrac{(x+5)^2}{(x+3)(x-4)(x+5)}, \dfrac{(x+3)(x-4)}{(x+3)(x-4)(x+5)},$
$\dfrac{(x+3)(x-4)}{(x+3)(x-4)(x+5)}$.

9. $\dfrac{(x-a)^2(x^2+a^2)}{x^4-a^4}, \dfrac{(x+a)^2(x^2+a^2)}{x^4-a^4},$
$\dfrac{x^2-a^2}{x^4-a^4}, \dfrac{x^2+a^2}{x^4-a^4}$.

10. $\dfrac{(1-x)^2}{(1-x^2)^2}, \dfrac{2(1-x^2)}{(1-x^2)^2}, \dfrac{(1+x)^2}{(1-x^2)^2}$.

EXERCISE 25

1. $\dfrac{11}{6x}$ 2. $\dfrac{bc+ca+ab}{abc}$ 3. $\dfrac{a^2+b^2+c^2}{abc}$.

4. $\dfrac{2x-3y+4z}{a}$. 5. $\dfrac{27xz+180x+25yz}{45yz}$.

6. $\dfrac{x^2+y^2+z^2}{xyz}$.

7. $\dfrac{2x^3+2a^2x+a}{2ax^2}$.

8. $\dfrac{3a^2cx+6ab^2+2bc^2x}{6abcx^2}$.

9. 1.

10. $\dfrac{13x+2}{12}$.

11. $\dfrac{22x-7}{105x}$.

12. $\dfrac{3y+2z+yz}{8y}$.

13. $\dfrac{a^3+b^3+c^3-3abc}{abc}$.

14. 0.

15. $\dfrac{2a}{a^2-b^2}$.

16. $\dfrac{a+b}{a-b}$.

17. $\dfrac{2}{(x+3)(x+5)}$.

18. $\dfrac{2y^3}{1-y^4}$.

19. $\dfrac{4(x-1)}{(x-2)^2(x+2)}$.

20. $\dfrac{7x}{x^2-16}$.

21. $\dfrac{a^2+b^2}{ab(a^2-b^2)}$.

23. $\dfrac{4x^2y^2}{x^4-y^2}$.

24. 0.

25. $\dfrac{2xz-yz-xy+2x^2}{xyz}$.

26. $\dfrac{x(a+x)}{a(x-a)}$.

27. $\dfrac{9b(a+3b)}{(a-2b)(2a+5b)}$.

28. $\dfrac{2(a+b)}{a-b}$.

29. $\dfrac{xy(4+xy)}{x^2-y^2}$.

30. $\dfrac{x-9}{(x^2-9)(x-3)}$.

31. 0.

32. $\dfrac{12xy}{4x^2-9y^2}$.

33. $\dfrac{x(x-4)}{x^2-1}$.

34. $\dfrac{16x}{1-x^2}$.

35. $\dfrac{3}{a+c}$.

36. $\dfrac{b-a}{b+a}$.

37. $\dfrac{8x^2}{1-x^4}$.

38. $\dfrac{8+8a-a^3}{2a(a+2)}$.

39. $\dfrac{4}{(x^2-4)(x^2+4)}$.

40 0.

42. $\dfrac{x+9}{(x^2-9)(x-2)}$.

43. $\dfrac{1}{(a-b)(b-c)}$.

44. $\dfrac{2}{x^2+10x+16}$.

45. $\dfrac{1}{(x-2)(x-3)}$.

46. $\dfrac{2a}{-(x-2a)(2x-a)(2x+a)}$.

47. $\dfrac{8}{(x+2)(x+10)}$.

48. $\dfrac{13}{(x+9)(x-4)}$.

49. $\dfrac{6}{(x+2)(x+5)(x+8)}$.

50. $\dfrac{2(x^3-6x^2+4x+8)}{(x^2-x-2)(x^2-5x-4)}$.

51. $\dfrac{14}{(x-7)(x+7)}$.

52. $\dfrac{1}{x^2-x+1}$.

53. $\dfrac{2(x-6)}{(x-1)(x-3)(x-5)}$.

54. $\dfrac{3}{(x-1)(x-3)}$.

55. $\dfrac{10}{(x-3)(x-4)(x+5)}$.

56. 0.

57. $\dfrac{3x+9}{x^3+2x^2-x-2}$.

58. $\dfrac{8}{(x+2)(x-3)(x-4)}$.

59. $\dfrac{7ab+3a-3b-3a^2}{ab(a^2-b^2)}$.

60. $\dfrac{5}{(x-3)(x-4)(x-5)}$.

61. $\dfrac{4x^3}{1+x^4+x^8}$.

62. $\frac{1}{1-2x}$.

63. $\frac{2(a+x)}{a^2+ax+x^2}$.

64. $\frac{3}{(x-1)(x^2+1)}$.

65. $\frac{x+3}{(x^2-1)^2}$.

67. $\frac{8x^7}{x^8-256y^8}$.

68. $\frac{1}{(x+2)^2}$.

69. $\frac{8x}{x^2-a^2}$.

70. $\frac{8x^7}{x^8-a^8}$.

71. 0. 73. 0. 74. 0. 75. 0.

76 0. 77. 0. 78. 0. 80 1.

81. -1. 82. $\frac{1}{abc}$. 83 $(a+b+c)$. 84. -1.

85. x. 86. 1. 87. -3. 88. 1.

89. $\frac{1}{(x-a)(x-b)(x-c)}$.

90. $\frac{x^2}{(x+a)(x+b)(x+c)}$.

92. 1. 93. 1. 94. 1. 95. 1.

EXERCISE 26

1. $\frac{1}{x}$.

2. 1

3. $\frac{1}{a-1}$.

4. 1

5. $\frac{2xy}{x^2+y^2}$.

6. $\frac{(c+a)(c-a)}{(a+b)(a-b)}$.

7. $\frac{a^4+3a^2+1}{a(a^2+2)}$

8 $\frac{x^4-x^2+1}{x^3}$

9. $\frac{4a^3x}{x^4-a^4}$.

10. x.

11. $\dfrac{2a^4}{(a^2+b^2)^2}$. 12. $\dfrac{b}{a}$.

13. $\dfrac{(a+b+c)^2}{2bc}$. 14. $\dfrac{a(b^2-a)}{b(b^2-2a)}$.

15. $\dfrac{3}{4(4x+25)}$. 16. $\dfrac{a+b}{ab}$.

17. $\dfrac{2(x^2+y^2)}{y^2}$. 18. $\dfrac{x^2-y^2}{2}$.

19. $-\dfrac{1}{xy}$. 20. x^2y^2.

Miscellaneous Exercise
II

1. $(3x^2+y^2+xy)(3x^2+y^2-xy)(9x^4+y^4-5x^2y^2)$.
2. $(ax-b)(bx-a)$.
3. $(x+y)(x+z)(x^2-xz+z^2)$.
4. $3(2a-3b)(3b-5c)(5c-2a)$.
5. $(2x^2+15x+26)(2x^2+15x+29)$.
6. $(x-y)(y-z)(z-x)(x+y+z)$.
7. $xy(x^2+y^2)(x+y)(x-y)$.
8. $(9+x^2)(3-x)(3+x)$. 9. $2(x+y)(16x+16y-1)$.
10. $(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$.
11. $(a+b)(a-b)(a^2+ab+b^2)(a^2-ab+b^2)$.
12. $(a-b)(c^2-ab)$.
13. $(1-2x+y)(1+4x^2+y^2+2x+2xy-y)$.
14. $(x^6+2)(x+1)(x-1)(x^2+x+1)(x^2-x+1)$.
15. $-(x-y)(y-z)(z-x)(x+y+z)$.
16. $(a+b)^3(a-b)$. 17. $(x-1)(x+1)(x-3)$.

18. $(x-3y+1)(x-y-1)$.   19. $(x-1)(x+2)(x+5)$.

20. $-3(a+b)(2a-b)(2b-a)$.   21. $3x-4$.   22. $2x^2-3$.

23. $x+2$.   24. $2x^2-2x+1$.   25. $3x-2y$.

26. $x+1$.   27. $a-1$.   28. $x+3$.

29. x^2+2x+3.   30. $4x^2+3$.

31. $(x-1)(x-3)(x+2)$ 32. $(x^6-1)(x^4-1)$

33. $ab(a+b)(a-b)(a^2+ab+b^2)(a^2-ab+b^2)$ or $ab\,(a^6-b^6)$.

34. $(a+b+c)(a+b-c)(a-b+c)$.

35. $(x-1)(x+1)(x^2+x+1)(x^2-x+1)$.

36. $(a-x)(a^2-x^2)(a^2+x^2)$.   37. $(x-a)^2(x-3a)(3x-7a)$.

38. $(x+2)(2x-1)(3x-1)(4x^2-3x+1)$

39. x^3-7x-6.   40. x^2+4x+3 ; x^2-4x-5.

41 $(4x+3)(3x-11)^3$; $(4x+3)^3(3x-11)$; $(4x+3)(3x-11)$.

42. $(x-1)(x+1)(4x-1)(3x^2+1)$; $x-1$.

43. $x^3(x-1)(x+1)(x^2+x+1)(x^2-x+1)$; x.

44. $(a+p)(a^2+p^2)(3a^2-5p^2)$; $a+p$.

46. $\pm\left(\frac{x^2}{2}-2x+\frac{a}{3}\right)$ 47. $\pm\left(\frac{x^2}{2}-\frac{2x}{3}-\frac{3}{4}.\right)$

48. $\pm\left(x^2-1+\frac{1}{x^2}.\right)$ 49. $\pm\left(x^3-2x+\frac{1}{x^3}.\right)$

50. $p=25$.   51. $x=10$.

52. $a=5$, $b=-4$, $c=2$ or $a=-3$, $b=4$, $c=-2$.

53. $\frac{1}{xyz}$.   54. $\frac{x^6-2x^5+8x^4-7x^3+24x^2-11x+46}{(x-2)(x-3)(x^2-2x+4)}$

55. 1.   56. x^4+2 57. -3.   58. 1.

59 1.   60. $\frac{24}{(x-5)(x+3)}$ 61. $\frac{8}{1-x^8}$.

62. $\frac{x+1}{x-1}$.   63. $\frac{a+b+c}{(b+c-a)(c+a-b)(a+b-c)}$.

64. $\frac{8a^3(3+16a^4)}{1-256a^8}$ 65. $\frac{5}{(x+1)(x+2)(x-3)}$.

EXERCISE 27

1. $2, -2$.
2. $4, -4$.
3. $5, -5$.
4. $\frac{b}{a}, -\frac{b}{a}$.
5. $10, -10$.
6. $\pm\sqrt{-1}$.
7. $3, -3$.
8. $2, -2$.
9. $2, -2$.
10. $6, -6$.

EXERCISE 28

1. $-5, +1$
2. $1, -\frac{7}{13}$.
3. $\frac{1}{2}, -\frac{4}{3}$.
4. $\frac{9}{2}, \frac{1}{3}$.
5. $\frac{9}{2}, -\frac{8}{11}$.
6. $2, -2$.
7. $-\frac{b}{a}$.
8. $0, 4$,
9. $-3, -5$
10. -3.
11. $1, 25$.
12. $-3, -4$.
13. $4, -8$
14. $5, -5$
15. $3, \frac{3}{4}$.
16. $\frac{3}{2}$.
17. 4.
18. $9, 8$.
19. $\frac{1}{2}, -\frac{1}{25}$
20. $\frac{4}{5}, -\frac{5}{12}$.
22. $4, \frac{1}{4}$.
23. $1, -1$.
24. $15, -4$.
25. $17, -\frac{1}{5}$.
27. $1, -4$
28. $0, 5$.
29. $5, \frac{6}{5}$.
30. $1, -\frac{17}{6}$.

EXERCISE 29

1. $1, -\frac{7}{5}$.
2. $\frac{3}{2}, -\frac{5}{6}$.
3. $\frac{1}{3}, -\frac{2}{9}$.
4. $-1, \frac{3}{4}$.
5. $14, \frac{9}{4}$.
6. $5a, -4a$.
7. $\frac{7b}{6}, -\frac{5b}{6}$.
8. $2, \frac{3}{2}$.
9. $6, 3\frac{1}{2}$.
10. $3, -3$
11. (i) $x^2+3x=0$.
(ii) $3x^2-5x+2=0$.
(iii) $42x^2-11x-3=0$.
(iv) $x^2-4x+4=0$.
(v) $x^2-5x+6=0$.

EXERCISE 30

1. 8 or -7.
2. -37 or 39.
3. 7, 5 or $-7, -5$.
4. 38 and 42.
5. 15, 12.
7. 13, 11 or $-13, -11$.
8. 12, 14 or $-12, -14$.
9. 14, 15 or $-14, -15$.
10. 6, 4 or $-6, -4$.
12. 4 dozens.

13. $4\frac{1}{2}$ *d*
15. £ 30.
16. Rs. 60 or Rs. 40.
17. 72.
19. 12, 4.
20. 5, 7.
22. 8 min., 4 min.
23. 12 min., 15 min.
25. 120, 80.
26. $1\frac{1}{2}$ ft.
28. 78.
29. 93.
31. 9 miles.
32. $27\frac{1}{2}$ miles

EXERCISE 31

1. $x=\pm 25, y=\pm 6$
2. $x=11$ or $3, y=3$ or 11.
3. $x=\frac{3}{4}$ or $\frac{1}{5}$; $y=\frac{1}{5}$ or $\frac{3}{4}$.
4. $x=2, y=3$; $x=1$; $y=6$.
5. $x=4$ or $1\cdot 6$; $y=2, 5$.
7. $y=-1, \frac{1}{2}$; $x=1, -2$.
8. $x=7, -5$; $y=5, -7$.
9. $x=2, -\frac{4}{3}$; $y=1, -\frac{2}{3}$.
10. $x=3, y=5$.
11. $x=3, 0$; $y=0, -9$.
12. $x=3, 1\frac{1}{3}$; $y=-2, -4\frac{1}{2}$.

Miscellaneous Exercise III

I

1. (*a*) $(25+x^2)(5+x)(5-x)$. (*b*) $a^2b(4a^2-3b)(3a^2+5b)$. (*c*) $(a-2b)(5a+3b)$.
2. x^2+x+2.
3. $(x-1)^3(x+1)(x^2+1)$.
4. $\pm(9x^2+33x+19)$.
5. 1.
6. (i) $x=6, y=-5$. (ii) $\pm\sqrt{3}$.
7. 9, 27.

II

1. (i) $(x^4+12x^2+16)(x^2+2x-4)(x^2-2x-4)$. (ii) $-4x(1+x^2)$ (iii) $(a-b)(c-d)$.
2. $2x+5$.
3. $(x-1)(2x+1)(2x-1)(3x-2)$.
4. $\pm\left(2x+1-\frac{3}{x}+\frac{1}{x^2}\right)$.
5. $\frac{-x-5}{x^2+4x+3}$.
6. $-\frac{7}{8}, 2$
7. 12, 4.

III

1. (i) $(x+3)(x-1)(x+2)$
   (ii) $(a^2+1)(b^2+1)(a+1)(a-1)(b+1)(b-1)$.

(iii) $(x+2)(x+6)(x^2+8x+10)$.

2. a. 3. $(a^4+4)(a^2-4)$.

4. $\pm\left(x^2-\dfrac{x}{2}-1.\right)$ 5 $\dfrac{32x^3}{16x^4-81y^4}$.

6 $-\frac{1}{2}$; $\frac{7}{3}$ 7. 21, -22.

IV

1. (i) $(a+b+c)(x^3y-2ab+3bc)$

(ii) $7ac(13bd-b-3d-8de)$.

2. $(x+y)(x-2y)$. 3. $190l^6n^5v^4s^5$.

4. (i) $\pm(x+2y)$, (ii) $\pm(5x+a)$.

5. $\dfrac{(x+1)^2}{(x+2)}$.

6 (i) ± 4. (ii) $\frac{2}{3}, -2\frac{1}{3}$. 7 14, 22 or $-14, -22$.

V

1. (i) $(7x+1)(49x^2-7x+1)$; (ii) $\left(2m+3-\dfrac{2}{m}\right)\left(2m-3-\dfrac{2}{m}\right)$

2. $3x-4$. 3. $(x+y+z)(x-y-z)(x+y-z)$.

4. $\pm(2x^2+3x-4)$ 5 $\dfrac{1}{x^2-1}$

6, (i) $3, -1$; (ii) $\frac{3}{2}, -\frac{3}{2}$. 7. 11, 9.

VI

1. (i) $3(2a-3)(a-1)(a-2)$.

(ii) $(p-q+s)(p^2+q^2+s^2+pq+qs-sp)$.

2. $x-1$. 3. $1575ab(a^3+b^3)(a^4-b^4)(a^2+ab+b^2)$.

4. $\pm\left(2x+1-\dfrac{3}{x}-\dfrac{1}{x^2}\right)$ 5. $\dfrac{2(a^2+b^2)}{a^2-b^2}$

6. (a) $x=\frac{5}{3}, -\frac{3}{2}$; (b) $3, -\frac{2}{3}$. 7. 5, 12. or $-5, -12$

VII

2. (i) $3(2x+y+z)(x+2y+z)(x+y+2z)$.

(ii) $(b+c)(c+a)(a+b)$

3. $(a+1)^2(a-2)(a-3)$. 4. $\pm(5x^2+4x-3.)$

5. 0 6 $-\frac{2}{3}, \frac{5}{11}$, (ii) $-\frac{5}{2}, \frac{4}{3}$ (iii) $7, -\frac{3}{2}$,

7. 12, 21.

VIII

1. 0

2. (i) $\left(\frac{x}{2}+2y+1\right)\left(\frac{x^2}{4}+4y^2+1-xy-2y-\frac{x}{2}\right)$.

(ii) $(a^2-b^2-2c^2)(a^4+b^4+4c^4+2c^2a^2+a^2b^2-2b^2c^2)$.

3. x^2-8x-2. 4. ± 2. 5. $\frac{a+1}{a^2}$

6. (i) $x=-\frac{1}{2}, \frac{2}{3}$. (ii) 4, -10 7. 15, 8.

IX

1. (a) $-3(x-4a)(2x-3a)(x+a)$. (b) $(1-xy)(1+xy)(1-x^2+y^2)$.

2. $x+2$. 3. $(4x^2+3)(x-2)(3x+1)$.

4. $\pm(6x^2-3x+2.)$ 5. (a) $\frac{1}{2}, -4$ (b) $\frac{2}{3}, -\frac{7}{6}$.

6. $\frac{4x^4}{x^4-16a^4}$. 7. $11, -14$.

X.

1. (a) $3(y+z)(y-z)(z+x)(z-x)(x+y)(x-y)$.

(b) $3(3x-1)(x-4)(5-4x)$.

2. 0. 3. x^2-3x+2.

4. 1. 5. 0. 6. 13ft. 7. 2, $-\frac{2}{3}$.

XI.

1. (a) $(x^2+4x+12)(x^2-4x+12)$.

(b) $(2x-y)(3x+y-3)$. (c) $(ab-4)(ab-11)$.

2. ± 14. 3. 8. 4. $\pm(x^2-1\frac{1}{2}x+\frac{1}{2}.)$

5. $\frac{x+1}{x+2}$ 6. (a) $x=a, -2a$. (b) $-\frac{7}{3}, \frac{5}{2}$. 7. 11, 4.

XII

2. a^2-a+1. 4. $\frac{a^2}{y^2}$. 5. $\pm(2x^2-2x-1.)$

6. $0, -\frac{13}{5}$. 7. 10, 6.

XIII.

1. x^2-xy+y^2.
2. $x-3$
3. x^6-a^6.
4 $\pm(3x^2+2x+1.)$
5. $\dfrac{8xy}{x^2-4y^2}$.
6. 14, 52 and 1914.
7. (a) $-1, -4$ (b) $3, 7$. (c) $-4, -\frac{1}{4}$.

XIV.

2. $(7x-2)(2x-11)$.
3. 10.
4 (i) $-\frac{7}{8}, \frac{11}{9}$. (ii) $-\frac{9}{7}, \frac{11}{8}$
5. $m=n=-4$.
6. $\dfrac{x-2}{(x-1)(x+3)}$.
7. $\pm(5x^2-2x-1)$

XV.

1, 6.
2. (i) $x(x+2)(x^2+2x+2)$.
(ii) $(x-y)(2x+3y+1)$
4 $a-b$.
5. $\pm\left(x^2-2+\dfrac{1}{x^2}\right)$
6. x^6-2x^4+2x-1.
7. $a+1$.

EXERCISE 32

1. -7. 2. 6 3. -5.
4. 15 6 $-1{\cdot}1$. 7. -10.
8. 13 9. 20 11. $1\frac{1}{17}$.

12. $\cdot 5$ 13 $4{\cdot}1$. 15 $\dfrac{a+b}{a-b}$. 16. $\frac{12}{13}$. 17. 1. 18. $\frac{3}{2}$.

19. $\dfrac{ab}{2(a+b)}$. 21. 6. 22 2 23. $4\frac{1}{2}$. 24. $3\frac{1}{2}$ 25. 3.

26 1. 27. $-\frac{5}{4}$. 28. 4. 29. -3. 30. -5. 32. $\frac{5}{3}$
33. $-\frac{5}{2}$. 34. $\frac{2}{5}$. 35. $1\frac{8}{11}$. 36. -1. 38. -9. 39 $\frac{1}{5}$.
41. $\frac{1}{2}$. 42 4.

EXERCISE 33

2. A gets Rs. 46; B, Rs. 30 and C, Rs. 16.
3 77 and 35. 4. 60, 42, 30, 21. 5. 57.

7 $25\frac{1}{5}$ years, $37\frac{4}{5}$ years. 8. 50, 30. 9. 44, 19, 11.
10. 22 years. 12. 8. 13. 10 days. 14. 22 days.
16. 48. 17. 234. 18. 28. 19. 72. 20. 63. 22. Rs. 78, as. 2. 23. 25s. 24. £$6\frac{2}{3}$. 25. 1100 26. $\frac{16}{27}$.
27. 116 miles. 28. 3·9 miles. 29. 5 miles. 30. 11 days.
31. 294, 147 and $73\frac{1}{2}$ days. 32. 3 hours. 33. 22 feet.
34. 10 books and 5 pictures. 35. 1863. 36. £48. 37. £160.
38. 1 lb. worth Rs. $1\frac{1}{4}$ per lb and 3 lbs. worth as. 12 per lb.
39. A, 52 miles : B, 42 miles

EXERCISE 34

1. $x=45, y=10$. 2. $x=1{\cdot}5, y=2{\cdot}4$. 3. $x=8, y=5$.
4. $x=5\frac{1}{2}, y=2\frac{1}{2}$. 5 $x=\frac{1}{4}, y=\frac{1}{5}$. 6. $x=12, y=6$.
8. $x=3, y=4$. 9. $x=2, y=3$. 10. $x=6, y=10$.
11. $x=\frac{1}{9}, y=3$. 12. $x=2, y=3$. 13. $x=\frac{1}{15}, y=18$.
14. $x=\frac{7}{5}, y=\frac{5}{2}$ 15 $x=3, y=1$.

16 $x=\dfrac{a_1b_2-a_2b_1}{c_1b_2-c_2b_1}, y=\dfrac{b_1a_2-b_2a_1}{c_1a_2-c_2a_1}$. 17 $x=3, y=2$.

18. $x=\dfrac{a^2-b^2}{Qa-Pb}, y=\dfrac{a^2-b^2}{Pa-Qb}$.

19. $x=\dfrac{l(l^2+lm+m^2)}{m(l+m)}, y=-\dfrac{l^2}{l+m}$. 20. $x=-1, y=5$.

EXERCISE 35

1. $x=5, y=2, z=7$. 2. $x=4, y=2, z=11$.
3. $x=10, y=20, z=5$. 4. $x=1, y=2, z=3$.
5. $x=\frac{14}{3}, y=\frac{11}{3}, z=\frac{11}{3}$. 6. $x=11, y=13, z=15$.
7 $x=\dfrac{a}{2}, y=\dfrac{b}{2}, z=\dfrac{c}{2}$. 8. $x=\frac{1}{4}, y=\frac{1}{3}, z=\frac{1}{2}$.
9 $x=2, y=-3, z=-4$. 10. $x=\frac{1}{12}, y=\frac{1}{12}, z=\frac{1}{12}$.
11. $x=1, y=\frac{1}{2}, z=\frac{1}{3}$. 12. $x=a, y=b, z=c$.
13. $x=4, y=-5, z=6$. 14. $x=-1, y=-2, z=-3$.
15. $x=\pm 2, y=\pm 3, z=\pm 4$. 16. $x=\frac{1}{2}, y=\frac{1}{3}, z=\frac{1}{4}$.

EXERCISE 36

1. 17, 10. 2. 36, 14. 3. 500, 250. 4. 3, 16. 81.

5. 32, 24. 6 10, 2. 7. $\frac{5}{12}$ 8. $\frac{7}{9}$.

9. $\frac{2}{3}$, $\frac{3}{4}$ 10. 1 horse $1\frac{1}{4}$ ton and 1 mule $\frac{3}{4}$ ton.

11. Rs 360, Rs 180.

12. A's age = 20 years; B's age = 64 years.

13. A's present age = 20 years; B's present age = 10 years and C's present age = 4 years.

14. A's present age is 57 years; B's present age is 19 years.

16. 72. 17. 97. 18. 39. 19. £1060. 20. £11.

21. $12\frac{1}{2}$ miles away from start.

22. 200 23. 6 sq. ft., $5\frac{1}{2}$ sq. ft.

25. 60 half crown and $ 22.

26. 40 eight anna pieces and 100 rupees.

27. 52 half anna pieces and 19 two anna pieces.

28. Man in 8 days and boy in 12 days.

29. 38 yds, 18 yds.

30. Each of the similar in 20 hours and the the other in 30 hours

31. 52 m. p. hr, 28 m. p. hr.

33. A got 543 votes; B got 364 votes.

34. 72, 24.

35. River, $3\frac{1}{2}$ m. p. hr and boat, $7\frac{1}{2}$ m. p. hr.

36. 17 camps and 60 scouts

37. 8 ft. and 12 ft. 38. 253.

EXERCISE 38

1. 3. 2. 8, 12. 3 15, 30.

4. 41 : 31. 5. 1 : 1. 6. 63, 72.

7 32 : 45. 8. 1 : 3 9. 1 : 3 or 2 : 1.

10. $\frac{pa}{p+q}, \frac{qa}{p+q}$. 16. 14 : 21. 19. The second.

20. 26 rupees 104 eight anna pieces, 117 two anna pieces

EXERCISE 39

1. 9. 2. $6b$. 3. bc 4. $\frac{12yz}{x}$.

5. $\frac{22qr}{p}$. 6. $21\frac{1}{3}$. 7. 25. 8. $\frac{y^2(x+y)^2}{x^2}$.

21. 0. 24. 0. 26 $x : y : z = 5 : 3 : 2$

27. $x : y : z = 6 : 3 : 4$ 29. $x=4, y=\frac{3}{4}$. 30. $6\frac{1}{2}$.

EXERCISE 40

2. $(-4, 4)$ 3. 10
4. 5, 2·23, 3·16 units respectively.
5. AB = 1·41; BC = 2·24; CD = 3·16; DA = 3·61 units.
6. OA = 1·41; OB = 2; OC = 2·22; OD = 2·22 units.
7. All these points lie on a straight line parallel to x axis.
8. All these points lie on a straight line parallel to y axis.
9. Area, 54 square units, co-ordinates. (7,5) and (2, 5)
10. 12·5 sq, units. 11. 2·5 cm
12. 1 sq. unit. 13 (2, 3)
15. (i) 41 sq. units. (ii) 41 sq. units. (iii) 12 sq. units.
    (iv) 85 sq. units. (v) 142·5 sq. units.
16. (i) 1·12." (ii) 1·1". (iii) 1 8"
17. ·64 sq. ins. 18. 1sq. in. 19. 1·05 sq. in. 20. Rect. ·39"
21. ·77 sq. ins. 22. 75 sq. ins. 23. 82sq. ins. 24. 1·09 sq. ins.

EXERCISE 41

40. $3x-2y=0$. 41. $9x+5y=11$
42. $9x-7y=3$. 43. $4x-3y+3=0$
44. $12x+12y=7$. 45. $3x-2y=22$.
46. $x+y+1=0$. 47. $3x-10y+20=0$.
48. $x+y+2=0$. 49. $7y-5x-2=0$.

EXERCISE 42

1. $x=2, y=3$. 2. $x=1, y=2$.
3. $x=2, y=4$. 4. $x=1\frac{3}{4}, y=2\frac{1}{4}$.

5. $x=4, y=3$.
6. $x=3, y=2$.
7. $x=7, y=7$.
8. $x=-2, y=-3$.
9. $x=12, y=18$.
10 $x=23, y=17$.
11. $x=11, y=5$.
12. $x=3, y=-2$.
13. $x=14, y=20$.
14. $x=14, y=20$.
15. $x=2, y=3$.
16. $x=2, y=0$.
17. $x=-3, y=2$.
18. $x=5, y=1$.
20. $x=\cdot 5, y=1\cdot 75$.
21. $24\cdot 2, x=\frac{8}{13}$.
22. $(\frac{7}{2}, \frac{8}{3})$ $(7, 5)$; $(7, -2)$
23. $(5, 4)$; $(2\cdot 8, 1\cdot 8)$; $(2, 3)$.
24. $(3, 2)$ $4x+3y=12$
25. $(0, \frac{7}{2})$, $(\frac{5}{2}, 0)$.

EXERCISE 43

1. 71°
2. 77°F, 10°C
3. 88·86°, 89·3°
4. Rs 50 ; 2400 copies
5. 1862, 1878 ; 21 millions. 25·1 millions.
6. 1834
7. In July 1936.
8. A, 2·16 millions, B, 1·16 millions.
9. bet. June. 25 and 30 ; bet. Jan. 9 and 13.
10. July 6 to 13, July 20 to 27.
14. 59·5 lbs, 90·5 lbs
16. 4′ 9″ ; 14 *y.* 7 *m.*
17. 500, 800.
18. 4·5 millions in 1900.

EXERCISE 44

1. Rs. 56, $1\frac{3}{4}$ mds.
2. Rs. $4\frac{4}{5}$, $13\frac{1}{8}$ Srs.
3. $13\frac{2}{3}$ tolas, Rs 775.
5. 11 lb., 10 kgm.
6. 4·8 c. ft. 268·75 lbs.
7. $37\frac{1}{2}$ miles, $2\frac{2}{3}$ hours.
8 $5\frac{3}{7}$ hours, 122·5 miles
9. $7\frac{1}{2}$ miles from P.
10. $1\frac{1}{2}$ hours, $13\frac{1}{2}$ miles.
12. 4 hours, 12 minutes after A's start ; $13\frac{1}{5}$ miles ;

$$2x-5y=6$$

14. At 60 miles from start ; at $4{\cdot}42\frac{6}{7}$ hours.
16. They meet at 17 17 hrs; 81 miles away from Allahabad.
17. They meet at 12.30 hrs; 10 miles from Khurja Jn.

18. 2 57 hrs; 25 miles from Calcutta

20 $11\frac{1}{4}$ days.

21. $4\frac{7}{13}$ hours.

22. 10 days.

23 $12\frac{10}{13}$ days.

24. 16 min

25. Rs 60

26. (i) Rs. 8440. (ii) 20 patients.

Miscellaneous Exercises IV

1. $\left(x-\frac{1}{x}+3\right)\left(x-\frac{1}{x}-1\right)$.

2. $64x^6-729y^6$, $4x^2-6xy+9y^2$.

3. $\frac{1}{abc}$

4. $x=\frac{-87\pm\sqrt{24817}}{16}$

5. The rate of the boat $=4$ miles p. h.
The rate of the stream$=2$ miles p. h.

6. $\pm(x^2+5x+5)$.

7. $\frac{39}{43}$.

9 $x=2, y=1$.

10. $a=-2, b=5$.

(2)

1. $(3a-4b)(a-2b)$.

2. $x(3x+2)$.

3. $\pm(2x^2+3x-5)$.

4. 1.

5. $x=\frac{4}{9}, \frac{1}{2}$

7. 2.

8. One number$=7$ or $5\frac{1}{2}$, and the other number $=5$ or $\frac{5}{2}$

9 $x=4, y=2$.

10. $x^3+x^2(-a+b+c)+x(-ab-ac+bc)-abc$.

(3)

1. x^8-y^8.

2. $x=\frac{a}{a-b}, y=\frac{b}{a-b}$.

3. $(x+2y)(x-2y)(x^2+4y^2)$.

4. $\frac{1}{(x-1)(x-3)}$.

5. 6.

6. $x=8, y=6$.

8. $3a^2-3a+2$.

9 2264.

10. $-1, 0, 6x^2-12x+8$.

(4)

1. $(xy+6z)(xy-12z)$.

2. $20x^4+96x^3+49x^2-237x-180$.

3. $2, -3$.
4. x^8+x^4+1.
5. 110.
6. 20.
7. 0.
8. $\frac{5}{4}$.
10. $\pm\sqrt{2(x^2+xy+y^2)}$.

(5)

1. $(a-b-c)(a-b+c-1)$.
2. $x=-\frac{1}{4}$.
3. Tin $=126$, Lead $=144$.
4. $12x^4-100x^3+195x^2+70x-72$.
5. $\dfrac{6(a^4-2)}{(a^4-1)(a^4-4)}$
6. 2.
8. $3x-2y-2=0$.
9. 80.
10. 2.

(6)

1. $2(b-c)(a+b)$.
2. 1.
4. 5 miles.
5. $x-2-\dfrac{1}{x}$.
6. $x=15, y=10, z=17$.
8. $x+1$.
9. $A=3, B=-1, C=2$.
10. $\dfrac{4}{7}$.

(7)

1. $-\dfrac{4}{9}$.
2. $\dfrac{7x+5}{(x^2-1)(x+2)}$
3. $x-2$.
4. One right angle.
5. $4ab(1+b)(1-b)(1+a)(1-a)$.
6. 132 yards and 110 yards.
7. $-\dfrac{a^6}{a+b}$.
8. $15-4x$.
10. $\dfrac{3x}{2y}-1+\dfrac{y}{2x}$.

(8)

1. $(2x-1-y)(4x^2+1+y^2+2x-y+2xy)$.
2. $\dfrac{2a^2b^2}{a^4-b^4}$
3. $x=2$.
4. 1.
5. $467\frac{23}{41}$.
6. $x=-\frac{5}{3}$.
7. -60.
8. x^3-7x-6.
10. 4 miles an hour.

(9)

1. $-(x-y)(y-z)(z-x)(x+y+z)$.
2. (i) $x=3, y=2$. (ii) $x=-1, y=5$.
3. 6, 8 or $-6, -8$.
7. $(-1, -1)$; $(-1, -3)$ and $(3, 2)$.
8. $2x^2-x-2$.
9. $x-8+\frac{1}{x}$.
10. $\frac{a}{x(y-x)(a^2-x^2)}$

(10)

1. 1.
2. 7.
4. 8.
6. (i) $(x+2)(x-2)(x^2+2x+4)(x^2-2x+4)$.
(ii) $(x-2)(x^2+2x-13)$.
7. £80, £248.
10. x^2-3x+2, $x^2-7x+10$.

EXERCISE 45

1. x^{m+n}.
2. x^{m-n}.
3. 3^4.
4. 3.
5. 2^{-1}.
6. 2^{11}.
7. 2^6.
8. x^{2m}.
9. y^{mn}.
10. 1.
11. 1.
12. 1.
13. 1.
14. 1.
15. 3.
16. $\sqrt[3]{27^2}=9$.
17. $\sqrt[3]{16^2}=4\sqrt[3]{4}$.
18. 125.
19. x^{-9}.
12. 2^8.
21. 2^{-2}.
22. a^3.
23. 36.
24. $\frac{1}{512000}$.
25. $\frac{1}{8}$.
26. x.
27. $x^{-\frac{2}{3}}$.
28. $a^6b^9c^{12}$.
29. $a^{-4}b^6c^{-8}$.
30. $(3p)^{-3r}$.
31. x^{-m}.
32. $a^{-\frac{1}{2}}b$.
33. $a^{-\frac{1}{3}}b^{\frac{2}{3}}$.

34. $\left(\dfrac{1}{a^{12}b^{11}}\right)^{\frac{1}{4}}$ 36 $x^{\frac{1}{6}}z^{-1}$.

37. $\frac{2}{3}$. 38. $\left(\dfrac{3}{10}\right)^n$, ·54. 39 8.

40. $\dfrac{1}{\sqrt{a}}+\dfrac{1}{\sqrt{b}}$ 41. 2. 42 1.

43. 32. 44. 2. 45. $\dfrac{44}{6075}$

48. $\sqrt[6]{\frac{1}{2}}$. 49. 1. 50. 25

EXERCISE 46

1. $x^{\frac{3}{2}}-2xy^{\frac{1}{2}}+2x^{\frac{1}{2}}y-y^{\frac{3}{2}}$. 2. $x^{\frac{7}{12}}y+x^{\frac{1}{3}}y^{\frac{2}{3}}-x^{\frac{1}{4}}y^{\frac{4}{3}}-y$.

3. $x-y$. 4. $x^{\frac{3}{2}}+y^{\frac{3}{2}}+z^{\frac{3}{2}}-3x^{\frac{1}{2}}y^{\frac{1}{2}}z^{\frac{1}{2}}$.

5. $x-y$ 7. $a^{\frac{1}{2}}+b^{\frac{1}{2}}$.

8. $a^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}$ 9 $a^{-\frac{3}{2}}+2a^{-1}b^{\frac{1}{2}}+4a^{-\frac{1}{2}}b+8b^{\frac{3}{2}}$.

10. $x^{\frac{2}{3}}+a^{\frac{1}{3}}x^{\frac{1}{3}}+a^{\frac{2}{3}}$.

11. $x^{\frac{4}{3}}y^{-\frac{4}{3}}+x^{\frac{2}{3}}y^{-\frac{2}{3}}+x^{-\frac{2}{3}}y^{\frac{2}{3}}+x^{-\frac{4}{3}}y^{\frac{4}{3}}$.

12. $a^{4m}-a^{3m}b^{m}+a^{2m}b^{2m}-a^{m}b^{3m}+b^{4m}$.

14. $2x^{-1}+7x^{-2}+3x^{-3}$. 15 $x^{-4}-3x^{-5}+4x^{-6}$.

17. $(64x^{\frac{4}{3}}-729)(3x^{\frac{2}{9}}+2)$ 18. $(x^{\frac{8}{3}}-1)(x^{\frac{8}{3}}+x^{\frac{4}{3}}-1)$.

20. $\pm(\sqrt{a}+\sqrt{2b}+2\sqrt{2c})$.

EXERCISE 47

1. $\sqrt{4}$. 2. $\sqrt{25}$. 5. $\sqrt{49}$

4. $\sqrt{a^2}$. 5. $\sqrt{c^2}$. 6. $\sqrt{a^6}$.

7. $\sqrt{d^{10}}$. 8 $\sqrt{a^6b^{10}}$. 9. $\sqrt{(a+b)^2}$.

10. $\sqrt{(x^2-y^2)^2}$. 11. $\sqrt{25x^6y^4}$. 12. $\sqrt{9x^8y^{12}z^{10}}$.

13. $\sqrt[5]{32}$. 14. $\sqrt[5]{243}$. 15. $\sqrt[5]{3125}$.

16. $\sqrt[5]{59049}$ 17. $\sqrt[5]{a^{10}}$ 18. $\sqrt[5]{a^{10}b^{10}}$

19. $\sqrt[5]{a^5b^5x^{\frac{5}{2}}}$. 20. $\sqrt[5]{a^{10}b^{15}c^{\frac{5}{3}}}$. 21. $\sqrt[5]{a^{25}b^{35}}$.

22. $\sqrt[5]{32a^{10}x^{15}z^{25}}$. 23 $\sqrt[n]{3^n}$. 24. $\sqrt[n]{5n}$.

25. $\sqrt[n]{a^{2n}}$. 26 $\sqrt[n]{c^{5n}}$. 27. $\sqrt[n]{a^{3n}b^{4n}}$.

28 $\sqrt[n]{x^{2n}y^{3n}z^{5n}}$. 29. $\sqrt[n]{a^{\frac{n}{2}}b^{\frac{n}{3}}c^{\frac{n}{5}}}$. 30 $\sqrt[n]{x^{\frac{n}{2}}y^{2n}z^{3n}}$.

31. $\sqrt[n]{p^n q^{\frac{4n}{3}} r^{\frac{2n}{3}}}$. 32. $\sqrt[n]{(a^2-x^2)^n}$. 33. $\sqrt[n]{(a^3-x^3)^{\frac{n}{2}}}$.

34. $\sqrt[n]{x^n y^{\frac{bn}{a}} z^{\frac{cn}{a}}}$. 35. $\sqrt[n]{a^2+x^{2n}y^{3n}}$. 36. $\sqrt[6]{x^3}, \sqrt[6]{a^4}$.

37. $\sqrt[6]{x^9}, \sqrt[6]{y^8}$. 38. $\sqrt[12]{a^{16}}, \sqrt[12]{x^{15}}$.

39. $\sqrt[10]{y^5}, \sqrt[10]{z^2}$ 40. $\sqrt[12]{a^{12}b^{24}}, \sqrt[12]{a^{12}c^4}, \sqrt[12]{b^6c^9}$.

41. $\sqrt[6]{(a-b)^3}, \sqrt[6]{(a^2-b^2)^2}$.

42. $\sqrt[12]{x^6y^6}, \sqrt[12]{x^9y^9}, \sqrt[12]{x^4y^4}$.

43. $\sqrt[12]{a^6b^{12}}, \sqrt[12]{x^8y^{12}}, \sqrt[12]{y^9z^{12}}$.

44. $\sqrt[12]{p^{16}q^{16}}, \sqrt[12]{p^4q^{12}}, \sqrt[12]{p^{36}q^{12}}$.

45. $\sqrt[6]{(x^2+y^2)^3}, \sqrt[6]{(x^3-y^3)^2}, \sqrt[6]{(x^2+xy+y^2)^3}$.

46. $\sqrt{12}$. 47. $\sqrt{45}$. 48. $\sqrt{175}$.

49. $\sqrt{a^4b}$. 50. $\sqrt[3]{a^6b}$. 51. $\sqrt[4]{a^{12}c}$.

52. $\sqrt{147x^5}$. 53. $\sqrt[3]{8x^{11}}$. 54. $\sqrt[3]{54x^8y^4}$.

55. $2\sqrt{5}$. 56. $3\sqrt{7}$. 57. $3\sqrt[3]{3}$.

58. $a\sqrt{a}$. 59. $a^2\sqrt{a}$. 60. $10\sqrt[3]{-6}$

61. $4ax^2\sqrt[3]{2a}$ 62. $2xy\sqrt[5]{5x}$. 64. $\sqrt{3}, \sqrt[3]{4}, \sqrt[4]{5}$.

65. $\sqrt[3]{4}, \sqrt{2}, \sqrt[4]{3}$. 66. $\sqrt[3]{6}, \sqrt{3}, \sqrt[4]{5}$.

67. $\sqrt{10}, \sqrt[3]{25}, \sqrt[4]{30}$. 68. $\sqrt[3]{a^2}, \sqrt{a}, \sqrt[3]{a}$.

69. $\sqrt{4x^8}, \sqrt[3]{2x^2}, \sqrt[6]{5x^3}$. 70. $\sqrt[3]{3x^4}, \sqrt[3]{2x^2}, \sqrt{x}$.

EXERCISE 48

1. $18\sqrt{2}$. 2. $20\sqrt{2}$. 3 $6\sqrt{3}$.

4. $9\sqrt[3]{2}$. 5 $6\sqrt[3]{2}$. 6. $3\sqrt{6}$.

7. $\sqrt{5}(6\sqrt{5}+4\sqrt{2}-\sqrt{3})$. 8. $\sqrt{7}(21\sqrt{2}-26\sqrt{3}$.

9. $24\sqrt{10}$.
10. 480.
11. 53.
12. 115.
13. $\frac{3}{2}$.
14. $\frac{4}{3}$.
15. $47\frac{1}{45}$.
16. $7\frac{7}{8}$.
17. $a^2-a+1+a^3\sqrt{2}+\sqrt[3]{2}+\sqrt[3]{4}$
18. 4.
19. 12, 10.
20. 4.
21. $2\sqrt{2}$.
22. $(\frac{3}{4}-\sqrt{5})$
23. 52.
24. $4x\sqrt{x^2-1}$.
25. $2x$.
26. 2.
27. 1154.
28. $\frac{3\sqrt{2}+2\sqrt{3}}{6}$.
29. $\frac{x+2\sqrt{xy}+y}{x-y}$.
30. $\frac{x-\sqrt{x^2-y^2}}{y}$.
31. $\frac{x^2+\sqrt{x^4-y^4}}{y^2}$.
32. $\frac{x+\sqrt{x^2-y^4}}{y^2}$.

EXERCISE 49

1. $\sqrt{\frac{7+\sqrt{29}}{2}}+\sqrt{\frac{7-\sqrt{29}}{2}}$.
2. $\sqrt{2(1+\sqrt{-1})}+\sqrt{2(1-\sqrt{-1})}$.
3. $\sqrt{\frac{1}{2}(11+\sqrt{13})}+\sqrt{\frac{1}{2}(12-\sqrt{13})}$.
4. $5\sqrt[4]{7}$.
5. $\sqrt[4]{8}\,[\sqrt{\tfrac{3}{2}}-\sqrt{\tfrac{1}{2}}\;]$
6. $\sqrt{2}-\sqrt{3}$
7. $\sqrt{\frac{7}{2}}+\sqrt{\frac{5}{2}}$.
8. $\sqrt[4]{3}\,[\sqrt{2}+1]$.
9. $\sqrt[4]{8}[\sqrt{\frac{3}{2}}+\sqrt{\frac{1}{2}}]$.
10. $\sqrt{2}+\sqrt{7}$.
11. $\sqrt{\frac{1}{2}(x+\sqrt{-1}y)}+\sqrt{\frac{1}{2}(x-\sqrt{-1})}$.
12. $\sqrt{\frac{1}{2}(a+b+\sqrt{2b^2+2ab})}+\sqrt{\frac{1}{2}(a+b-\sqrt{2b^2+2ab})}$.
13. $\sqrt{a+\sqrt{-1}b}+\sqrt{a-\sqrt{-1}b}$

14 $\sqrt{\frac{1}{2}(x^2+\sqrt{x^4-4x^2y^2-4y^2})}+\sqrt{\frac{1}{2}(x^2-\sqrt{x^4-4x^2y^2-4y^2})}$.

15 $\sqrt{\frac{1}{2}(x+4a+\sqrt{x^2-24ax+16a^2})}$

$$-\sqrt{\frac{1}{2}(x+4a-\sqrt{x^2-24ax+16a^2})}.$$

EXERCISE 50

1. 7.	2. 6.	3. 2.	4. 1.
5. 4.	6. 5.	7. 12.	8. $\frac{2}{3}$.
9. 5.	10. -3.	11. $2a$.	12. $\frac{81}{a}$
13. $\pm 3a$.	15. $\frac{a}{2}\sqrt{\frac{13}{3}}$.	16. $\frac{25a}{24}$.	17. 1, -1.
18. $\frac{5}{8}$.	19. 5.	20. 2.	

U. P. BOARD 1949

१. (क) (य+२) (य^2 − २य+४) (ख) (य^2+य र+र^2) (य^2−यर+र^2) (ग) (य+१७) (य+६) २. (क) य−१ (ख) य^3+य^2 —४य—४. ३. $\frac{\text{क}}{(\text{क}+\text{ख}-\text{ग})(\text{क}+\text{ख}+\text{ग})}$, (ब) $\frac{१}{\text{क ख ग}}$,

४. (क) ± (२क^2+३क+$\frac{३}{\text{क}}$) ५. (क) य = —३, २ (ख) य = $\frac{१}{२}$

६. ८० रु० ७. (क) (य+२) (य+६) (य^2+८य+१०) (ख) (य+र+ल) (य^2+र^2+ल^2 − यर—रल—लय) ८. (क) ४१ (ख) ४ ६. $\frac{२०}{२१}$ १०. (०,—४), (०, $\frac{३}{८}$), (—५,—$\frac{१}{२}$)

१३.२४० लड़के १४.१३२ गज

1950

१. (क) ८य (ख) (२य+५) (२य−१) (ग) (य+५) (य^2—५य+२५) २. (क) य (य^2+य+१) (ख) य^{4}र^2—य^{2}र^4

३. (क) ० (ख) १ ४. (क) $\pm$ (य^२ − ३ − $\frac{१}{य}$) ५. (क) —१०९ (ख) य = ३, र = ४ ६. ९, ७, ७. (क) (२ + य) (४ — २य + य^२) (२ − य) (४ + २य + य^२) (ख) (य — १) (२य + ३) (२य — १) ८. (क) ६य^४ — य^२ − १ (ख) ९.६ ९. ५०८ १०. (−४, ३), (३, ४), (−१, —२) ११ (क) ३ (ख) ८$\sqrt{५}$ १२. ५% १३. ३ मील प्र० घंटा १४. १५ ग०, २५ ग०

1951

१. (क) (य^२ + र^२) (य + र) (य − र) (ख) (क + ३) (१३क — २) (ग) (३य — ४र) (९य^२ + १२यर + १६र^२) २. (क) य^२ + ३य + ६ (ख) ५ (य + २) (३य — १) (२य + १) ३ (क) ० (ख) $\frac{१}{२}$ ४. (क) $\pm$(२य — ३ — $\frac{२}{य}$) ५. (क) य = ५ (ख) य = $\frac{१}{३}$, र = $\frac{१}{४}$ ६. (क) य = $\frac{१}{३}$ (ख) य = $\frac{१}{२}$, — $\frac{१}{३}$ ७. (क) (कय + २७खर) (कय — ३खर) (ख) (क + $\frac{ख}{३}$ + ग) (क^२ + $\frac{ख^२}{९}$ + ग२ — $\frac{कख}{३}$ — कग — $\frac{खग}{३}$)

८. २० मनुष्य, १२० रु० ९. (क), अ = ४ (ख) ३$\frac{३}{४}$ १०. (क) (२, ०) (ख) (१, −३$\frac{१}{३}$,) (१, ०), (६, ०) ११. ५८ : ८२ वर्ष १२. ४॥ मील, चाल = ५ मी० प्र० घण्टा १३. ४, ५ ६ १४. ६० मील

1952

१. (क) (३य — १) (४य + १) (ख) (९क + ८ख) (९क — ८ख) (ग) (य — १)(य^२ + य + १) २. (क) य — ३ (ख) (य + १) (य + २) (य + ३) (य — २) ३. $\frac{क^२ + २कख — ख^२}{क^२ + ख^२}$ (ख) ० ४. (क) $\pm$(य^२ — ३ + $\frac{१}{य^२}$) ५. (क) य = ५ (ख) य = ५, र = $\frac{१}{५}$ ६. (क) य = ४, −$\frac{३}{२}$

(ख) $\frac{—ख \pm \sqrt{ख^२ — ४कग}}{२क}$ ७. (क) (य^२ + य + १०)(८य^२ + ८य + ५) (ख) (य—१) (य—२)(य—५)
८. १५. १७, ४६ ९. (क) ६३ रु० १० आ० ४ पा० (ख) ३८ अंक
१०. (क) य = ७, र = ६ (ख) र = मय + ग ११. (ख) य = ३, र = ४, ल = ५ १२. ३५ मी० प्र० घन्टा, १३२ ग० १३. ४ ग०

1953

१. (क) (४क^२ + ख^२) (२क + ख) (२क—ख) (ख) (य + ७) (२य — ३) (ग) य(य^२ + ३य + ३) २. (क) य — २ (ख) ६य(य + १) (य—३) (य—४) ३. (क) १. (ख) $\frac{१}{कखग}$ ४. (क) $\pm\left(य^२ - ४ + \frac{१}{य^२}\right)$
५, (क) य = ३ (ख) य = ५, र = ३ ६. (क) य = $\frac{१}{२}$, य = $\frac{३}{२}$ (ख) य = $\frac{१}{२}$, य = $-\frac{१}{५}$ ७. (क) (य + ६)(य + २)(य^२ + ८य + १०) (ख) (क — ख — २ग)(क — ३ख + २ग) ८. $\frac{५}{७}$ ९, (क) लघु० २ (ख) २८ १०, य = १, र = २ ११. (क) ० (ख) लम्बाई = ४८ गज चौड़ाई = १४ गज १२, २४ दिन १३. ३ वर्ष १४. १२० ग० लम्बाई ४० गज चौड़ाई [Read the second line as: उसके बाहर चारों ओर ५ गज चौड़ी सड़क बनाने का खर्चा ४ आ० ६ पा० प्रति वर्ग गज की दर से]

RAJPUTANA UNIVERSITY 1948

1. (i) $(a+b+1)(3a+3b-2)$ (ii) $(x-y-1)$ $(x^2+y^2+1+xy+x-y)$ 2. (i) $x=4,\ y=2$ (ii) $x=\frac{5}{3}$ (iii) $x=5.\ \frac{5}{2}$ 3. (a) 1 (b) $\pm\left(x+2+\frac{1}{x}\right)$ 4. (a) 4 (b) Rs. 2700, x men
6. 4.5 million in 1900.

1949

1. (i) $(x+y-5)(x-y+1)$ (ii) $(x^2+10x+21)(x^2+10x+19)$ (iii) $(a-b)$ $(a+b-1)$ 2. (a) 66 (b) $x-1$ 3 (i) $x=\pm 8$ (ii) $x=2\frac{8}{17}, y=-12$ 4 (a) 5 (b) 10 yds., 7 yds.

1950

1 (i) $(x+y+z)(x-y-z+1)$ (ii) $3(a-b)(b-c)(c-a)$ (iii) $(x-1)(x-2)(x-3)$ 2. (b) $\pm\left(x-\frac{1}{x}-2\right)$

3. (i) $x=4\frac{2}{3}$. $y=3\frac{2}{3}$. $z=3\frac{2}{3}$ (ii) $x=\frac{1}{\sqrt{2}}$

1951

1 (i) $(x+2)(x+3)(x-5)$ (ii) $(2x^2-9x+11)(2x^2-9x+3)$ (iii) $(2x-1-y)(4x^2+1+y^2+2x-y+2xy)$ 2. (i) $x=6$, $y=10$ (ii) $x=4\frac{1}{2}$ 3 (b) $\pm\left(\frac{x}{y}-\frac{y}{x}-1\right)$ 4 (a) Rs $256\frac{1}{9}$ and Rs $508\frac{8}{9}$ (b) 10 miles and 12 miles per hour respectively. 5. $42\frac{2}{9}$°C, 68°F.

1952

1. (i) $(2a+2b-c)(2a-4b+c)$ (ii) $3(x-y)(y-z)(z-x)$ (iii) $(a+b+c)(ab+bc+ca)$ 2. (i) $x=5$. (ii) $x=2$ $y=4$, $z=6$. (iii) $x=\pm\frac{a}{\sqrt{2}}$ 3. (*a*) 3 4. (*a*) 200 miles per hour (*b*) father 42 years, son 12 years. 5. (*a*) (2, 1), (0, 0), (0, 5) (*b*) 65 lbs. at the age of 9, 102·5 lbs. at the age of 15, 120 lbs. at the age of 17 years 2 months.

1953

1. (i) $(x^4-x^2y^2+y^4)(x^2+xy+y^2)(x^2-xy+y^2)$ (ii) $(x^2-4x-16)(x^2-4x-4)$ 2. (*a*) $\pm(3x^2-\frac{xy}{3}+3y^2)$. (*b*) ± 35 (*c*) x^2-3x-2. 3 (i) $\frac{8a^7}{a^8-1}$ (ii) 1. (iii) $4x\sqrt{x^2-1}$.

4. (*a*) (i) $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ (ii) -7. (iii) $x=\frac{1}{5}$, $y=\frac{1}{7}$; (iii) $x=16$ $y=8$. 5. (*b*) $a=4$, $b=9$ (*c*) 25 digits.

RAJPUTANA UNIVERSITY

1953

1. Resolve into factors any *three* of the following :—

(i) $x^8+x^4y^4+y^8$. (ii) $(x^2-4x)(x^2-4x-20)+64$.

(iii) $(a+b)(b+c)(c+a)+abc$. (iv) $2x^4+x^3-6x^2+x+2$.

(v) $x^2-4bx-(a+5b)(a+b)$.

2. Do any *two* parts of the following :—

(*a*) Find the square root of :—

$$9x^4-2x^3y+\frac{163x^2y^2}{9}-2xy^3+9y^4.$$

(*b*) If $a^2+b^2+c^2=13$, $ab+bc+ca=6$, find the value of $a^3+b^3+c^3-3abc$.

(*c*) Find the H. C. F. of :—

$$x^4-39x-22 \text{ and } 11x^4-39x^3-8,$$

3. Simplify any *two* of the following: —

(i) $\dfrac{1}{a-1}+\dfrac{1}{a+1}+\dfrac{2a}{a^2+1}+\dfrac{4a^3}{a^4+1}-\dfrac{8a^7}{a^8-1}$.

(ii) $\left(\dfrac{x^a}{x^b}\right)^{a^2+ab+b^2}\times\left(\dfrac{x^b}{x^c}\right)^{b^2+bc+c^2}\times\left(\dfrac{x^c}{x^a}\right)^{c^2+ac+a^2}$.

(iii) $\dfrac{x+\sqrt{x^2-1}}{x-\sqrt{x^2-1}}-\dfrac{x-\sqrt{x^2-1}}{x+\sqrt{x^2-1}}$.

4. (*a*) Solve any *two* of the following equations:—

(i) $ax^2+bx+c=0$.

(ii) $\dfrac{x+8}{x+6}+\dfrac{x+10}{x+8}=\dfrac{x+7}{x+5}+\dfrac{x+11}{x+9}$.

(iii) $$\left.\begin{aligned}\frac{1}{3x}-\frac{1}{7y}&=\frac{2}{3}\\ \frac{1}{2x}-\frac{1}{3y}&=\frac{1}{6}\end{aligned}\right\}$$

$$\text{(iv)}\quad \left.\begin{aligned} 2^x &= 4^y \\ (\sqrt{3})^{x-y} &= 81 \end{aligned}\right\}$$

5. Do any *two* parts:—

(*a*) If $\dfrac{a}{b+c}=\dfrac{b}{c+a}=\dfrac{c}{a+b}$ show that $a=b=c$, supposing $a+b+c$ is not equal to zero.

(*b*) Determine the value of a and b in order that the expression $ax^3+bx^2-58x-15$ may be divisible by $x^2+2x-15$.

(*c*) Find the number of digits in 2^{80} having given $\log 2=\cdot 3010$.